Random Networks Nutshell

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Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number, 2024-2025

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networks

Generalized Random Networks Strange friends



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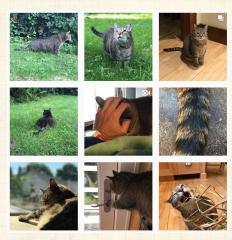
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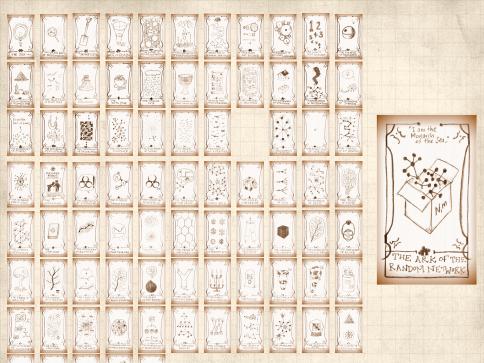
References

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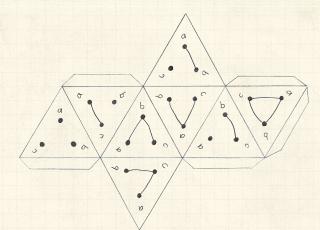
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Random network generator for N = 3:



Get your own exciting generator here
As N ↗, polyhedral die rapidly becomes a ball...

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Pure, abstract random networks:

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Pure, abstract random networks:

Solution Consider set of all networks with N labelled nodes and m edges.

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Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.

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 - Sometimes equiprobability is a good assumption, but it is always an assumption.

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 - Sometimes equiprobability is a good assumption, but it is always an assumption.
 - Known as Erdős-Rényi random networks or ER graphs.

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🗞 Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

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 \bigotimes Limit of m = 0: empty graph.

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$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N(N-1)}$$

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 \bigotimes Given *m* edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.

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Siven m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. The PoCSverse Random Networks Nutshell 2 of 74

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How to build standard random networks:



 \mathcal{A} Given N and m.

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🚳 Two probablistic methods

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How to build standard random networks:

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 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.



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How to build standard random networks:

- \clubsuit Given N and m.
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 - Connect each of the ^N₂ pairs with appropriate probability *p*.
 Useful for theoretical work.
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Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.

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 - Best for adding relatively small numbers of links (most cases).

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 - 🝞 Best for adding relatively small numbers of links (most cases).
 - \bigcirc 1 and 2 are effectively equivalent for large N.

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A few more things:



For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2}$$

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A few more things:



For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

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A few more things:



For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$



So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

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Random networks

A few more things:



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Nhich is what it should be...

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$$= \frac{2}{N} p \frac{1}{2} N(N-1) = \frac{2}{N} p \frac{1}{2} \mathcal{N}(N-1) = p(N-1).$$

Nhich is what it should be... \mathfrak{R} If we keep $\langle k \rangle$ constant then $p \propto 1/N \to 0$ as $N \to \infty$.

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Next slides: Example realizations of random networks The PoCSverse Random Networks Nutshell 14 of 74

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Next slides: Example realizations of random networks $\Im N = 500$

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Next slides:

Example realizations of random networks

- $\bigotimes N = 500$
- \bigotimes Vary *m*, the number of edges from 100 to 1000.

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Next slides:

Example realizations of random networks

- N = 500 Vary m, the number of edges from 100 to 1000.
- \bigotimes Average degree $\langle k \rangle$ runs from 0.4 to 4.

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Next slides:

Example realizations of random networks

- $\bigotimes N = 500$
- \bigotimes Vary *m*, the number of edges from 100 to 1000.
- \bigotimes Average degree $\langle k \rangle$ runs from 0.4 to 4.
- look at full network plus the largest component.

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Random networks: examples for N=500











m = 100 $\langle k \rangle = 0.4$

m = 260

(k) = 1.04

m = 200 $\langle k \rangle = 0.8$

 $\langle k \rangle = 0.92$

m = 240 $\langle k \rangle = 0.96$

m = 250 $\langle k \rangle = 1$

m = 1000 $\langle k \rangle = 4$

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m = 280(k) = 1.12 m = 300 $\langle k \rangle = 1.2$

