Mechanisms for Generating Power-Law Size Distributions, Part 3

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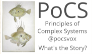
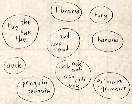
Principles of Complex Systems, Vols. 1 & 2
CSYS/MATH 300 and 303, 2021–2022| @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont





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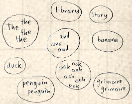

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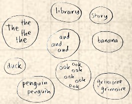
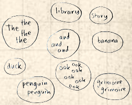


Outline

Rich-Get-Richer Mechanism

- Simon’s Model
- Analysis
- Words
- Catchphrases
- First Mover Advantage

References



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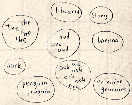

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grimoire grimoire

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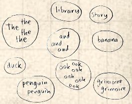
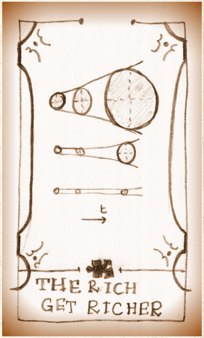



THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END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THE RICH GET RICHER

Aggregation:

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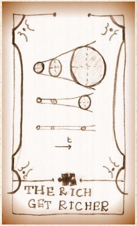

Aggregation:

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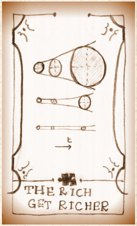
- Mechanism: Random addition and subtraction



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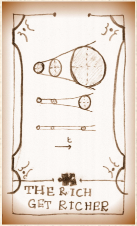

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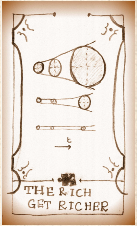

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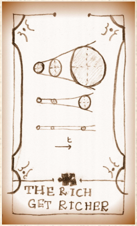

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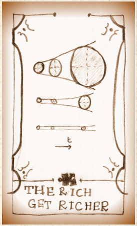
- Competing mechanisms (trickiness)



Pre-Zipf's law observations of Zipf's law

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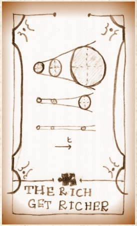

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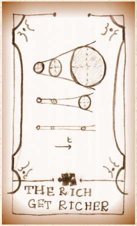

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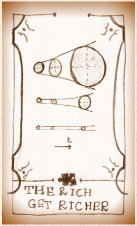

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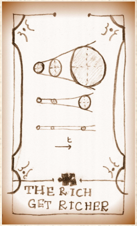
- 1926: Lotka [9]:
  # Scientific papers per author (Lotka's law)



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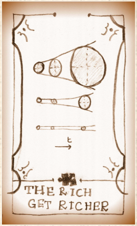

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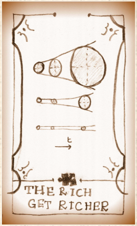

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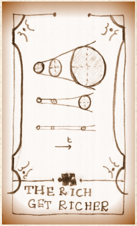

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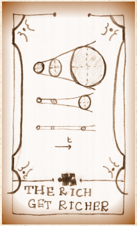

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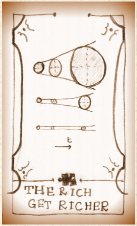
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.



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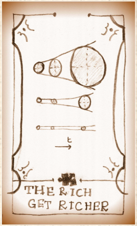

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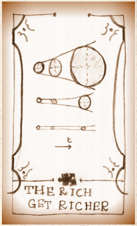

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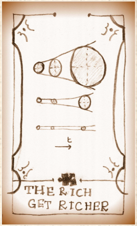

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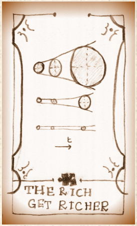

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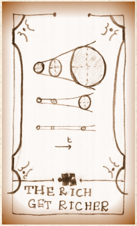

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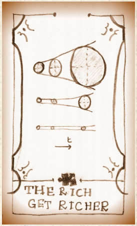
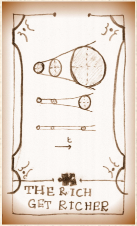
- 1978 Nobel Laureate in Economics (his Nobel bio is here).



Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$



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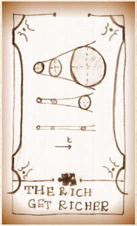

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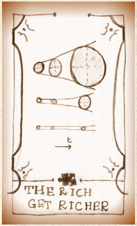

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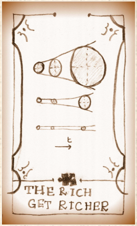

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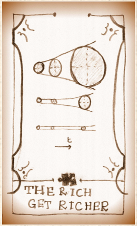

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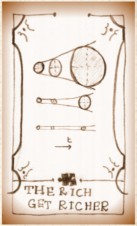
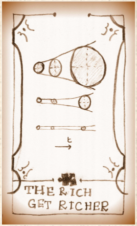


Random Competitive Replication:

Example: Words appearing in a language



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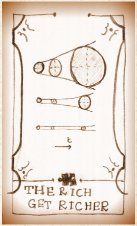

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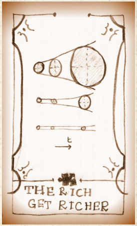

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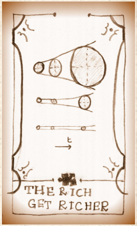

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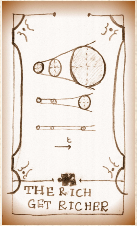

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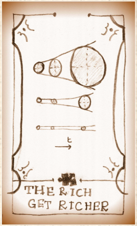

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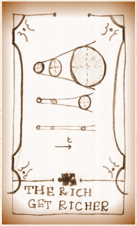
Note: This is a terrible way to write a novel.



For example:

- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
$6/21$	ook
$4/21$	the
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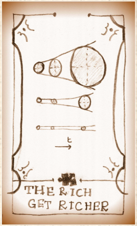

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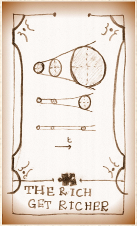

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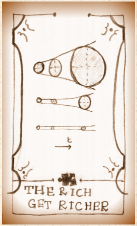

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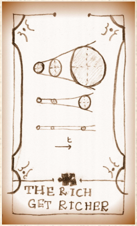

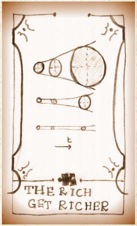

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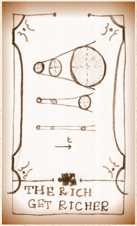

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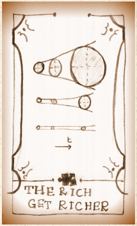

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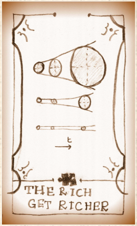

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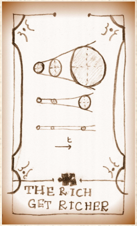

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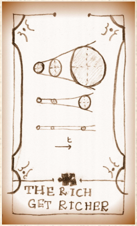
- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.



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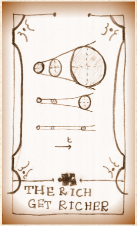

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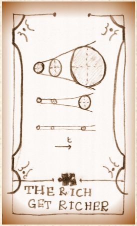

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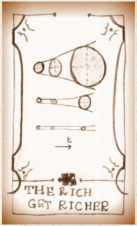

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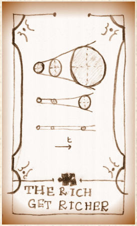

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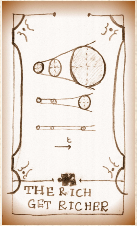

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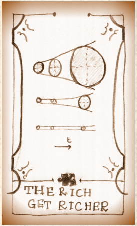

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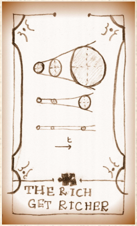

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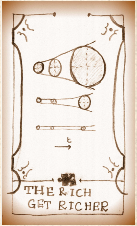

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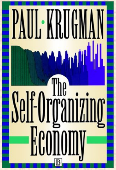
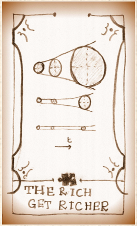
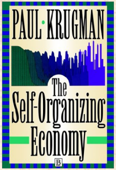
     (But mechanism for selection is not as simple...)



"The Self-Organizing Economy" by Paul Krugman (1996). [8]





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Ch. 3: An Urban Mystery, p. 46

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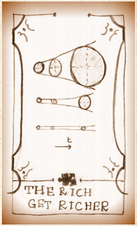

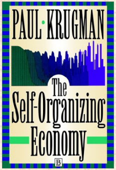

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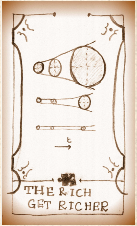

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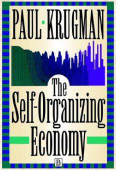

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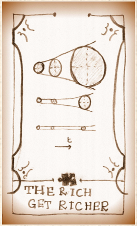

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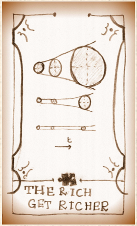
[2] Let's use π for probability because π's not special, right guys?

