PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

Mechanisms for Generating
Power-Law Size Distributions, Part 3

Last updated: 2021/10/06, 20:25:47 EDT

Principles of Complex Systems, Vols. 1 & 2
CSYS/MATH 300 and 303, 2021–2022 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

These slides are brought to you by:

Sealie & Lambie
Productions

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

These slides are also brought to you by:

PoCS
@pocsvox

Power-Law Mechanisms, Pt. 3

Rich-Get-Richer Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

Special Guest Executive Producer

On Instagram at pratchett_the_cat ⬀

Outline

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

The Boggoracle Speaks:

THE RICH
GET RICHER

Aggregation:

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

- Random walks represent additive aggregation
- Mechanism: Random addition and subtraction
- Compare across realizations, no competition.
- Next: Random Additive/Copying Processes involving Competition.
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
- Competing mechanisms (trickiness)
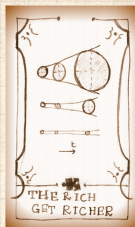
Pre-Zipf's law observations of Zipf's law

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

- 1910s: Word frequency examined re Stenography🔗 (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup🔗 [6].
- 1910s: Felix Auerbach🔗 pointed out the Zipfitude of city sizes in "Das Gesetz der Bevölkerungskonzentration" ("The Law of Population Concentration") [1].
- 1924: G. Udny Yule [15]:
  # Species per Genus (offers first theoretical mechanism)
- 1926: Lotka [9]:
  # Scientific papers per author (Lotka's law)

THE RICH GET RICHER
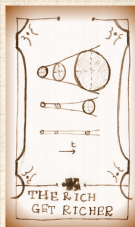
Theoretical Work of Yore:

- 1949: Zipf's "Human Behaviour and the Principle of Least-Effort" is published. [16]
- 1953: Mandelbrot [10]: Optimality argument for Zipf's law; focus on language.
- 1955: Herbert Simon [14, 16]: Zipf's law for word frequency, city size, income, publications, and species per genus.
- 1965/1976: Derek de Solla Price [4, 13]: Network of Scientific Citations.
- 1999: Barabasi and Albert [2]: The World Wide Web, networks-at-large.

THE RICH GET RICHER
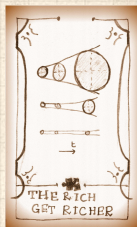
Herbert Simon ☐ (1916–2001):

🎲 Political scientist (and much more)

🎲 Involved in Cognitive Psychology, Computer Science, Public Administration, Economics, Management, Sociology

🎲 Coined 'bounded rationality' and 'satisficing'

🎲 Nearly 1000 publications (see Google Scholar ☐)

🎲 An early leader in Artificial Intelligence, Information Processing, Decision-Making, Problem-Solving, Attention Economics, Organization Theory, Complex Systems, And Computer Simulation Of Scientific Discovery.

🎲 1978 Nobel Laureate in Economics (his Nobel bio is here ☐).

THE RICH
GET RICHER
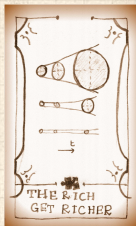
Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$

2. At time $t = 2, 3, 4, ...$, add a new elephant in one of two ways:

   - With probability ρ, create a new elephant with a new flavor
     = Mutation/Innovation

   - With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy.
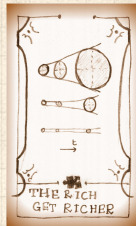   = Replication/Imitation

   - Elephants of the same flavor form a group

Random Competitive Replication:

Example: Words appearing in a language

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
  = Mutation/Innovation

- With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word
  = Replication/Imitation

Note: This is a terrible way to write a novel.

THE RICH
GET RICHER

For example:

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References
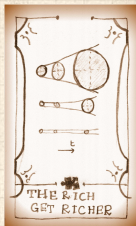
- 21 words used

- next word is new with prob ρ

- next word is a copy with prob $1-\rho$

  prob: next word:

  $6/21$ ook

  $4/21$ the

  $3/21$ and

  $2/21$ penguin

  $\vdots$

  $1/21$ library

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References
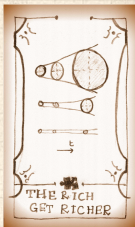
Some observations:

- Fundamental Rich-get-Richer story;
- Competition for replication between individual elephants is random;
- Competition for growth between groups of matching elephants is not random;
- Selection on groups is biased by size;
- Random selection sounds easy;
- Possible that no great knowledge of system needed (but more later ...).