m = 500 $\langle k \rangle = 2$

m = 230

Some visual examples

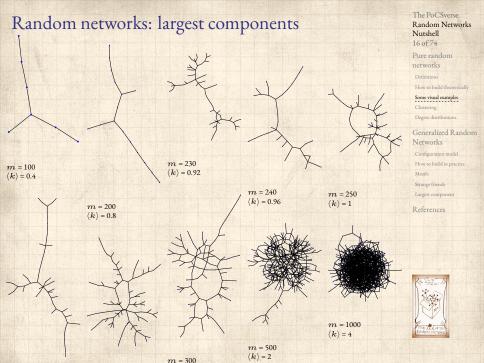
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Random networks: examples for N=500



m = 250

m = 250

 $\langle k \rangle = 1$

 $\langle k \rangle = 1$



m = 250

 $\langle k \rangle = 1$



 $m = 250 \ \langle k \rangle = 1$

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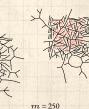
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 $\langle k \rangle = 1$

m = 250

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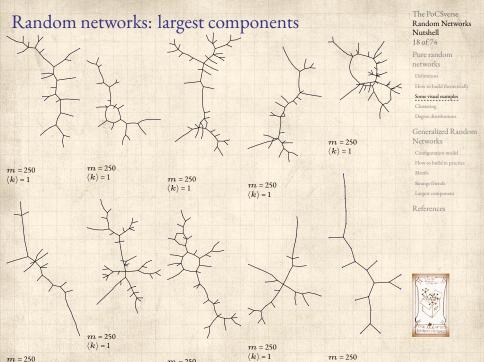
m = 250 $\langle k
angle$ = 1

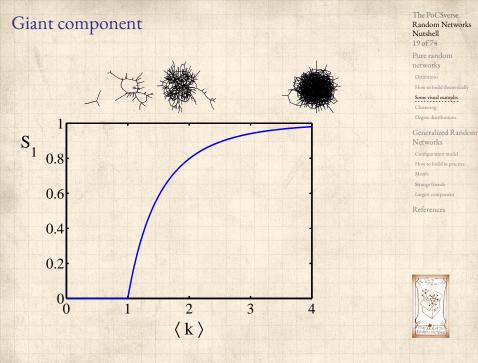
m = 250 $\langle k \rangle = 1$

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line for construction method 1, what is the clustering coefficient for a finite network?

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For construction method 1, what is the clustering coefficient for a finite network?

🗞 Consider triangle/triple clustering coefficient: [6]

$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

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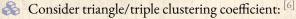
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For construction method 1, what is the clustering coefficient for a finite network?



$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

Recall: C_2 = probability that two friends of a node are also friends.

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For construction method 1, what is the clustering coefficient for a finite network?

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Recall: C_2 = probability that two friends of a node are also friends. Or: C_2 = probability that a triple is part of a triangle.

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For construction method 1, what is the clustering coefficient for a finite network?

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 $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$

Recall: C_2 = probability that two friends of a node are also friends. Or: C_2 = probability that a triple is part of a triangle. 😤 For standard random networks, we have simply that

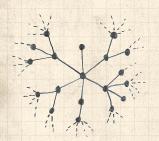
$$C_2=p$$

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So for large random networks $(N \rightarrow \infty)$, clustering drops to zero.

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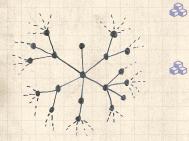
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So for large random networks $(N \to \infty)$, clustering drops to zero.

Key structural feature of random networks is that they locally look like

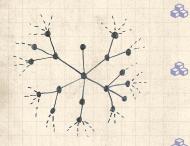
pure branching networks

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So for large random networks $(N \rightarrow \infty)$, clustering drops to zero.

Key structural feature of random networks is that they locally look like pure branching networks

\lambda No small loops.

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8	Recall P_k = probability that a randomly selected noc	de has
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- Recall P_k = probability that a randomly selected node has degree k.
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- Each connection occurs with probability p, each non-connection with probability (1 p).

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🚳 Therefore have a binomial distribution 🗹 :

$$P(k;p,N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$

 \mathfrak{S} What happens as $N \to \infty$?

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- \bigotimes What happens as $N \to \infty$?

🛞 We must end up with the normal distribution right?

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- \mathfrak{S} What happens as $N \to \infty$?

We must end up with the normal distribution right?

So If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$.



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- \mathfrak{S} What happens as $N \to \infty$?

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- \mathfrak{S} What happens as $N \to \infty$?

He must end up with the normal distribution right?

- \bigotimes But we want to keep $\langle k \rangle$ fixed...