Outline

Rich-Get-Richer Mechanism

- Simon's Model
- Analysis
- Words
- Catchphrases
- First Mover Advantage

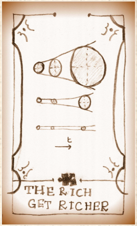
References



Random Competitive Replication:

Definitions:

- k_i = size of a group i



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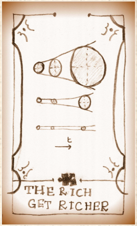

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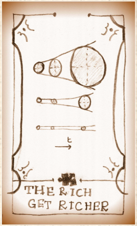

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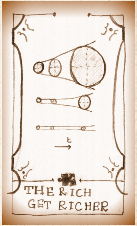

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First: $\sum_k kN_{k,t} = t =$ number of elephants at time t

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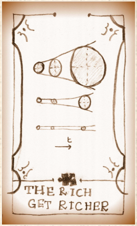
$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:



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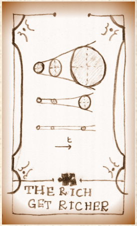

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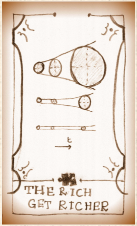

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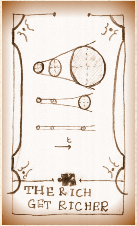

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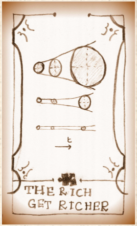
$$P_k(t) = \frac{kN_{k,t}}{t}.$$



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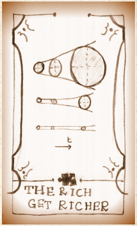

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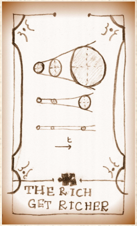

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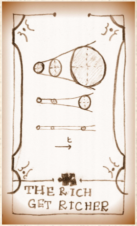

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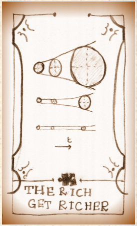

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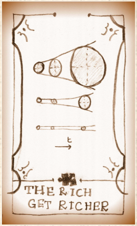

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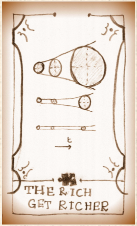

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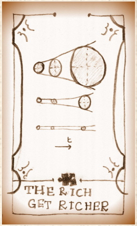

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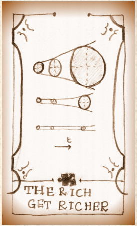

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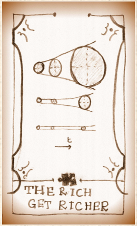

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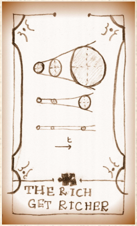

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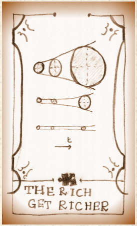

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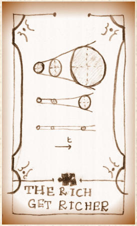

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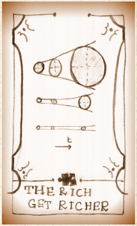

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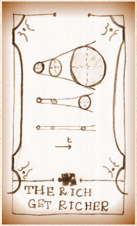
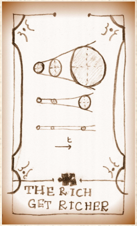


Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t}\right)$$



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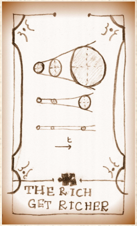

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Assume distribution stabilizes: $N_{k,t} = n_k t$
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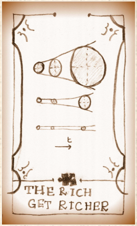

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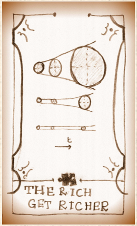

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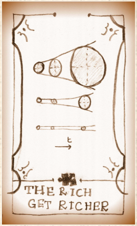

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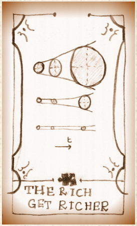

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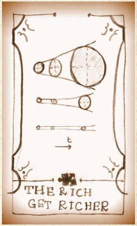
- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k \cancel{t}}{\rho \cancel{t}} = \frac{n_k}{\rho}.$$



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$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t} \right)$$

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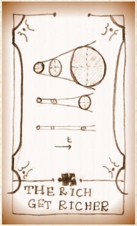

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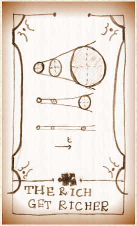

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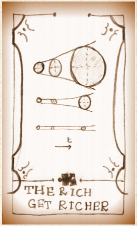

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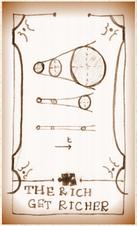

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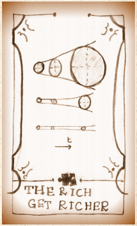

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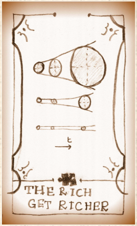

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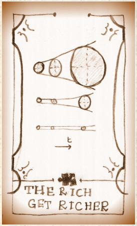

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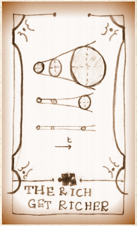

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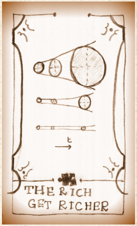

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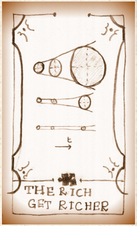

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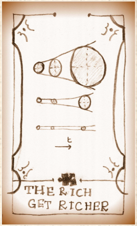
  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$



Micro-to-Macro story with ρ and γ measurable.