Your free set of tofu knives:

- Related to Pólya's Urn Model ⬀, a special case of problems involving urns and colored balls ⬀.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.

THE RICH
GET RICHER

Random Competitive Replication:

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
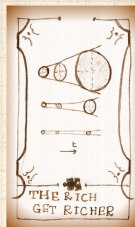First Mover Advantage

References
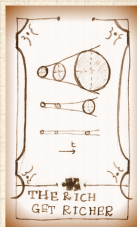
Some observations:

🎲 Steady growth of system: +1 elephant per unit time.

🎲 Steady growth of distinct flavors at rate ρ

🎲 We can incorporate
   1. Elephant elimination
   2. Elephants moving between groups
   3. Variable innovation rate ρ
   4. Different selection based on group size
      (But mechanism for selection is not as simple...)

THE RICH
GET RICHER
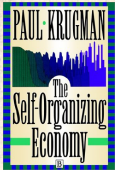
"The Self-Organizing Economy" 🅰 🗗
by Paul Krugman (1996). [8]

Ch. 3: An Urban Mystery, p. 46

"...Simon showed—in a completely impenetrable exposition!—that the exponent of the power law distribution should be ..."[1, 2]

_____

[1] Krugman's book was handed to the Deliverator by a certain Álvaro Cartea 🗗 many years ago at the Santa Fe Institute Summer School.

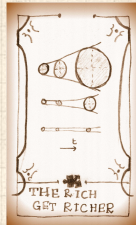[2] Let's use π for probability because π's not special, right guys?

Random Competitive Replication:

Definitions:

🧊 k_i = size of a group i

🧊 $N_{k,t}$ = # groups containing k elephants at time t.

Basic question: How does $N_{k,t}$ evolve with time?

First: $\sum_k k N_{k,t} = t$ = number of elephants at time t

Random Competitive Replication:

PoCS
@pocsvox
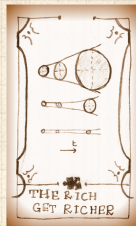
**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

- $N_{k,t}$ size k groups
- $\Rightarrow kN_{k,t}$ elephants in size k groups
- t elephants overall

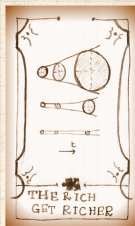$$P_k(t) = \frac{kN_{k,t}}{t}.$$

THE RICH
GET RICHER

Random Competitive Replication:

$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:
   $N_{k,t+1} = N_{k,t} - 1$
   Happens with probability $(1-\rho)kN_{k,t}/t$

2. An elephant belonging to a group with $k-1$ elephants is replicated:
   $N_{k,t+1} = N_{k,t} + 1$
   Happens with probability $(1-\rho)(k-1)N_{k-1,t}/t$

Random Competitive Replication:
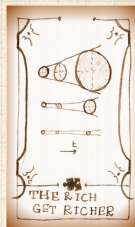
Special case for $N_{1,t}$:

1. The new elephant is a new flavor:
   $N_{1,t+1} = N_{1,t} + 1$
   Happens with probability ρ

2. A unique elephant is replicated:
   $N_{1,t+1} = N_{1,t} - 1$
   Happens with probability $(1 - \rho)N_{1,t}/t$
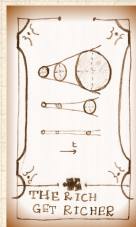

THE RICH
GET RICHER

Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho) \left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t} \right)$$

For $k = 1$:

$$\langle N_{1,t+1} - N_{1,t} \rangle = (+1)\rho + (-1)(1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

THE RICH
GET RICHER
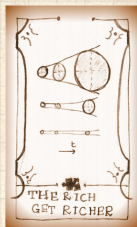
Random Competitive Replication:

Assume distribution stabilizes: $N_{k,t} = n_k t$

(Reasonable for t large)

- ♟ Drop expectations
- ♟ Numbers of elephants now fractional
- ♟ Okay over large time scales

- ♟ For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k t}{\rho t} = \frac{n_k}{\rho}.$$
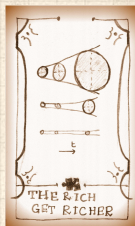
Random Competitive Replication:

Stochastic difference equation:

