

## Fundamentals

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Principles of Complex Systems, Vols. 1, 2, & 3D  
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Center  
Santa Fe Institute | University of Vermont

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## No really, that's a lot of data

Unit	Size	What it means
Bit (b)	1 or 0	Short for "binary digit", after the binary code (1 or 0) computers use to store and process data
Byte (B)	8 bits	Enough information to create an English letter or number in computer code. It is the basic unit of computing
Kilobyte (KB)	1,000, or $2^{10}$ bytes	From "thousand" in Greek. One page of typed text is 2KB
Megabyte (MB)	1,000KB; $2^{20}$ bytes	From "large" in Greek. The complete works of Shakespeare total 5MB. A typical pop song is about 4MB
Gigabyte (GB)	1,000MB; $2^{30}$ bytes	From "giant" in Greek. A two-hour film can be compressed into 1-2GB
Terabyte (TB)	1,000GB; $2^{40}$ bytes	From "monster" in Greek. All the catalogued books in America's Library of Congress total 15TB
Petabyte (PB)	1,000TB; $2^{50}$ bytes	All letters delivered by America's postal service this year will amount to around 5PB. Google processes around 1PB every hour
Exabyte (EB)	1,000PB; $2^{60}$ bytes	Equivalent to 10 billion copies of <i>The Economist</i>
Zettabyte (ZB)	1,000EB; $2^{70}$ bytes	The total amount of information in existence this year is forecast to be around 1.2ZB
Yottabyte (YB)	1,000ZB; $2^{80}$ bytes	Currently too big to imagine

The prefixes are set by an intergovernmental group, the International Bureau of Weights and Measures. Yotta and Zetta were added in 1991; terms for larger amounts have yet to be established.  
Source: *The Economist*

## Overboard

Internet-scale data sets can be overly **exciting**.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- "The Unreasonable Effectiveness of Data," Halevy et al. <sup>[7]</sup>.
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" <sup>[20]</sup>

But:

- For scientists, description is only part of the battle.
- We still need to **understand**.

## Outline

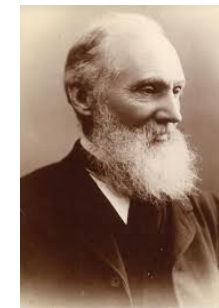
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- Modeling
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## Estimates of internet data, past, present, future:<sup>1</sup>

Created:

- 2010: ~ 2 zettabytes
  - 2014: ~ 12 zettabytes
  - 2022: ~ 97 zettabytes
  - 2023: ~ 120 zettabytes
  - 2024: ~ 150 zettabytes
  - 2025: ~ 180 zettabytes
- Video accounts for about 50% of all data.

## Basic Science ≈ Describe + Explain:



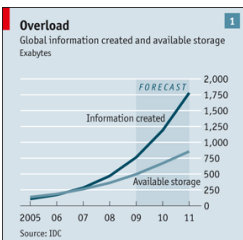
Lord Kelvin (possibly):

- "To measure is to know."
- "If you cannot measure it, you cannot improve it."

Bonus:

- "X-rays will prove to be a hoax."
- "There is nothing new to be discovered in physics now, All that remains is more and more precise measurement."
- "My beard will always be cool."

Around 2010: Data becomes **Big Data** because it's about us:



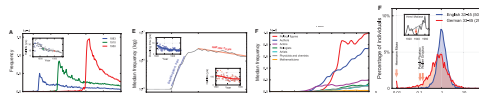
Big Data Science over time:

- Vera C. Rubin Observatory: 200,000 photos, 1.28 petabytes/year.
- Large Hadron Collider, 2024: 1 petabyte/second.
- ~ 2 trillion photos taken in 2024
- Over 10 billion photos shared each day
- Large language models , 2024: Up to  $10^{13}$  tokens.

- Exponential growth: ~ 60% per year.

