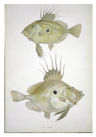
P o C S  What's The Story?

Principles of Complex Systems, Vols. 1 and 2
CSYS/MATH 6701, 6713
University of Vermont, Fall 2025
"We never have a hard time passing, it's not a big deal."
Assignment 02

It's Always Sunny in Philadelphia: Charlie Work, S10E04
Episode links: IMDB, Fandom, TV Tropes.

Due: Wednesday, September 10, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse/assignments/02/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python. And it's going to be a no for the catachrestic Excel. Please do not use any kind of AI thing unless directed. The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via **Brightspace** (which is not to be confused with the death vortex of the same name, just a weird coincidence). Again: One PDF document per assignment only.

Overall points: 30 points.

1. (3 points)

   Use a back-of-an-envelope scaling argument to show that maximal rowing speed V increases as the number of oarspeople N as $V \propto N^{1/9}$.

   Assume the following:

   (a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner [1] shows that shell width is roughly proportional to shell length ℓ.

   Shell dimensions and performances.

No. of oarsmen	Modifying description	Length, l (m)	Beam, b (m)	l/b	Boat mass per oarsman (kg)	Time for 2000 m (min) I	II	III	IV
8	Heavyweight	18.28	0.610	30.0	14.7	5.87	5.92	5.82	5.73
8	Lightweight	18.28	0.598	30.6	14.7				
4	With coxswain	12.80	0.574	22.3	18.1				
4	Without coxswain	11.75	0.574	21.0	18.1	6.33	6.42	6.48	6.13
2	Double scull	9.76	0.381	25.6	13.6				
2	Pair-oared shell	9.76	0.356	27.4	13.6	6.87	6.92	6.95	6.77
1	Single scull	7.93	0.293	27.0	16.3	7.16	7.25	7.28	7.17

   (b) The resistance encountered by a shell is due largely to drag on its wetted surface.

   (c) Drag force is proportional to the product of the square of the shell's speed (V^2) and the area of the wetted surface ($\propto \ell^2$ due to shell isometry).

   (d) Power $\propto$ drag force $\times$ speed (in symbols: $P \propto D_f \times V$).

   (e) Volume displacement of water by a shell is proportional to the number of oarspeople N (i.e., the team's combined weight).

   (f) Assume the depth of water displacement by the shell grows isometrically with boat length ℓ.

   (g) Power is proportional to the number of oarspeople N.

2. (6 points overall: 3 points for men's rowing, 3 points for women's rowing)

   Find the modern day world record times for 2000 meter races and see if this scaling still holds up.

   Do so for men's and women's records for 2000 meters where there are at least two points of comparison. And only consider the open category (not lightweight).

   In general, there are three classes:

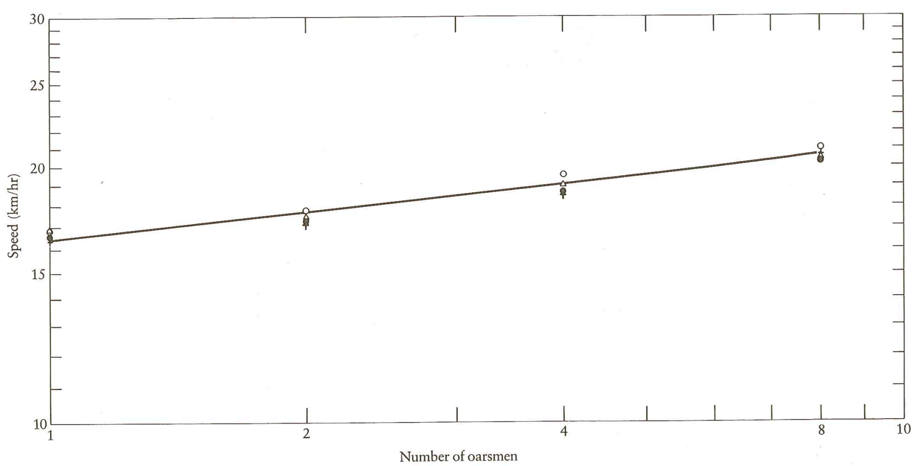
- Coxless sculling (N=1, 2, 4 oarshumans),
- Coxless rowing (2, 4 oarshumans),
- Coxed rowing (2, 4, 8 oarshumans).