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho) \left((k-1) \frac{N_{k-1,t}}{t} - k \frac{N_{k,t}}{t} \right)$$

becomes

$$n_k(t+1) - n_k t = (1-\rho) \left((k-1) \frac{n_{k-1} t}{t} - k \frac{n_k t}{t} \right)$$

$$n_k(t + 1 - t) = (1-\rho) \left((k-1) \frac{n_{k-1} t}{t} - k \frac{n_k t}{t} \right)$$

$$\Rightarrow n_k = (1-\rho) \left((k-1) n_{k-1} - k n_k \right)$$

$$\Rightarrow n_k \left(1 + (1-\rho)k \right) = (1-\rho)(k-1) n_{k-1}$$
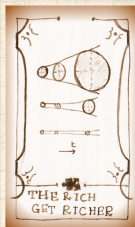
Random Competitive Replication:

We have a simple recursion:

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-\rho)}{1+(1-\rho)k}$$

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

- Interested in k large (the tail of the distribution)
- Can be solved exactly.

  Insert question from assignment 4 $\boxed{\nearrow}$

- For just the tail: Expand as a series of powers of $1/k$

  Insert question from assignment 4 $\boxed{\nearrow}$

  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
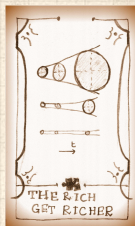Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

🎲 Micro-to-Macro story with ρ and γ measurable.

$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

🎲 Observe $2 < \gamma < \infty$ for $0 < \rho < 1$.

🎲 For $\rho \simeq 0$ (low innovation rate):

$$\gamma \simeq 2$$

🎲 'Wild' power-law size distribution of group sizes, bordering on 'infinite' mean.

🎲 For $\rho \simeq 1$ (high innovation rate):

$$\gamma \simeq \infty$$

🎲 All elephants have different flavors.

🎲 Upshot: Tunable mechanism producing a family of universality classes.

THE RICH
GET RICHER

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References
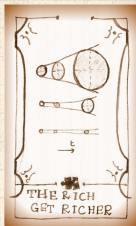
🎲 Recall Zipf's law: $s_r \sim r^{-\alpha}$
(s_r = size of the rth largest group of elephants)

🎲 We found $\alpha = 1/(\gamma - 1)$ so:

$$\alpha = \frac{1}{\gamma - 1} = \frac{1}{\cancel{1} + \frac{1}{(1-\rho)} - \cancel{1}} = 1 - \rho.$$

🎲 $\gamma = 2$ corresponds to $\alpha = 1$

🎲 We (roughly) see Zipfian exponent [16] of $\alpha = 1$ for many real systems: city sizes, word distributions, ...

🎲 Corresponds to $\rho \to 0$, low innovation.

🎲 Still, other quite different mechanisms are possible...

🎲 Must look at the details to see if mechanism makes sense... more later.

THE RICH
GET RICHER
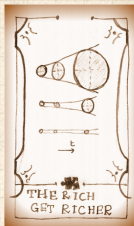
We had one other equation:

$$\langle N_{1,t+1} - N_{1,t} \rangle = \rho - (1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

As before, set $N_{1,t} = n_1 t$ and drop expectations

$$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$

$$n_1 = \rho - (1-\rho)n_1$$

Rearrange:

$$n_1 + (1-\rho)n_1 = \rho$$

$$\boxed{n_1 = \frac{\rho}{2 - \rho}}$$

THE RICH
GET RICHER

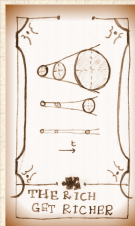So... $N_{1,t} = n_1 t = \dfrac{\rho t}{2 - \rho}$

- Recall number of distinct elephants = ρt.
- Fraction of distinct elephants that are unique (belong to groups of size 1):

$$\frac{1}{\rho t} N_{1,t} = \frac{1}{\rho t} \frac{\rho t}{2 - \rho} = \frac{1}{2 - \rho}$$

(also = fraction of groups of size 1)
- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$
- Model works well for large and small k #awesome