So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1) = \text{constant.}$

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \rightarrow \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

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 \mathfrak{S} This is a Poisson distribution \mathfrak{C} with mean $\langle k \rangle$.

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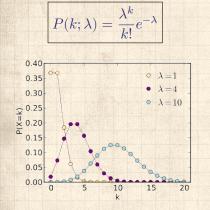
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40

 $\lambda > 0$ $k = 0, 1, 2, 3, \dots$ 🚳 Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence. 🔗 e.g.: phone calls/minute, horse-kick deaths. 🔧 'Law of small numbers'

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🚳 The variance of degree distributions for random networks turns out to be very important.

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- The variance of degree distributions for random networks turns out to be very important.
- Solution Similar to one for finding $\langle k \rangle$ we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Variance is then

 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$

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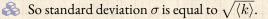
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Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

So standard deviation σ is equal to √⟨k⟩.
Note: This is a special property of Poisson distribution and can trip us up...



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🚳 So... standard random networks have a Poisson degree distribution

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🚳 So... standard random networks have a Poisson degree distribution



 \mathfrak{G} Generalize to arbitrary degree distribution P_k .

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🚳 So... standard random networks have a Poisson degree distribution

- Generalize to arbitrary degree distribution P_k .
- Also known as the configuration model. [6]

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Generalized Random Networks

Configuration model



- 🚳 So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution P_k .
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- 🗞 Can generalize construction method from ER random networks.



Generalized Random Networks

Configuration model



- So... standard random networks have a Poisson degree distribution
- \mathfrak{S} Generalize to arbitrary degree distribution P_k .
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 - Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

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Generalized Random Networks

Configuration model



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 - Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

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- 👶 But we'll be more interested in
 - 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
 - 2. Examining mechanisms that lead to networks with certain degree distributions.

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Coming up:

Example realizations of random networks with power law degree distributions:

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Coming up:

Example realizations of random networks with power law degree distributions:

3 N = 1000.

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Coming up:

Example realizations of random networks with power law degree distributions:

- $\bigotimes N = 1000.$
- $\label{eq:prod} \$eq P_k \propto k^{-\gamma} \text{ for } k \geq 1.$

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 \bigotimes Vary exponent γ between 2.10 and 2.91.

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- 🗞 Again, look at full network plus the largest component.

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- \bigotimes Vary exponent γ between 2.10 and 2.91.
- 🗞 Again, look at full network plus the largest component.
- 🚳 Apart from degree distribution, wiring is random.

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Random networks: examples for N=1000









 $\gamma = 2.1$ (k) = 3.448

 $\gamma = 2.55$

(k) = 1.712

 $\gamma = 2.19$ (k) = 2.986

 $\gamma = 2.64$

 $\langle k \rangle = 1.6$

 $\gamma = 2.28$ (k) = 2.306.pdf

 $\gamma = 2.73$

 $\langle k \rangle = 1.862.pdf$

 $\gamma = 2.37$ (k) = 2.504

 $\gamma = 2.82$

(k) = 1.386

 $\gamma = 2.46$ (k) = 1.856

 $\gamma = 2.91$

 $\langle k \rangle = 1.49$





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Random networks: largest components











 $\gamma = 2.1$ $\langle k \rangle = 3.448$ $\gamma = 2.19$ $\langle k \rangle = 2.986$ $\gamma = 2.28$ $\langle k \rangle = 2.306$ $\gamma = 2.37$ $\langle k \rangle = 2.504$ $\gamma = 2.46$ $\langle k \rangle = 1.856$











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Generalized random networks:

Arbitrary degree distribution P_k .

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Generalized random networks:

- Arbitrary degree distribution P_k .
- \bigotimes Create (unconnected) nodes with degrees sampled from P_k .

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Generalized random networks:

- \bigotimes Arbitrary degree distribution P_k .
- \mathfrak{B} Create (unconnected) nodes with degrees sampled from P_k .
- 🛞 Wire nodes together randomly.

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Generalized random networks:

- \bigotimes Arbitrary degree distribution P_k .
- \mathfrak{B} Create (unconnected) nodes with degrees sampled from P_k .
- 🛞 Wire nodes together randomly.
- 🗞 Create ensemble to test deviations from randomness.