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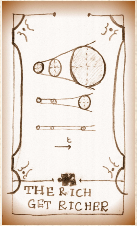

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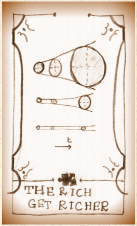

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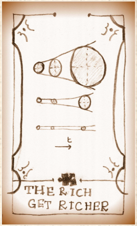

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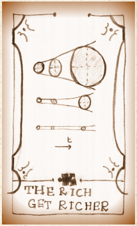

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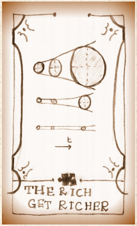

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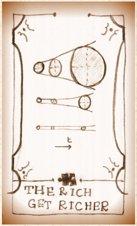

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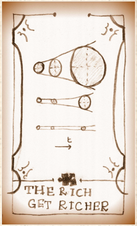
- Upshot: Tunable mechanism producing a family of universality classes.



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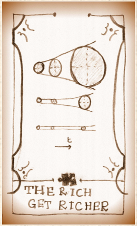

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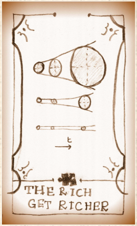

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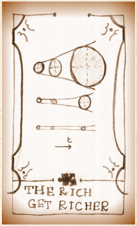

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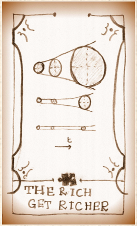

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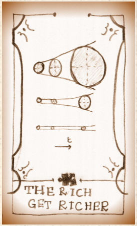

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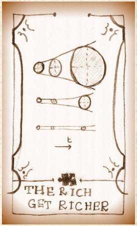

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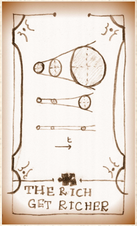

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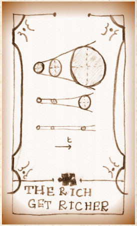

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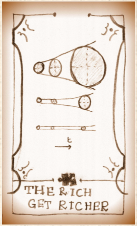

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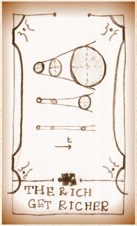

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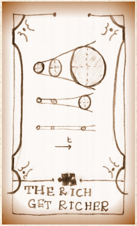
- Rearrange:
$$n_1 + (1-\rho) n_1 = \rho$$



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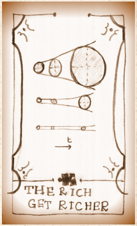
- $$n_1 = \rho - (1-\rho) n_1$$
- Rearrange:
$$n_1 + (1-\rho) n_1 = \rho$$
- $$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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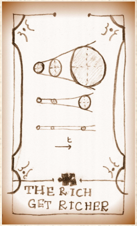

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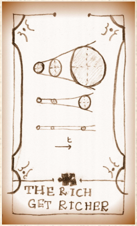
- Recall number of distinct elephants = ρt.

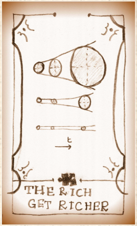


So... $$N_{1,t} = n_1 t = \frac{\rho t}{2-\rho}$$

- Recall number of distinct elephants = ρt.
- Fraction of distinct elephants that are unique (belong to groups of size 1):

$$\frac{1}{\rho t} N_{1,t} = \frac{1}{\cancel{\rho t}} \frac{\cancel{\rho t}}{2-\rho} = \frac{1}{2-\rho}$$

(also = fraction of groups of size 1)



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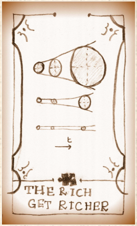

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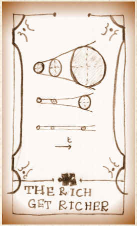

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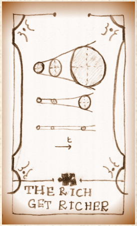

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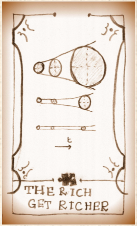
- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$