THE RICH
GET RICHER

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**
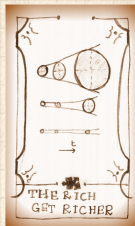
Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References
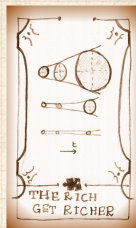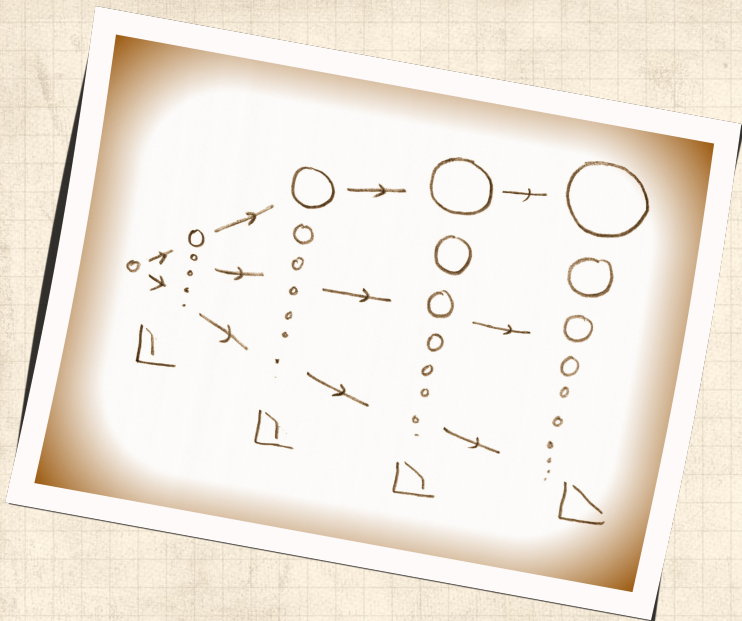
Words:

From Simon [14]:

Estimate $\rho_{\text{est}} =$ # unique words/# all words
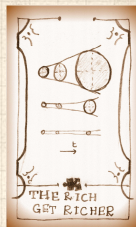
For Joyce's Ulysses: $\rho_{\text{est}} \simeq 0.115$

N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870

THE RICH
GET RICHER

Evolution of catch phrases:

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

- ♣ Yule's paper (1924) [15]:
  "A mathematical theory of evolution, based on the conclusions of Dr J. C. Willis, F.R.S."
- ♣ Simon's paper (1955) [14]:
  "On a class of skew distribution functions" (snore)

From Simon's introduction:

It is the purpose of this paper to analyse a class of distribution functions that appear in a wide range of empirical data—particularly data describing sociological, biological and economic phenomena.

Its appearance is so frequent, and the phenomena so diverse, that one is led to conjecture that if these phenomena have any property in common it can only be a similarity in the structure of the underlying probability mechanisms.

THE RICH
GET RICHER
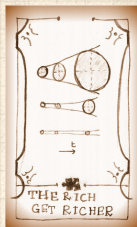
Evolution of catch phrases:

Derek de Solla Price:

- First to study network evolution with these kinds of models.
- Citation network of scientific papers
- Price's term: Cumulative Advantage
- Idea: papers receive new citations with probability proportional to their existing # of citations
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
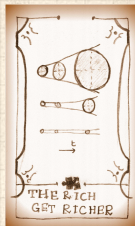 2. Selection mechanism is more complicated

THE RICH
GET RICHER

Evolution of catch phrases:

PoCS
@pocsvox
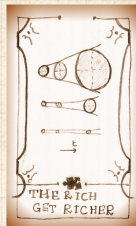
**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

Robert K. Merton: the Matthew Effect ☑

- Studied careers of scientists and found credit flowed disproportionately to the already famous

  From the Gospel of Matthew:
  "For to every one that hath shall be given...
  (Wait! There's more....)
  but from him that hath not, that also which he seemeth to have shall be taken away.
  And cast the worthless servant into the outer darkness; there men will weep and gnash their teeth."

- (Hath = suggested unit of purchasing power.)

- Matilda effect: ☑ women's scientific achievements are often overlooked
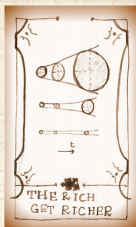
Evolution of catch phrases:

Merton was a catchphrase machine:

1. Self-fulfilling prophecy
2. Role model
3. Unintended (or unanticipated) consequences
4. Focused interview → focus group
5. Obliteration by incorporation ☐ (includes above examples from Merton himself)

And just to be clear...

Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.