## Big Data—Culturomics:

"Quantitative analysis of culture using millions of digitized books" by Michel et al., *Science*, 2011 <sup>[10]</sup>



<http://www.culturomics.org/> and [Google Books ngram viewer](#)

Barney Rubble:



"Characterizing the Google Books corpus: Strong limits to inferences of socio-cultural and linguistic evolution"   
Pechenick, Danforth, and Dodds,  
PLoS ONE, **10**, e0137041, 2015. <sup>[11]</sup>

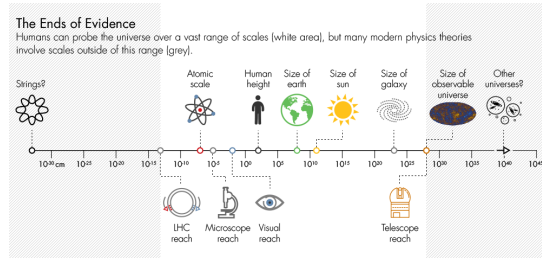
## A brief history of measuring time:

- Megaliths for Big Time
- Sundials, 1500 BC, Egypt (solid for over 2000 years)
- Escapements (200s), Hourglasses (1300s?), Pendulum clocks (Galileo, 1500s)
- Chronometers, 1700s:
  - "Longitude: The True Story of a Lone Genius Who Solved the Greatest Scientific Problem of His Time"   
by Dava Sobel (2007). <sup>[17]</sup>
  - Billionths of a second accuracy: Atomic clocks (Lord Kelvin, 1879)



## Limits of testability and happiness in Science:

From [A Fight for the soul of Science](#) in Quanta Magazine (2016/02):



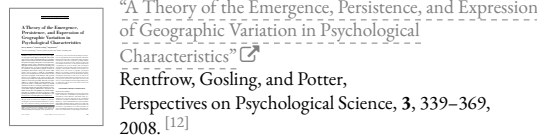
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## The conceptual trapping pit of a single scale:

- Lure of simplicity: Comparisons and rankings are easy.
- A single scale measure is very appealing, very hard to resist ... and hard to push back against when widely adopted.
- Examples:
  - Grade point average (GPA)
  - College rankings, City rankings, Country rankings, Wine scores, Michelin Guide, Yelp scores, Amazon ratings, ...
  - Body Mass Index (BMI)
  - Intelligence Quotient (IQ)<sup>2</sup>
  - Effective temperature
  - Price for all things: One dimension of belief
  - Salary!
  - stock market valuation for corporations
  - Complexity of civilizations [18]
  - A 1-d axis for political ideologies (a spatial metaphor trap, thanks France!)

<sup>2</sup>Phrenology was at least a 2-d map; see also palmistry

## Personality distributions:

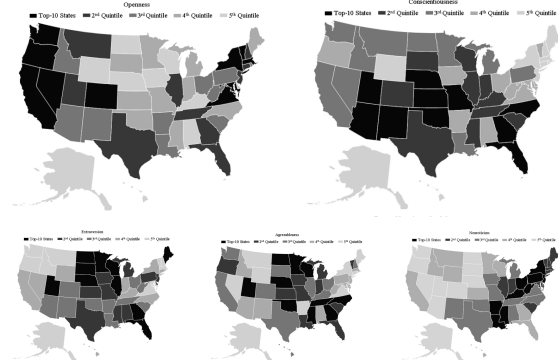


### Five Factor Model (FFM):

- Extraversion [E]
- Agreeableness [A]
- Conscientiousness [C]
- Neuroticism [N]
- Openness [O]

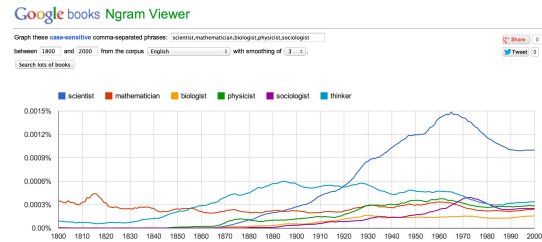
“...a robust and widely accepted framework for conceptualizing the structure of personality... Although the FFM is not universally accepted in the field...” [12]  
**A concern:** self-reported data.  
**Bigger concern:** mass manipulation.

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## The Newness of being a Scientist (1833 on):



Etymology [here](#).

“Scientists are the people who ask a question about a phenomenon and proceed to **systematically** go about answering the question themselves. They are by nature curious, creative and well organized.”