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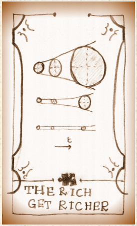

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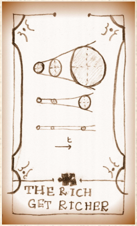

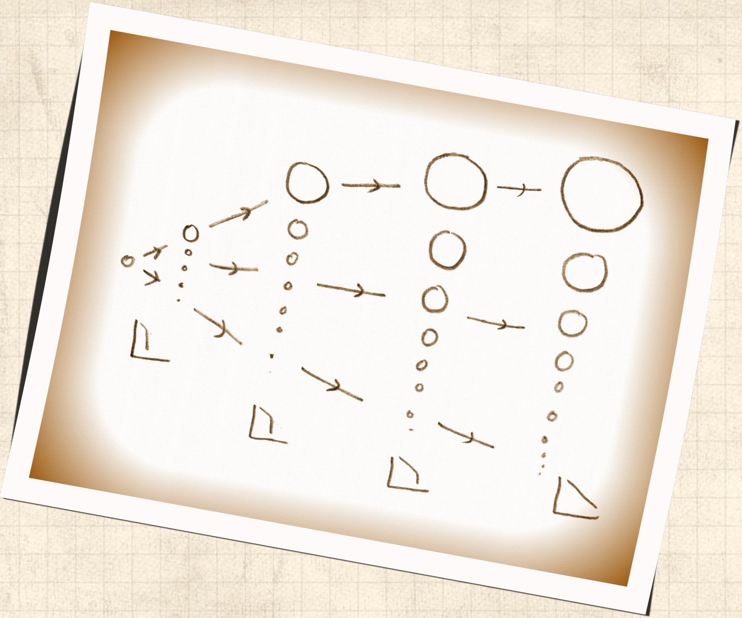

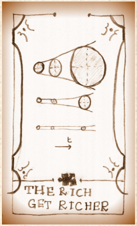
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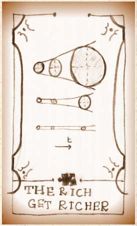
THE RICH
GET RICHER

Outline



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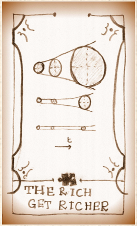
From Simon [14]:

Estimate $\rho_{\text{est}} = \#$ unique words/# all words



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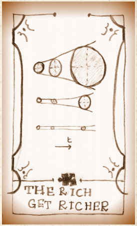

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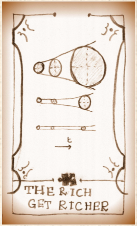
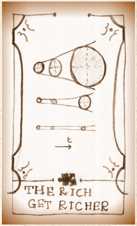
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870





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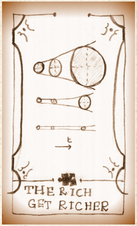

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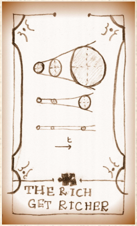

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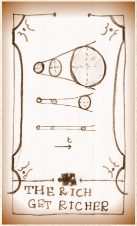

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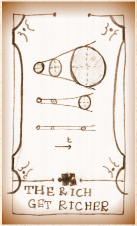

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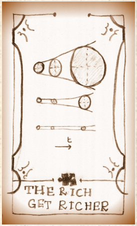

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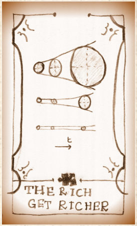

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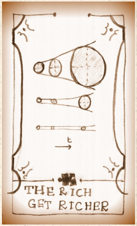

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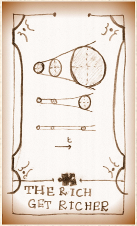

Evolution of catch phrases:

Derek de Solla Price:

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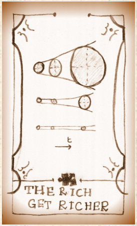

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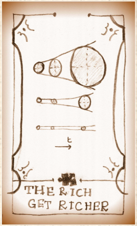

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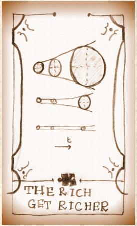

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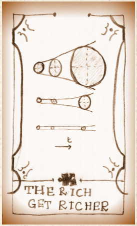

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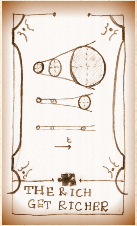

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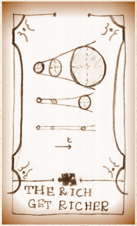
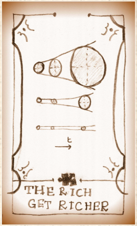
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
  2. Selection mechanism is more complicated



Evolution of catch phrases:

Robert K. Merton: the Matthew Effect

- Studied careers of scientists and found credit flowed disproportionately to the already famous