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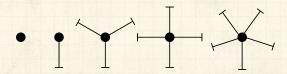
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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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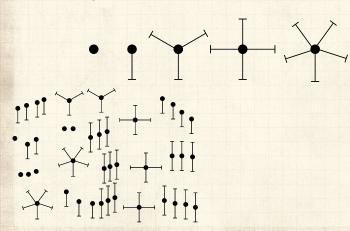
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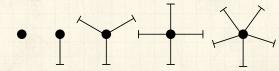
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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



.⊥ Landomly select stubs (nc The PoCSverse Random Networks Nutshell 35 of 74

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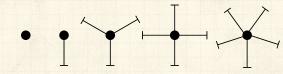
References

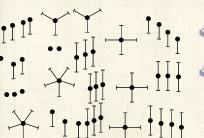


Randomly select stubs (not nodes!) and connect them.

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs. The PoCSverse Random Networks Nutshell 35 of 74

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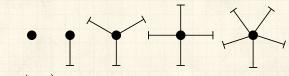
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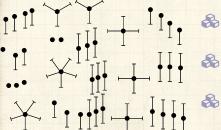


Building random networks: Stubs

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs.

Initially allow self- and repeat connections.

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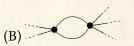


Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





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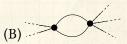


Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful: we can't change the degree of any node, so we can't simply move links around.

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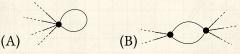
Largest component



Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



Being careful: we can't change the degree of any node, so we can't simply move links around.

Simplest solution: randomly rewire two edges at a time.

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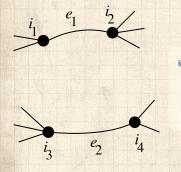
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Randomly choose two edges. (Or choose problem edge and a random edge) The PoCSverse Random Networks Nutshell 37 of 74

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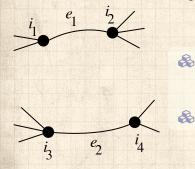
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Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

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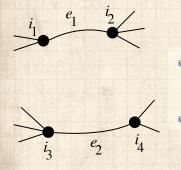
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Check to make sure edges are disjoint.



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Rewire one end of each edge.



ez.

e'

3

Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

Rewire one end of each edge.

Node degrees do not change.

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Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

Rewire one end of each edge.

🗞 Node degrees do not change.

Works if e₁ is a self-loop or repeated edge. The PoCSverse Random Networks Nutshell 37 of 74

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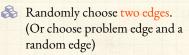
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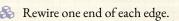


ez.

e'



Check to make sure edges are disjoint.



- 🗞 Node degrees do not change.
 - Works if e_1 is a self-loop or repeated edge.
 - Same as finding on/off/on/off 4-cycles. and rotating them.

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Phase 2:



🛞 Use rewiring algorithm to remove all self and repeat loops.

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Phase 2:

🗞 Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

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Phase 2:

loops. When the termina algorithm to remove all self and repeat loops.

Phase 3:

- Randomize network wiring by applying rewiring algorithm liberally.
- \bigotimes Rule of thumb: # Rewirings $\simeq 10 \times$ # edges ^[4].

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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks. The PoCSverse Random Networks Nutshell 39 of 74

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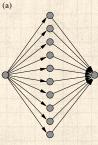
Largest component



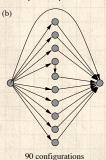
Random sampling

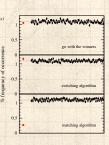
Problem with only joining up stubs is failure to randomly sample from all possible networks.

Example from Milo et al. (2003)^[4]:



1 configuration





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 \bigotimes What if we have P_k instead of N_k ?

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 \bigotimes What if we have P_k instead of N_k ? A Must now create nodes before start of the construction algorithm.

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 \mathbb{R} What if we have P_k instead of N_k ?

Must now create nodes before start of the construction algorithm.

Generate N nodes by sampling from degree distribution P_k .

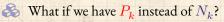
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- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k.
 Easy to do exactly numerically since k is discrete.

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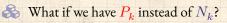
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together.



- Must now create nodes before start of the construction algorithm.
- \bigotimes Generate N nodes by sampling from degree distribution P_k . Easy to do exactly numerically since k is discrete. \bigotimes Note: not all P_k will always give nodes that can be wired

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ldea of motifs ^[7] introduced by Shen-Orr, Alon et al. in 2002.

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Idea of motifs ^[7] introduced by Shen-Orr, Alon et al. in 2002.
 Looked at gene expression within full context of transcriptional regulation networks.

