# Chaotic Contagion:
# The Idealized Hipster Effect

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## Principles of Complex Systems, Vols. 1 & 2
## CSYS/MATH 300 and 303, 2021–2022| @pocsvox

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Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont


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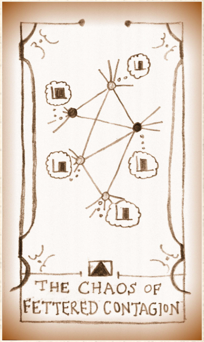
# Outline


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE END


THE CHAOS OF FETTERED CONTAGION

# Chaotic Contagion on Networks:



"Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability"
Dodds, Harris, and Danforth,
Phys. Rev. Lett., **110**, 158701, 2013. [1]

"Dynamical influence processes on networks: General theory and applications to social contagion"
Harris, Danforth, and Dodds,
Phys. Rev. E, **88**, 022816, 2013. [2]

A. Mandel, conference at Urbana-Champaign, 2007:

"If I was a younger man, I would have stolen this from you."

# Outline

# Chaotic contagion:



- What if individual response functions are not monotonic?

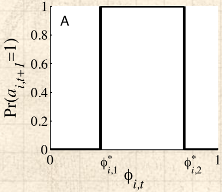
# Chaotic contagion:



- What if individual response functions are not monotonic?
- Consider a simple deterministic version:

- Node $i$ has an 'activation threshold' $\phi_{i,1}$

  ...and a 'de-activation threshold' $\phi_{i,2}$

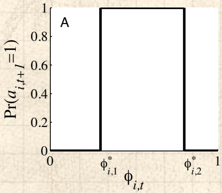


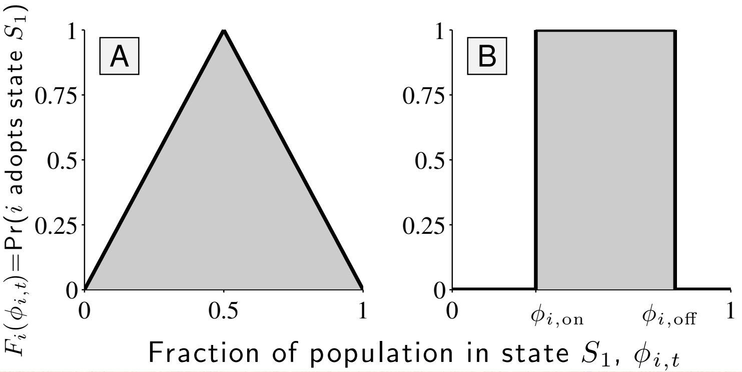
# Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:

- Node $i$ has an 'activation threshold' $\phi_{i,1}$

  ...and a 'de-activation threshold' $\phi_{i,2}$
- Nodes like to imitate but only up to a limit—they don't want to be like everyone else.




A
B
1
0.75
0.5
0.25
0
0
0.5
1
$\phi_{i,\mathrm{on}}$
$\phi_{i,\mathrm{off}}$
$F_i(\phi_{i,t})$=Pr($i$ adopts state $S_1$)
Fraction of population in state $S_1$, $\phi_{i,t}$

# Chaotic contagion



## Definition of the tent map:

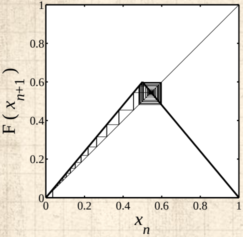
$$F(x) = \begin{cases} rx \text{ for } 0 \le x \le \frac{1}{2}, \\ r(1-x) \text{ for } \frac{1}{2} \le x \le 1. \end{cases}$$

- The usual business: look at how $F$ iteratively maps the unit interval $[0, 1]$.

# The tent map



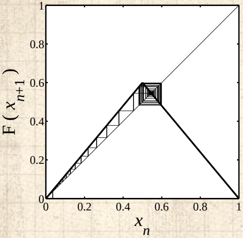
Effect of increasing $r$ from 1 to 2.

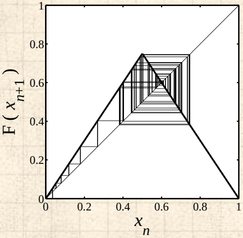


# The tent map



Effect of increasing $r$ from 1 to 2.