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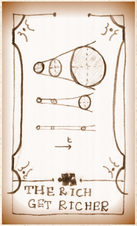

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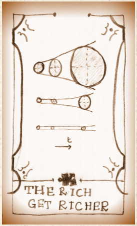

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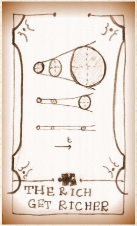

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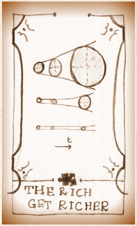

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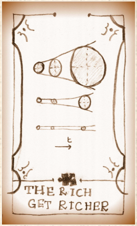

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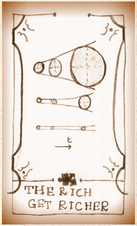
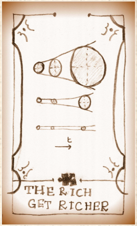
- (Hath = suggested unit of purchasing power.)
- Matilda effect: women's scientific achievements are often overlooked



Evolution of catch phrases:

Merton was a catchphrase machine:



Evolution of catch phrases:

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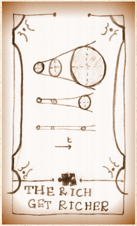
1. Self-fulfilling prophecy



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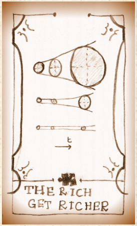

Merton was a catchphrase machine:

1. Self-fulfilling prophecy
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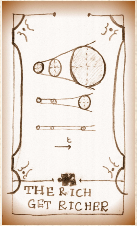

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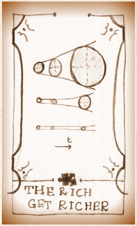

Merton was a catchphrase machine:

1. Self-fulfilling prophecy
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3. Unintended (or unanticipated) consequences
4. Focused interview $\rightarrow$ focus group

Evolution of catch phrases:

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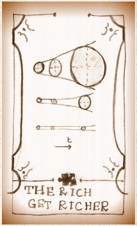
5. Obliteration by incorporation (includes above examples from Merton himself)



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Merton was a catchphrase machine:

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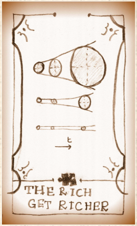

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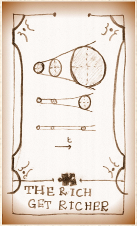

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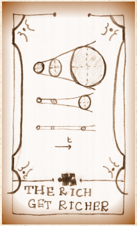

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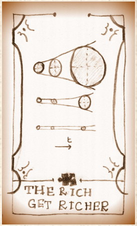

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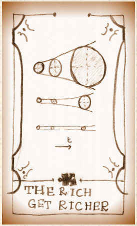

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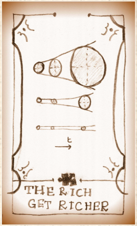

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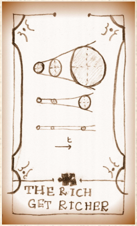

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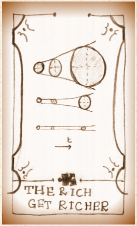

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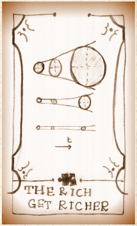

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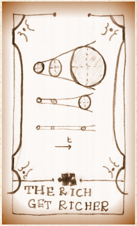
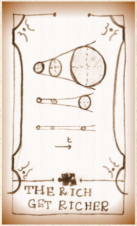
- "Scale-free networks" = food on the table for physicists

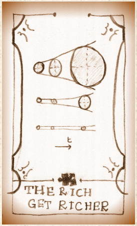


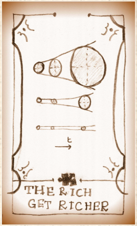
Outline



Another analytic approach: [5]

- Focus on how the nth arriving group typically grows.





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$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

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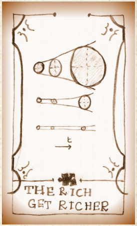
- First mover is a factor $1/\rho$ greater than expected.



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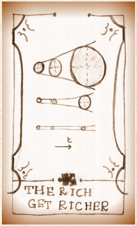

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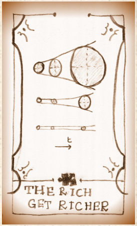
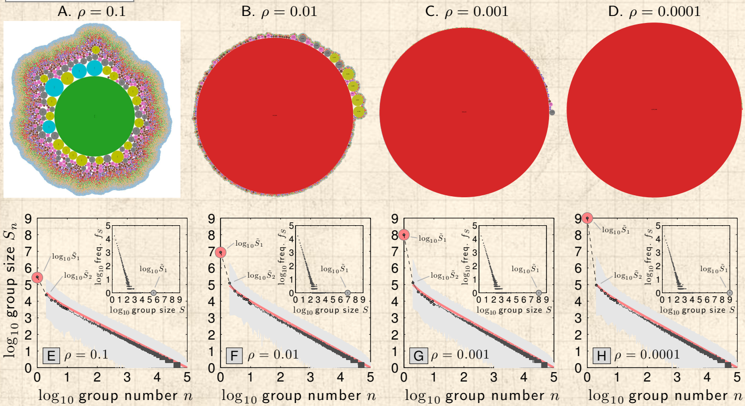
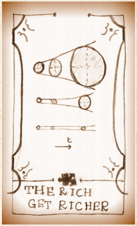
- Appears that this has been missed for 60 years ...