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Idea of motifs ^[7] introduced by Shen-Orr, Alon et al. in 2002.
Looked at gene expression within full context of transcriptional regulation networks.
Specific example of Escherichia coli.
Directed network with 577 interactions (edges) and 424 operons (nodes).

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- Looked at gene expression within full context of transcriptional regulation networks.
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- Solution Looked for certain subnetworks (motifs) that appeared more or less often than expected

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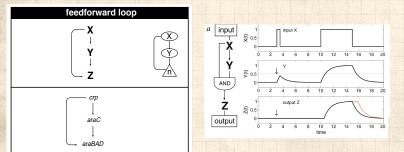
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 $\bigotimes Z$ only turns on in response to sustained activity in X.



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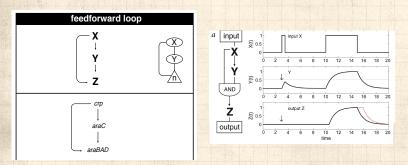
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Z only turns on in response to sustained activity in X.
Turning off X rapidly turns off Z.



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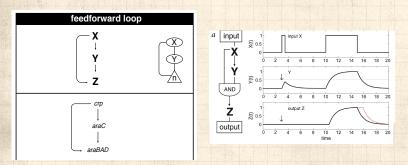
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Z only turns on in response to sustained activity in X.
Turning off X rapidly turns off Z.
Analogy to elevator doors.



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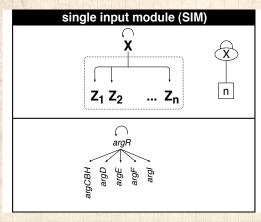
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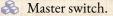
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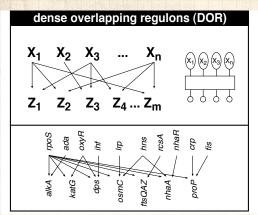
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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.



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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

For more, see work carried out by Wiggins *et al.* at Columbia.

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The edge-degree distribution:



 \bigotimes The degree distribution P_k is fundamental for our description of many complex networks

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The degree distribution ${\cal P}_k$ is fundamental for our description of many complex networks

Again: P_k is the degree of randomly chosen node.



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- The degree distribution P_k is fundamental for our description of many complex networks
- Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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 $Q_k \propto k P_k$

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Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_k}{\langle k \rangle}.$$

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Normalized form:

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 $Q_k \propto k P_k$

Big deal: Rich-get-richer mechanism is built into this selection process.

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Probability of randomly selecting a node of degree k by choosing from nodes:

 $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

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Probability of randomly selecting a node of degree k by choosing from nodes:

 $\begin{array}{l} P_1=3/7, P_2=2/7, P_3=1/7, \\ P_6=1/7. \end{array}$

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16.$ The PoCSverse Random Networks Nutshell 49 of 74 Pure random networks Diffictions How to build theoretically Some visual examples. Clustering Dagree distributions Generalized Bandom

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Probability of randomly selecting a node of degree k by choosing from nodes:

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Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16.$

> Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$\begin{split} R_0 &= 3/16 \; R_1 = 4/16, \\ R_2 &= 3/16, R_5 = 6/16. \end{split}$$

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 \bigotimes For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.

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- For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:
 - R_k = probability that a friend of a random node has k other friends.



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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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R

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Solution Equivalent to friend having degree k + 1.

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R

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
 Useful variant on Q_k:

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?



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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}$$

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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

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$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

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$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^\infty \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

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(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad \text{(using } j = k+1\text{)}$$

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$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}\left((k+1)^2-(k+1)\right)P_{k+1}$$

(where we have sneakily matched up indices)

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$$=\frac{1}{\langle k\rangle}\left(\langle k^2\rangle-\langle k\rangle\right)$$

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 $\begin{aligned} & \bigotimes \text{ Note: our result, } \langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right) \text{, is true for all} \\ & \text{ random networks, independent of degree distribution.} \end{aligned}$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution. 🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution. 🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



Therefore:

$$\left\langle k \right\rangle_R = rac{1}{\left\langle k \right\rangle} \left(\left\langle k \right\rangle^2 + \left\langle k \right\rangle - \left\langle k \right\rangle \right)$$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution. 🚳 For standard random networks, recall

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A Therefore:

$$\left\langle k\right\rangle _{R}=\frac{1}{\left\langle k\right\rangle }\left(\left\langle k\right\rangle ^{2}+\left\langle k\right\rangle -\left\langle k\right\rangle \right) =\left\langle k\right\rangle$$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution. 🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



Therefore:

$$\left\langle k \right\rangle_R = rac{1}{\left\langle k \right\rangle} \left(\left\langle k \right\rangle^2 + \left\langle k \right\rangle - \left\langle k \right\rangle \right) = \left\langle k \right\rangle$$

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Again, neatness of results is a special property of the Poisson distribution.