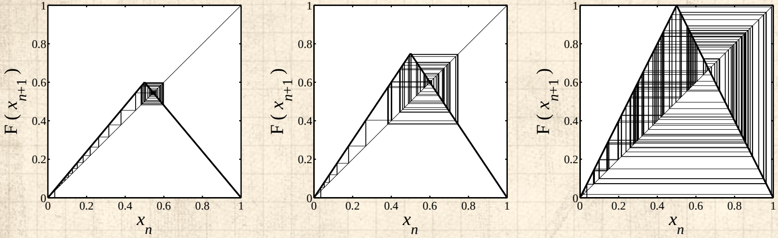


# The tent map



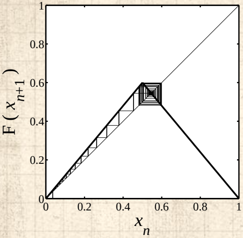
Effect of increasing $r$ from 1 to 2.

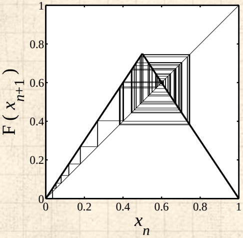


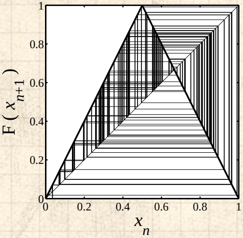
# The tent map

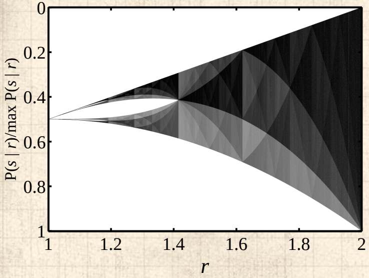


Effect of increasing $r$ from 1 to 2.



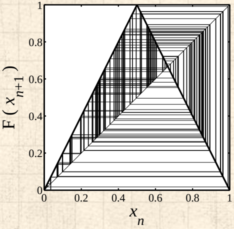




## Orbit diagram:

Chaotic behavior increases as map slope $r$ is increased.

# Chaotic behavior

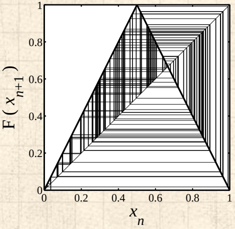
Take $r = 2$ case:



# Chaotic behavior



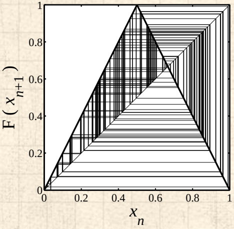
Take $r = 2$ case:




- What happens if nodes have limited information?

# Chaotic behavior

Take $r = 2$ case:

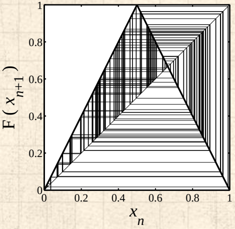



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.

# Chaotic behavior

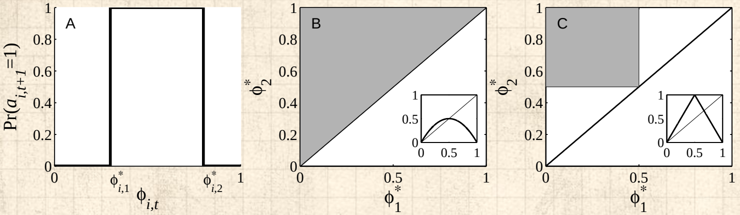


Take $r = 2$ case:



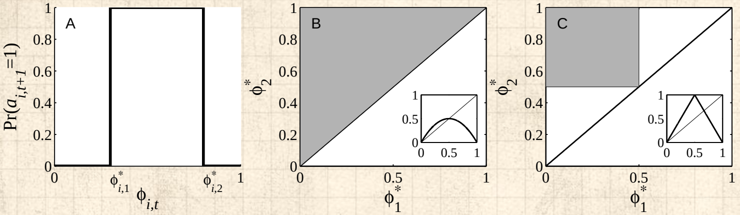

- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

# Two population examples:




- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.