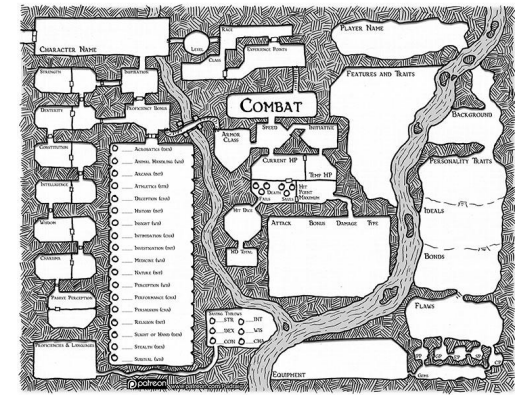
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## Please do not measure complex systems with one number:



- This is **real**—someone having some fun.
- Obtained from [this tweet](#).
- Sadness for Buckingham (if Buckingham has no sense of humor).

## Dungeons & Dragons' full embrace of complexity:



From [here](#).

## Dungeons & Dragons—Two alignment axes for character:



Law-Chaos (vertical) and Good-Evil (horizontal).

From this [Reddit thread](#), where, naturally, the choices are enthusiastically debated.

## Emergence:

### The Wikipedia on Emergence (2006):

“In philosophy, systems theory and the sciences, emergence refers to the way complex systems and patterns arise out of a multiplicity of relatively simple interactions. ... emergence is central to the physics of complex systems and yet very controversial.”

### Wikipedia, 2016:

In philosophy, systems theory, science, and art, emergence is a process whereby larger entities arise through interactions among smaller or simpler entities such that the larger entities exhibit properties the smaller/simpler entities do not exhibit.

The philosopher G. H. Lewes first used the word explicitly in 1875.

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## Emergence:

There's no tornado in a water molecule,  
no financial collapse in a dollar bill,  
no love in a carbon atom.

### Examples:

- Fundamental particles ⇒ Life, the Universe, and Everything
- Genes ⇒ Organisms
- Neurons etc. ⇒ Brain ⇒ Thoughts
- People ⇒ Religion, Collective behaviour
- People ⇒ The Web
- People ⇒ Language, and rules of language
- ? ⇒ time; ? ⇒ gravity; ? ⇒ reality.

“The whole is more than the sum of its parts” –Aristotle

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## Emergence:

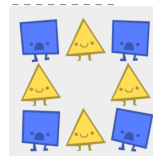
Thomas Schelling (Economist/Nobelist):



“Micromotives and Macrobehavior” [15]

- Segregation [13, 16]
- Wearing hockey helmets [14]
- Seating choices

Vi Hart and Nicky Case's Polygon-themed visualization:



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## Reductionism

- “people don't eat nutrients, they eat foods, and foods can behave very differently than the nutrients they contain.”
- Studies suggest diets high in fruits and vegetables help prevent cancer.
- So... find the nutrients responsible and eat more of them
- But “in the case of beta carotene ingested as a supplement, scientists have discovered that it actually increases the risk of certain cancers. Oops.”

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## Emergence:

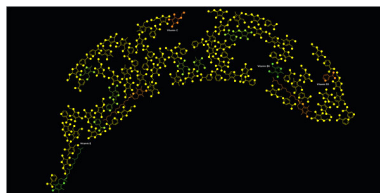
Friedrich Hayek (Economist/Philosopher/Nobelist):

- Markets, legal systems, political systems are emergent and not designed.
- 'Taxis' = made order (by God, Sovereign, Government, ...)
- 'Cosmos' = grown order
- Archetypal limits of hierarchical and decentralized structures.
- Hierarchies arise once problems are solved. [5]
- Decentralized structures help solve problems.
- Dewey Decimal System versus tagging.

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## The emergence of taste:

- Molecules ⇒ Ingredients ⇒ Taste
- See Michael Pollan's article on nutritionism in the New York Times, January 28, 2007.



nytimes.com

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## Reductionism

### Thyme's known antioxidants:

4-Terpineol, alanine, anethole, apigenin, ascorbic acid, beta carotene, caffeic acid, camphene, carvacrol, chlorogenic acid, chrysoeriol, eriodictyol, eugenol, ferulic acid, gallic acid, gamma-terpinene isochlorogenic acid, isoeugenol, isothymonin, kaempferol, labiatic acid, lauric acid, linalyl acetate, luteolin, methionine, myrcene, myristic acid, naringenin, oleanolic acid, p-coumaric acid, p-hydroxy-benzoic acid, palmitic acid, rosmarinic acid, selenium, tannin, thymol, tryptophan, ursolic acid, vanillic acid.