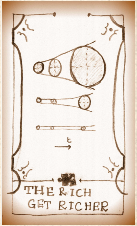


"Simon's fundamental rich-get-richer model entails a dominant first-mover advantage"
Dodds et al.,
Physical Review E, **95**, 052301, 2017. [5]





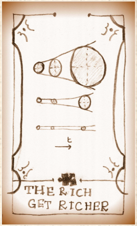
See visualization at paper's online app-endices



Alternate analysis:

- Evolution of the nth arriving group's size:

$$\left\langle S_{n,t+1} - S_{n,t} \right\rangle = (1 - \rho_t) \cdot \frac{S_{n,t}}{t} \cdot (+1).$$



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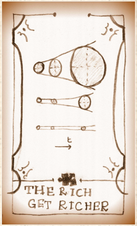
- For $t \geq t_n^{\text{init}}$, fix $\rho_t = \rho$ and shift t to $t-1$:

$$S_{n,t} = \left[1 + \frac{(1-\rho)}{t-1} \right] S_{n,t-1}.$$

where $S_{n,t_n^{\text{init}}} = 1$.



Betafication ensues:

$$
\begin{aligned}
S_{n,t} &= \left[1 + \frac{(1-\rho)}{t-1}\right]\left[1 + \frac{(1-\rho)}{t-2}\right]\cdots\left[1 + \frac{(1-\rho)}{t_n^{\text{init}}}\right]\cdot 1 \\
&= \left[\frac{t+1-\rho}{t-1}\right]\left[\frac{t-\rho}{t-2}\right]\cdots\left[\frac{t_n^{\text{init}}+1-\rho}{t_n^{\text{init}}}\right] \\
&= \frac{\Gamma(t+1-\rho)\Gamma(t_n^{\text{init}})}{\Gamma(t_n^{\text{init}}+1-\rho)\Gamma(t)} \\
&= \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}.
\end{aligned}
$$

The first mover is really different:

- The issue is t_n^{init} in

$$S_{n,t} = \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}$$

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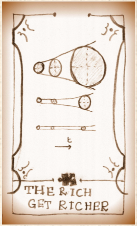

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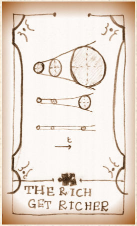

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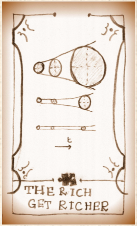

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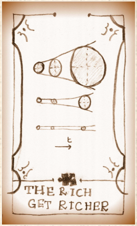

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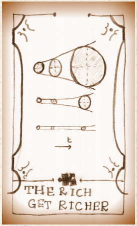
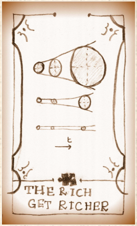
- Note: Does not apply to Barabási-Albert model.





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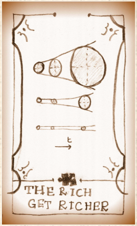
$$= \prod_{i=0}^{\tau-1} \left[1 - (1-\rho)\frac{k}{t+i}\right] \cdot (1-\rho)\frac{k}{t+\tau}$$

$$= k\frac{B(\tau, t)}{B\left(\tau, t-(1-\rho)\right)}\frac{1-\rho}{t+\tau} \propto \frac{\tau^{-(1-\rho)k}}{t+\tau} \sim \tau^{-(2-\rho)k}.$$



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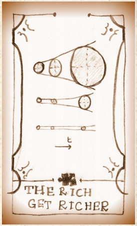
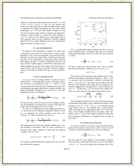
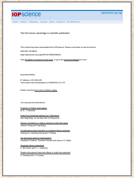
- Upshot: nth arriving group starting at size 1 will on average wait for an infinite time to replicate.