THE RICH
GET RICHER

Evolution of catch phrases:

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References
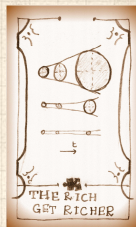
- Barabasi and Albert [2]—thinking about the Web
- Independent reinvention of a version of Simon and Price's theory for networks
- Another term: "Preferential Attachment"
- Considered undirected networks (not realistic but avoids 0 citation problem)
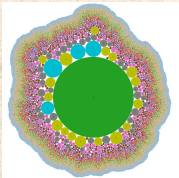- Still have selection problem based on size (non-random)
- Solution: Randomly connect to a node (easy) ...
- ...and then randomly connect to the node's friends (also easy)
- "Scale-free networks" = food on the table for physicists

THE RICH GET RICHER

Another analytic approach: [5]

⬧ Focus on how the nth arriving group typically grows.

⬧ Analysis gives:

$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

⬧ First mover is a factor $1/\rho$ greater than expected.

⬧ Because ρ is usually close to 0, the first element is truly an elephant in the room.

⬧ Appears that this has been missed for 60 years ...

THE RICH
GET RICHER
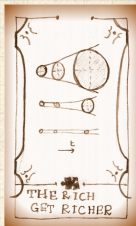
"Simon's fundamental rich-get-richer model
entails a dominant first-mover advantage" 🔗
Dodds et al.,
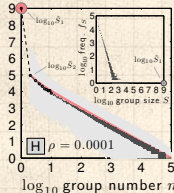Physical Review E, **95**, 052301, 2017. [5]

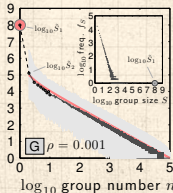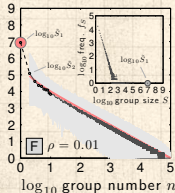A. $\rho = 0.1$ B. $\rho = 0.01$ C. $\rho = 0.001$ D. $\rho = 0.0001$

E. $\rho = 0.1$ F. $\rho = 0.01$ G. $\rho = 0.001$ H. $\rho = 0.0001$

THE RICH
GET RICHER
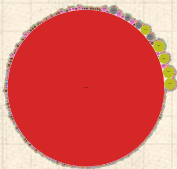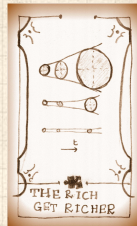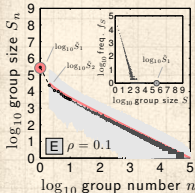
🧩 See visualization at paper's online app-endices 🔗

Alternate analysis:

♟ Evolution of the nth arriving group's size:

$$\langle S_{n,t+1} - S_{n,t} \rangle = (1 - \rho_t) \cdot \frac{S_{n,t}}{t} \cdot (+1).$$

♟ For $t \geq t_n^{\text{init}}$, fix $\rho_t = \rho$ and shift t to $t - 1$:

$$S_{n,t} = \left[1 + \frac{(1 - \rho)}{t - 1} \right] S_{n,t-1}.$$

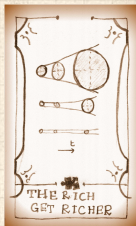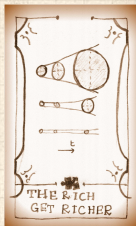where $S_{n,t_n^{\text{init}}} = 1$.

THE RICH
GET RICHER

Betafication ensues:

$$S_{n,t} = \left[1 + \frac{(1-\rho)}{t-1}\right]\left[1 + \frac{(1-\rho)}{t-2}\right]\cdots\left[1 + \frac{(1-\rho)}{t_n^{\text{init}}}\right] \cdot 1$$

$$= \left[\frac{t+1-\rho}{t-1}\right]\left[\frac{t-\rho}{t-2}\right]\cdots\left[\frac{t_n^{\text{init}}+1-\rho}{t_n^{\text{init}}}\right]$$

$$= \frac{\Gamma(t+1-\rho)\Gamma(t_n^{\text{init}})}{\Gamma(t_n^{\text{init}}+1-\rho)\Gamma(t)}$$

$$= \frac{\text{B}(t_n^{\text{init}}, 1-\rho)}{\text{B}(t, 1-\rho)}.$$

THE RICH
GET RICHER

The first mover is really different:

🎲 The issue is t_n^{init} in

$$S_{n,t} = \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}$$

🎲 For $n \geq 2$ and $\rho \ll 1$, the nth group typically arrives at $t_n^{\text{init}} \simeq \left[\frac{n-1}{\rho}\right]$

🎲 But $t_1^{\text{init}} = 1$ and the scaling is distinct in form.

🎲 Simon missed the first mover by working on the size distribution.

🎲 Contribution to $P_{k,t}$ of the first element vanishes as $t \to \infty$.