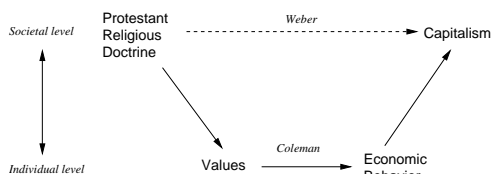


[cnn.com]

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## Emergence:

James Coleman in *Foundations of Social Theory*:



- Understand macrophenomena arises from microbehavior which in turn depends on macrophenomena. [4]
- More on Coleman here.

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## Reductionism

### Reductionism and food:

- Pollan: “even the simplest food is a hopelessly complex thing to study, a virtual wilderness of chemical compounds, many of which exist in complex and dynamic relation to one another...”
- “So ... break the thing down into its component parts and study those one by one, even if that means ignoring complex interactions and contexts, as well as the fact that the whole may be more than, or just different from, the sum of its parts. This is what we mean by reductionist science.”

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## Reductionism

“It would be great to know how this all works, but in the meantime we can enjoy thyme in the knowledge that it probably doesn't do any harm (since people have been eating it forever) and that it may actually do some good (since people have been eating it forever) and that even if it does nothing, we like the way it tastes.”

Gulf between theory and practice (see baseball and bumblebees).

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## This is a Collateralized Debt Obligation:



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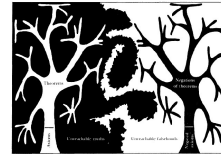
### Higher complexity:

- Many system scales (or levels) that interact with each other.
- Potentially much harder to explain/understand.

### Even mathematics: [6]



Gödel's Theorem [7]: we can't prove every theorem that's true ...



"Gödel, Escher, Bach" [8]

- Suggests a **strong form of emergence**: Some phenomena cannot be analytically deduced from elementary aspects of a system.

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Listen to Steve Strogatz, Hod Lipson, and Michael Schmidt (Cornell) in the last piece on Radiolab's show 'Limits' [9] (April 5, 2010). Starts at 50:30.



(El Bibliomata/flickr)

Dr. Steve Strogatz wonders if we've reached the limits of human scientific understanding, and should soon turn the reins of research over to robots. Cold, calculating robots. Then, Dr. Hod Lipson and Michael Schmidt walk us through the workings of a revolutionary computer program that they developed—a program that can deduce mathematical relationships in nature, through simple observation. The catch? As Dr. Guroi Suel explains, the program gives answers to complex biological questions that we humans have yet to ask, or even to understand.

TAGS: mind bending

Pair with some slow TV [10]

Bonus: Mike Schmidt's talk on Eureka [11] at VCSC's increasingly ancient 2011 TEDx event "Big Data, Big Stories." [12]

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- "The Universe is made of stories, not of atoms."



- From "The Speed of Darkness" (1968) by Muriel Rukeyser [13]
- Quoted by Metatron in Supernatural, Meta Fiction, S9E18.

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### Emergence:

Roughly speaking, there are **two types** of emergence:

#### I. Weak emergence:

System-level phenomena is different from that of its constituent parts yet can be connected theoretically.

#### II. Strong emergence:

System-level phenomena fundamentally cannot be deduced from how parts interact.

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### Definitions

"Self-organization [14] is a process in which the internal organization of a system, normally an open system, increases in complexity without being guided or managed by an outside source." (also: Self-assembly)

#### Examples:

- Molecules/Atoms liking each other → Gases, liquids, and solids.
- Spin alignment → Magnetization.
- Protein folding.
- Imitation → Herding, flocking, mobs, ...

Fundamental question: how likely is 'complexification'?

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## (Sir Terry) Pratchett's Narrativium [15]:



- "The most common element on the disc, although not included in the list of the standard five: earth, fire, air, water and surprise. It ensures that everything runs properly as a story."
- "A little narrativium goes a long way: the simpler the story, the better you understand it. Storytelling is the opposite of reductionism: 26 letters and some rules of grammar are no story at all."