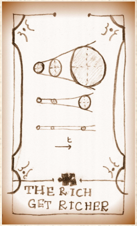
Related papers:



"Organization of Growing Random Networks"
Krapivsky and Redner,
Phys. Rev. E, **63**, 066123, 2001. [7]



"The first-mover advantage in scientific publication"
M. E. J. Newman,
Europhysics Letters, **86**, 68001, 2009. [11]



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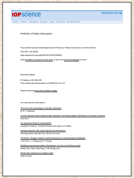
Prediction of highly cited papers

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The effect of the initial network configuration on preferential attachment

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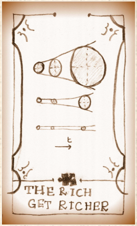
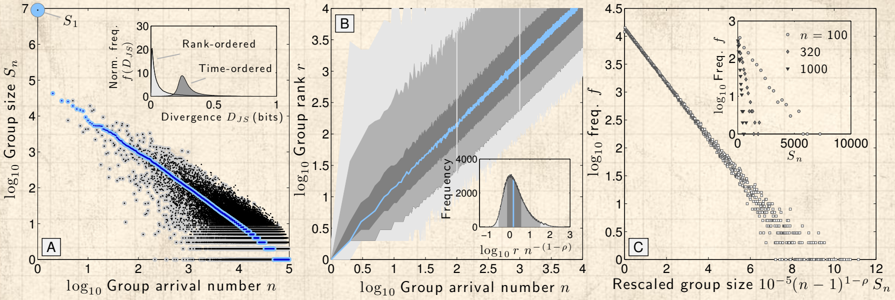
Berset and Medo,
The European Physical Journal B, **86**, 1–7, 2013. [3]

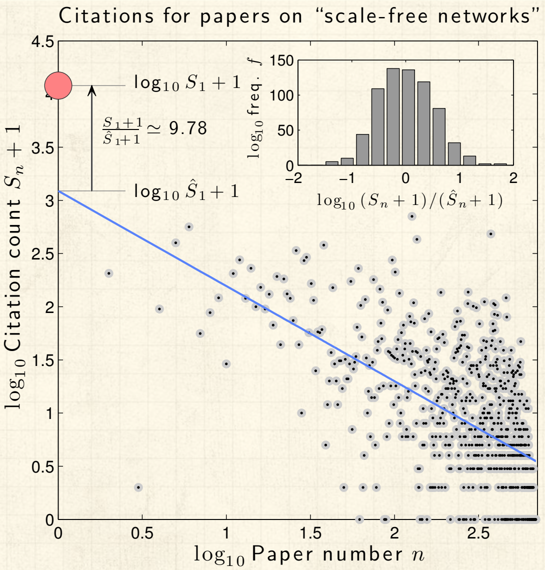


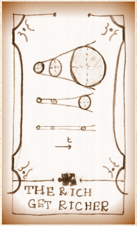
Arrival variability:



- Any one simulation shows a high amount of disorder.
- Two orders of magnitude variation in possible rank.
- Rank ordering creates a smooth Zipf distribution.
- Size distribution for the nth arriving group show exponential decay.

Self-referential citation data:

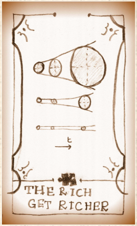




More mattering:

Rich-get-richerness in social contagion:

- We love to rank everyone, everything: Top n lists.



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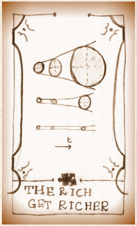

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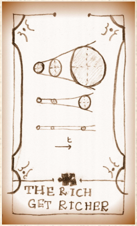

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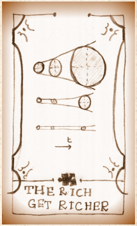
- Black-box ranking algorithms make ranking opaque.



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- Black-box ranking algorithms make ranking opaque.
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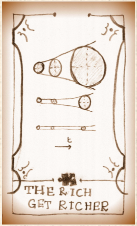

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Rich-get-richerness in social contagion:

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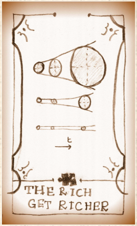

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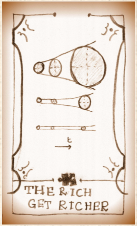

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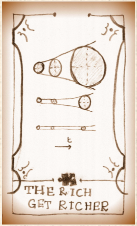

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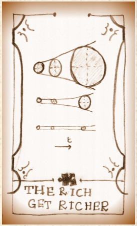
- What if a healthier Facebook is just ...



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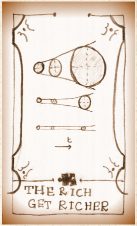

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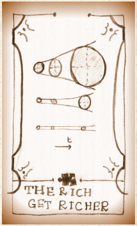

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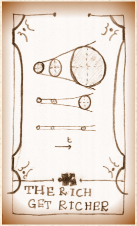

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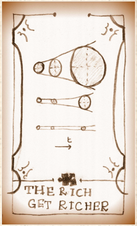

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