🎲 Note: Does not apply to Barabási-Albert model.

THE RICH
GET RICHER

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3
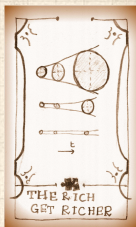
Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

Variability:

🎲 The probability that the nth arriving group, if of size $S_{n,t} = k$ at time t, first replicates at time $t + \tau$:

$$\mathbf{Pr}(S_{n,t+\tau} = k+1 \,|\, S_{n,t+i} = k \text{ for } i = 0, ..., \tau-1)$$

$$= \prod_{i=0}^{\tau-1} \left[1 - (1-\rho)\frac{k}{t+i} \right] \cdot (1-\rho)\frac{k}{t+\tau}$$

$$= k\frac{B(\tau, t)}{B\left(\tau, t-(1-\rho)\right)} \frac{1-\rho}{t+\tau} \propto \frac{\tau^{-(1-\rho)k}}{t+\tau} \sim \tau^{-(2-\rho)k}.$$

🎲 Upshot: nth arriving group starting at size 1 will on average wait for an infinite time to replicate.

THE RICH
GET RICHER

Related papers:
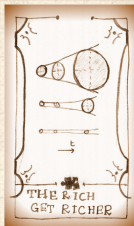
PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

"Organization of Growing Random
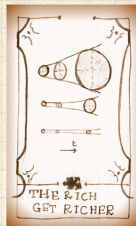Networks" ⬀
Krapivsky and Redner,
Phys. Rev. E, **63**, 066123, 2001. [7]

"The first-mover advantage in scientific
publication" ⬀
M. E. J. Newman,
Europhysics Letters, **86**, 68001, 2009. [11]

Related papers:

PoCS
@pocsvox

**Power-Law
Mechanisms, Pt. 3**

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

"Prediction of highly cited papers" 🔗
M. E. J. Newman,
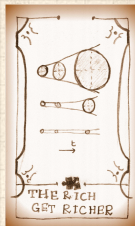Europhysics Letters, **105**, 28002, 2014. [12]

"The effect of the initial network
configuration on preferential
attachment" 🔗
Berset and Medo,
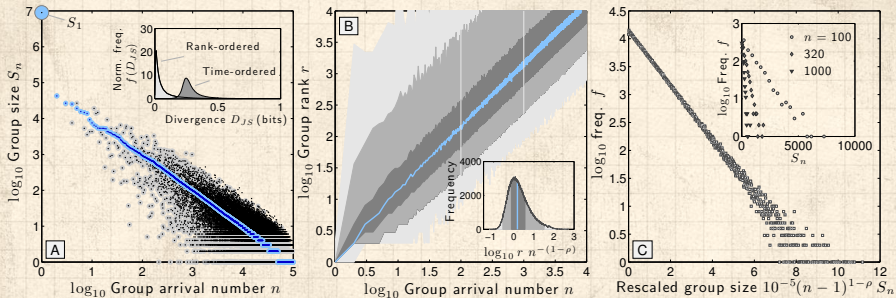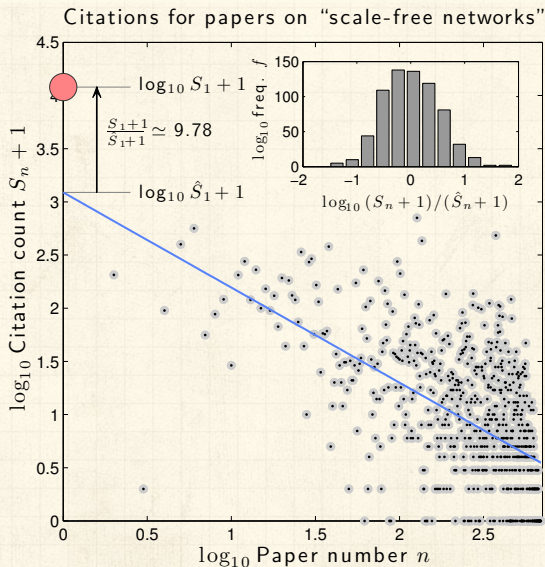The European Physical Journal B, **86**, 1–7,
2013. [3]

Arrival variability:

- Any one simulation shows a high amount of disorder.
- Two orders of magnitude variation in possible rank.
- Rank ordering creates a smooth Zipf distribution.
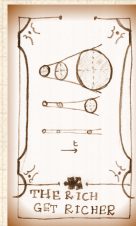- Size distribution for the nth arriving group show exponential decay.