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Again, neatness of results is a special property of the Poisson distribution.

So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total 1 friends...



 $\underset{k}{\bigotimes}$ In fact, R_k is rather special for pure random networks ...

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 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

 $R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$

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Two reasons why this matters

Reason #1:

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Reason #1:



Average # friends of friends per node is

 $\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$

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Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.



Strange friends



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🚷 Three peculiarities:

1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.

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- 3. Your friends really are different from you...^[3, 5]

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- 4. See also: class size paradoxes (nod to: Gelman)

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More on peculiarity #3:

 \clubsuit A node's average # of friends: $\langle k \rangle$

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More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ The PoCSverse Random Networks Nutshell 55 of 74

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More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

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$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2}$$

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So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as its friends.

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So only if everyone has the same degree (variance= $\sigma^2 = 0$) can a node be the same as its friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend. The PoCSverse Random Networks Nutshell 55 of 74

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"Generalized friendship paradox in complex networks: The case of scientific collaboration" Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014.^[2]

Your friends really are monsters #winners:¹



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¹Some press here C [MIT Tech Review].



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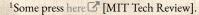
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- So ther horrific studies: your connections on Twitter have more followers than you, your sexual partners more partners than you, ...
- The hope: Maybe they have more enemies and diseases too.

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(Big) Reason #2:

 $\langle k \rangle_R$ is key to understanding how well random networks are connected together.

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(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.



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- 🗞 Note: Component = Cluster

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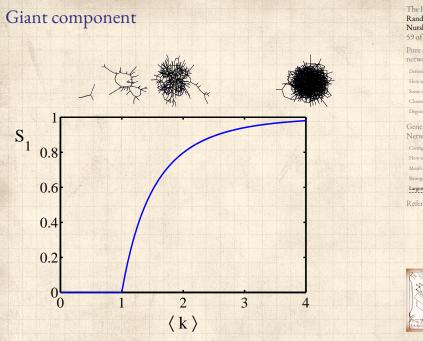
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Giant component:

A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

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Again, see that the second moment is an essential part of the story.

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- $\ref{eq: eq: alpha}$ Equivalent statement: $\langle k^2
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Spreading on Random Networks

🚳 For random networks, we know local structure is pure branching.

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- For random networks, we know local structure is pure branching.
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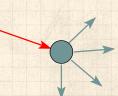
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- For random networks, we know local structure is pure branching.
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 Failure:





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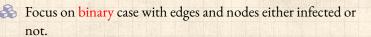
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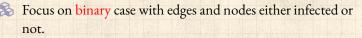
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References



First big question: for a given network and contagion process, can global spreading from a single seed occur?



We need to find: ^[1] **R** = the average # of infected edges that one random infected edge brings about.
Call **R** the gain ratio.

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prob. of connecting to a degree k node

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outgoing infected edges

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 $\langle k \rangle$ prob. of

connecting to

a degree k node

outgoing infected edges

 B_{k1} Prob. of infection

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outgoing infected edges

 $(1 - B_{k1})$

Prob. of no infection

lour global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

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🙈 Case 1–Rampant spreading:

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💑 Good: This is just our giant component condition again.

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🚳 Case 2—Simple disease-like:

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 \bigcirc Case 2—Simple disease-like: If $B_{k1} = \beta < 1$

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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

 \mathfrak{S} A fraction (1- β) of edges do not transmit infection.

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 \clubsuit Aka bond percolation \square .

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 \mathfrak{S} Resulting degree distribution \tilde{P}_k :

$$\tilde{P}_k = \beta^k \sum_{i=k}^\infty \binom{i}{k} (1-\beta)^{i-k} P_i.$$

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Giant component for standard random networks:

 $\label{eq:Recall} \bigotimes \ {\rm Recall} \ \langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$

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 $\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$

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$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

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Giant component for standard random networks: Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

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So Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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 \bigotimes When $\langle k \rangle < 1$, all components are finite.



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line example of a continuous phase transition 🖉.

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So Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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 \mathfrak{F} Fine example of a continuous phase transition \mathbb{C} .

