P o C S What's The Story?

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number
University of Vermont, Fall 2024
"And that's why you always leave a note"
Assignment 09

George Bluth, Arrested Development, Pier Pressure, S1E010.
Episode links: Wikipedia, IMDB, Fandom, TV Tropes.

Due: Monday, October 28, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse/assignments/09/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python, R, and/or Julia. **Please do not use any kind of AI thing.** The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via Brightspace (which is not to be confused with the death vortex of the same name).

Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace four days after the due date for this assignment (normally a Friday). For teams, please list all team member names clearly at the start.

Main thing this week: Continue with your projects.

First videos are due Friday, October 25.

1. (3 points)

   Show that the Gini coefficient G for our idealized power-law size distribution of wealth is:

   $$G = \begin{cases} 1 & \text{if } 1 < \gamma \leq 2, \\ \frac{1}{1+2(\gamma-2)} & \text{if } \gamma > 2. \end{cases} \quad (1)$$

   Having developed a sense of what values of γ mean, and because of the simplicity of the relationship between G and γ, we can convert a real-world wealth distribution's value of G to γ for the equivalent idealized power-law size distribution:

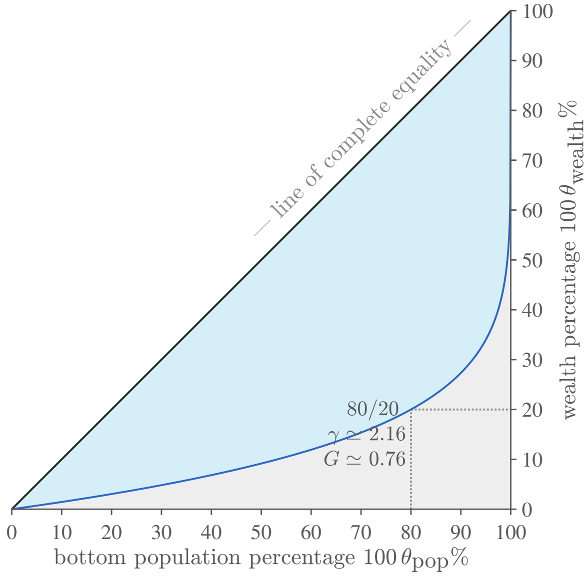
   $$\begin{array}{ll} \gamma \leq 2 & \text{if } G = 1, \\ \gamma = \frac{1}{2}\left(\frac{1}{G} + 3\right) & \text{if } G < 1. \end{array} \quad (2)$$

   For example, what does a Gini coefficient of $1/2$ mean for an idealized power law?

   Eq. 2 gives $\gamma = 5/2$, which we recognized as coming from the Bad Place of finite mean and infinite variance.

   The Gini coefficient is a general measure of inequality that can be determined for any wealth distribution.

   Graphically, using the 80/20 rule example repeated below, the Gini coefficient is the area of the blue region divided by the area of the triangle (1/2) in the distribution's $\theta_{\text{wealth}} = f(\theta_{\text{pop}})$ plot.



We can compute the area of the gray region, and subtract that from 1/2 to find the area of the blue region.

Use the expression for the curve f (you will determine this form in a later assignment):

$$\theta_{\mathrm{wealth}} = f(\theta_{\mathrm{pop}}) = 1 - (1 - \theta_{\mathrm{pop}})^{(\gamma-2)/(\gamma-1)}. \tag{3}$$