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### Emergence:

- Reductionist techniques can explain weak emergence.
- Magic explains strong emergence. [2]
- But: maybe magic should be interpreted as an inscrutable yet **real mechanism** that cannot ever be **simply described**.
- Gulp.

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### Tools and techniques:

- Differential equations, difference equations, linear algebra, stochastic models.
- Statistical techniques for comparisons and descriptions.
- Methods from statistical mechanics and computer science.
- Machine learning (but beware the black box).
- Computer modeling, everything from
  - Artisanal toy models
  - to kitchen sink models.

#### Key advance (more soon):

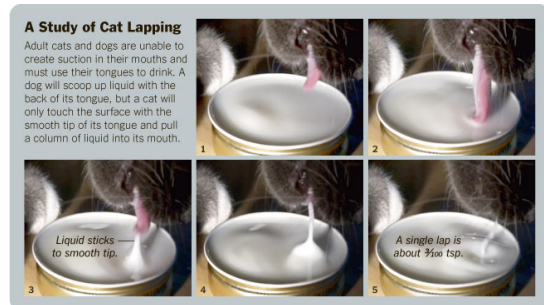
- Representation of complex interaction patterns as complex networks.
- The driver: **Massive amounts of Data**

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# Rather silly but great example of real science:

“How Cats Lap: Water Uptake by *Felis catus*”

Reis et al., *Science*, 2010.

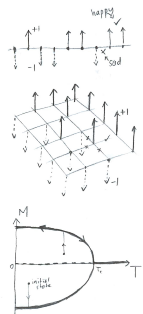


Source: Science THE NEW YORK TIMES; IMAGES FROM VIDEO BY ROMAN STOCKER, SUNGWAN JUNG, JEFFREY M. ARNSTOFF AND FERRO M. REIS

Amusing interview [here](#)

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## The Ising Model of a ferromagnet:



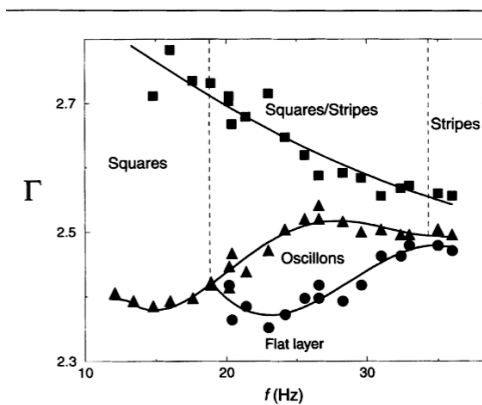
- Each atom is assumed to have a local spin that can be up or down:  $S_i = \pm 1$ .
- Spins are assumed to be arranged on a lattice.
- In isolation, spins like to align with each other.
- Increasing temperature breaks these alignments.
- The drosophila of statistical mechanics.
- Criticality: Power-law distributions at critical points.

## Example 2-d Ising model simulation:

<https://mattbierbaum.github.io/ising.js/>

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## Phase diagrams

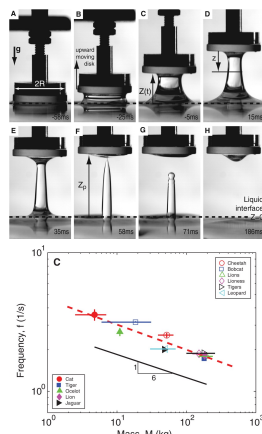


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## Another great, great moment in scaling:

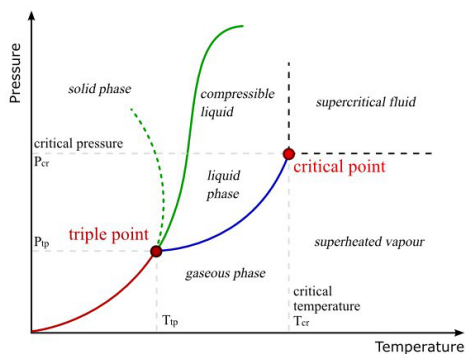
$$f \sim M^{-1/6}$$

The balance of inertia and gravity yields a prediction for the lapping frequency of other felines. Assuming isometry within the Felidae family (i.e., that lapping height  $H$  scales linearly with tongue width  $R$  and animal mass  $M$  scales as  $R^3$ ), the finding that  $f \sim M^{-1/6}$  is of order one translates to the prediction  $f \sim R^{-1/2} \sim M^{-1/6}$ . Isometry or marginally positive allometry among the Felidae has been demonstrated for skull (20, 21) and limb bones (22). Although variability by function can lead to departures from isometry in interspecific scalings (23), reported variations within the Felidae (23, 24) only minimally affect the predicted scaling  $f \sim M^{-1/6}$ . We tested this  $\sim 1/6$  power-law dependence by measuring the lapping frequency for eight species of felines, from videos acquired at the Zoo New England or available on YouTube (16). The lapping frequency was observed to decrease with animal mass as  $f = 4.6 M^{-0.161} \pm 0.004$  ( $\text{min}^{-1} \cdot \text{kg}^{-0.161}$ ) (Fig. 4C), close to the predicted  $M^{-1/6}$ . This close agreement suggests that the domestic cat's inertia- and gravity-controlled lapping mechanism is conserved among felines.



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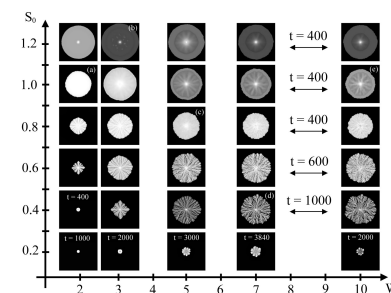
## Phase diagrams



Qualitatively distinct macro states.

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## Phase diagrams



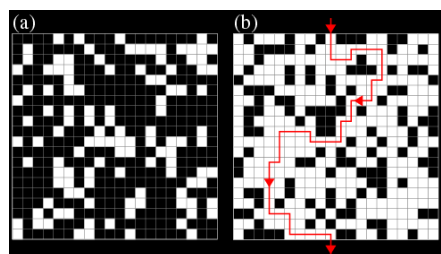
$W_0$  = initial wetness,  $S_0$  = initial nutrient supply  
<http://math.arizona.edu/~lega/HydroBact.html>

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Statistical Mechanics is “a science of collective behavior.”

Simple rules give rise to collective phenomena.

## Percolation:

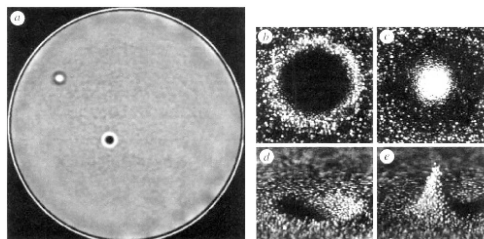


Snared from Michael Gastner's page on percolation [no longer online]

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References

## Phase diagrams

Oscillons, bacteria, traffic, snowflakes, ...



Umbanhowar et al., *Nature*, 1996 [19]

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## Ising model

### Analytic issues:

- 1-d: simple (Ising & Lenz, 1925)
- 2-d: hard (Onsager, 1944)
- 3-d: extremely hard...
- 4-d and up: simple.

See lower and upper critical dimension in statistical physics.  
Also: [Curse and Blessing of Dimensionality](#)

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## Historical surprise:

- 🌀 Origins of Statistical Mechanics are in the studies of people... (Maxwell and co.)
- 🌀 Now physicists are using their techniques to study everything else including people...
- 🌀 See Philip Ball's "Critical Mass" <sup>[1]</sup>

## Beyond Statistical Mechanics:

- 🌀 Analytic approaches have their limits, especially in evolutionary, algorithm-rich systems.
- 🌀 Algorithmic methods and simulation techniques will continue to rise in importance.

## Nutshell

- 🌀 The central concepts **Complexity** and **Emergence** are reasonably well defined.
- 🌀 There is no general theory of Complex Systems.
- 🌀 But the problems exist...  
Complex (Adaptive) Systems abound...
- 🌀 And the observation of Universality of dynamical systems, statistical mechanics, and other quantitative areas means not everything is special and different.
- 🌀 Framing from the Manifesto: Science's focus is moving to Complex Systems **because it finally can**.
- 🌀 We use whatever tools we need.
- 🌀 Science  $\approx$  Describe + Explain.

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