Disappointingly, there are no coxed 2s and 4s for women, only coxed 8s. You will only be able to analyze sculling and coxless teams for women's records.

Because the range is extremely small (1–8 oarspeople), and the scaling is very weak (1/9), the measured scalings will have a poor foundation. We have much more confident if would have data for boats with N=10^k oarspeople with $k = 0$ to, say 6. We'll need some funding.

In any case, see what you can find.

The figure below shows data from McMahon and Bonner.



3. (3 points) Finish the calculation for the platypus on a pendulum problem so show that a simple pendulum's period τ is indeed proportional to $\sqrt{\ell/g}$.

   Basic plan from lectures: Create a matrix A where ijth entry is the power of dimension i in the jth variable, and solve by row reduction to find basis null vectors.

   In lectures, we arrived at:

$$A\vec{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

   You only have to take a few steps from here.

   From Lecture 3: the Buckingham π theorem (20 minutes).

4. (3 points)

   Show that the maximum speed of animals $V_{\max}$ is proportional to their length L [2]. Here are five dimensionful parameters:

   - $V_{\max}$, maximum speed.
   - ℓ, animal length.
   - ρ, organismal density.
   - σ, maximum applied force per unit area of tissue.
   - b, maximum metabolic rate per unit mass (b has the dimensions of power per unit mass).

   And here are the three dimensions: L, M, and T.

   Use a back-of-the-envelope calculation to express $V_{\max}/\ell$ in terms of ρ, σ, and b.

   Note: It's argued in [2] that $V_{\max}/\ell$ only depends on these latter three parameters. Moreover, the claim is that these three paramters vary little across all organisms (we're mostly thinking about running organisms here), and so finding $V_{\max}/\ell$ as a function of them indicates that $V_{\max}/\ell$ is also roughly constant.

5. (3 points) Use the Buckingham π theorem to reproduce G. I. Taylor's finding the energy of an atom bomb E is related to the density of air ρ and the radius of the blast wave R at time t:

   $$E = \text{constant} \times \rho R^5/t^2. \tag{2}$$

   In constructing the matrix, order parameters as E, ρ, R, and t and dimensions as L, T, and M.

6. (12 points overall) Use the Buckingham π theorem to derive Kepler's third law, which states that the square of the orbital period of a planet is proportional to the cube of its semi-major axis.

   Let's shed some enlightenment and assume circular orbits.

   Parameters:

   - Planet's mass m;
   - Sun's mass M_{sun};
   - Orbital period τ;
   - Orbital radius r;
   - Graviational constant G.

   (a) (3 points) What are the dimensions of these five quantities?

(b) (3 points) You will find that there are two dimensionless parameters using the Buckingham π theorem, and that you can choose one to be $\pi_2 = m/M_{\text{sun}}$. Find the other dimensionless parameter, π_1.

(c) (3 points) Now argue that $\tau^2 \propto r^3$.

Some help:

Because we have two dimensionless parameters, we can first state $f(\pi_1, \pi_2) = \text{constant}$.

Because we are working on the back of an envelope and making outrageous claims in general, we can assume that as $\pi_1 \to 0$, $f(\pi_1, \pi_2) \to g(\pi_2)$. That is, we do not expect the masses of the relatively very small planets to matter.

(d) (3 points) For our solar system's nine (9) planets (yes, Pluto is on the team here), plot τ^2 versus r^3, and using basic linear regression (in log-log space) report on how well Kepler's third law holds up.

References

[1] T. A. McMahon and J. T. Bonner. On Size and Life. Scientific American Library, New York, 1983.

[2] N. Meyer-Vernet and J.-P. Rospars. How fast do living organisms move: Maximum speeds from bacteria to elephants and whales. American Journal of Physics, pages 719–722, 2015. pdf