 \bigotimes We say $\langle k \rangle = 1$ marks the critical point of the system.

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$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

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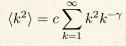
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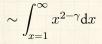
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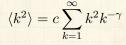
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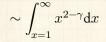
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$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty}$$

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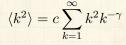
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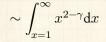
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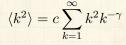
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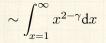
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$$\propto \left. x^{3-\gamma} \right|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

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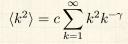
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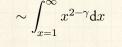
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Random networks with skewed P_k : \bigotimes e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3, k \geq 1,$ then





$$\propto x^{3-\gamma} \big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

So giant component always exists for these kinds of networks.



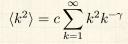
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Random networks with skewed P_k : so e.g., if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3, k \ge 1$, then



$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

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So giant component always exists for these kinds of networks.
 Cutoff scaling is k⁻³: if γ > 3 then we have to look harder at (k)_R.

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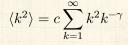
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Random networks with skewed P_k : e.g. if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3, k \ge 1$, then



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$$\sim \int_{x=1} x^{2-\gamma} \mathrm{d}x$$

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So giant component always exists for these kinds of networks.
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$$\mathbb{R}$$
 How about $P_k = \delta_{kk_0}$?

f



And how big is the largest component?

3 Define S_1 as the size of the largest component.

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And how big is the largest component?

- \mathfrak{S} Define S_1 as the size of the largest component.
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- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

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So So

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- Befine δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 S_1$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$



Substitute in Poisson distribution...

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🚳 Carrying on:

 $\pmb{\delta} = \sum_{k=0}^\infty P_k \delta^k$

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🚳 Carrying on:

$${\color{black} \boldsymbol{\delta}} = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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🚳 Carrying on:

$$\frac{\delta}{\delta} = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$=e^{-\langle k
angle}\sum_{k=0}^{\infty}rac{(\langle k
angle\delta)^k}{k!}$$

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$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta}$$

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$$=e^{-\langle k
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angle(1-\delta)}$$

Solution Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}$$

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 \clubsuit We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}.$

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We can figure out some limits and details for S₁ = 1 − e^{-⟨k⟩S₁}.
First, we can write ⟨k⟩ in terms of S₁:

$$\langle k\rangle = \frac{1}{S_1} {\rm ln} \frac{1}{1-S_1}$$

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 $\begin{aligned} & \& & \text{We can figure out some limits and details for} \\ & S_1 = 1 - e^{-\langle k \rangle S_1}. \\ & \& & \text{First, we can write } \langle k \rangle \text{ in terms of } S_1: \end{aligned}$

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

$$\label{eq:stars} \$ \ \ \, \operatorname{As}\left\langle k\right\rangle \to 0, S_1\to 0.$$

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$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

$$\begin{split} & \bigotimes \ \, \operatorname{As}\,\langle k\rangle \to 0, S_1 \to 0. \\ & \bigotimes \ \, \operatorname{As}\,\langle k\rangle \to \infty, S_1 \to 1. \end{split}$$

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🛞 We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}.$ \mathfrak{S} First, we can write $\langle k \rangle$ in terms of S_1 :

 $\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$

 \Re As $\langle k \rangle \to 0, S_1 \to 0.$ \mathfrak{k} As $\langle k \rangle \to \infty$, $S_1 \to 1$. \mathfrak{S} Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

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We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}.$

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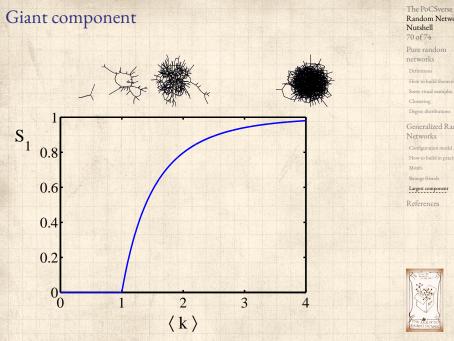
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Turns out we were lucky ...

like the second second

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Turns out we were lucky ...

Our dirty trick only works for ER random networks.
 The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.



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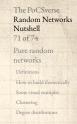
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 But we know our friends are different from us...



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Turns out we were lucky...

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 The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- 🚳 But we know our friends are different from us...
- So Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.

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- We can sort many things out with sensible probabilistic arguments...

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- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generatingfunctionology.^[9]

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