Self-referential citation data:

Citations for papers on "scale-free networks"

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

More mattering:

Rich-get-richerness in social contagion:

- 🎲 We love to rank everyone, everything: Top n lists.
- 🎲 People, wealth, sports, music, movies, books, schools, cities, countries, dogs (13/10) ⧉, ...
- 🎲 Gameable: payola ⧉, astroturfing ⧉, sockpuppetry ⧉, John Barron ⧉ (the sockpuppet hype man ⧉), ...
- 🎲 Black-box ranking algorithms make ranking opaque.
- 🎲 Black boxes are gameable but takes money and commensurate skill.
- 🎲 Black box algorithms can make things spread rampantly.[1]
- 🎲 No "regramming" is a positive feature of Instagram (also: Pratchett the Cat ⧉)
- 🎲 What if a healthier Facebook is just ...

THE RICH
GET RICHER

References I

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
Mechanism
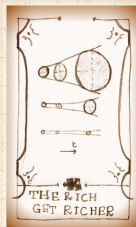Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage
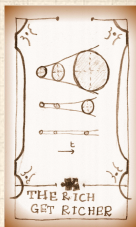
References

[1] F. Auerbach.
Das gesetz der bevölkerungskonzentration.
Petermanns Geogr. Mitteilungen, 59:73–76, 1913.

[2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf

[3] Y. Berset and M. Medo.
The effect of the initial network configuration on
preferential attachment.
The European Physical Journal B, 86(6):1–7, 2013.
pdf

[4] D. J. de Solla Price.
Networks of scientific papers.
Science, 149:510–515, 1965. pdf

References II

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage

References

[5] P. S. Dodds, D. R. Dewhurst, F. F. Hazlehurst, C. M.
     Van Oort, L. Mitchell, A. J. Reagan, J. R. Williams,
     and C. M. Danforth.
     Simon's fundamental rich-get-richer model
     entails a dominant first-mover advantage.
     Physical Review E, 95:052301, 2017. pdf ☑

[6] J.-B. Estoup.
     Gammes sténographiques: méthode et exercices
     pour l'acquisition de la vitesse.
     Institut Sténographique, 1916.

[7] P. L. Krapivsky and S. Redner.
     Organization of growing random networks.
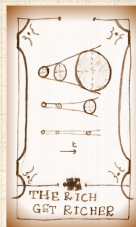   Phys. Rev. E, 63:066123, 2001. pdf ☑

THE RICH
GET RICHER

References III

[8] P. Krugman.
     The Self-Organizing Economy.
     Blackwell Publishers, Cambridge, Massachusetts,
     1996.

[9] A. J. Lotka.
     The frequency distribution of scientific
     productivity.
     Journal of the Washington Academy of Science,
     16:317–323, 1926.

[10] B. B. Mandelbrot.
     An informational theory of the statistical
     structure of languages.
     In W. Jackson, editor, Communication Theory,
     pages 486–502. Butterworth, Woburn, MA, 1953.
     pdf ↗

References IV

PoCS
@pocsvox

Power-Law
Mechanisms, Pt. 3

Rich-Get-Richer
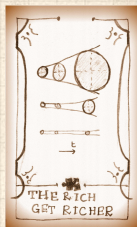Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage
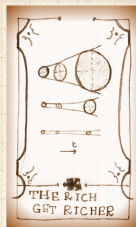
References

[11] M. E. J. Newman.
The first-mover advantage in scientific
publication.
Europhysics Letters, 86:68001, 2009. pdf

[12] M. E. J. Newman.
Prediction of highly cited papers.
Europhysics Letters, 105:28002, 2014. pdf

[13] D. D. S. Price.
A general theory of bibliometric and other
cumulative advantage processes.
Journal of the American Society for Information
Science, pages 292–306, 1976. pdf

[14] H. A. Simon.
On a class of skew distribution functions.
Biometrika, 42(3-4):425–440, 12 1955. pdf

THE RICH
GET RICHER

References V

[15] G. U. Yule.
     A mathematical theory of evolution, based on the
     conclusions of Dr J. C. Willis, F.R.S.
     Phil. Trans. B, 213:21–87, 1925. pdf🗗

[16] G. K. Zipf.
     Human Behaviour and the Principle of
     Least-Effort.
     Addison-Wesley, Cambridge, MA, 1949.