# Two population examples:

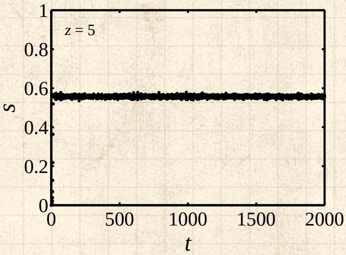



- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

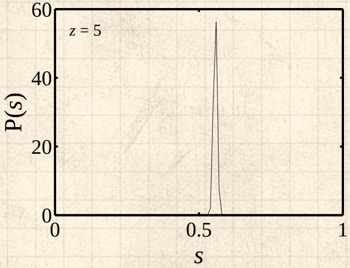
# Outline

# Invariant densities—stochastic response functions





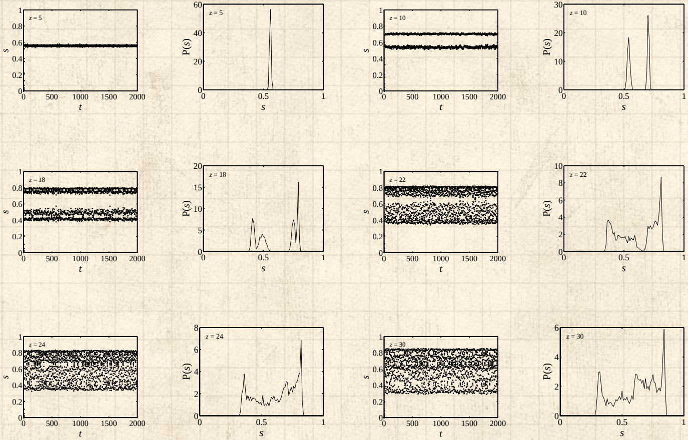

activation time series



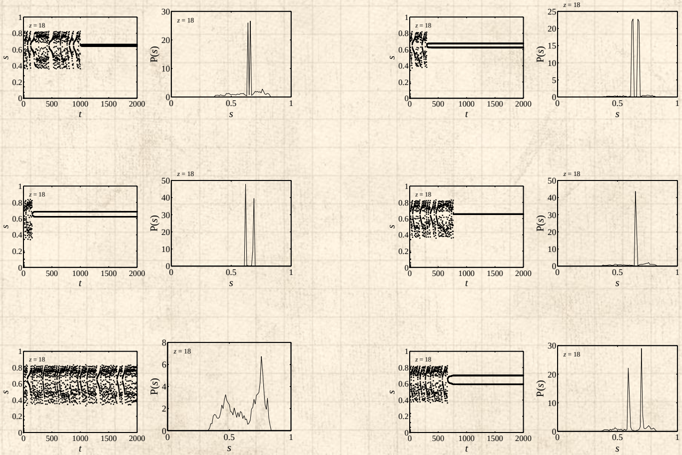

activation density

# Invariant densities—stochastic response functions

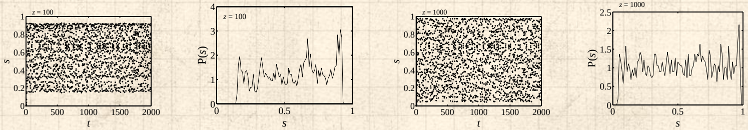




# Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



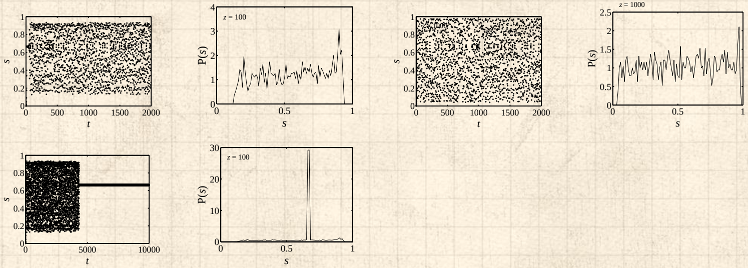
# Invariant densities—stochastic response functions




Trying out higher values of $\langle k \rangle$...

# Invariant densities—deterministic response functions

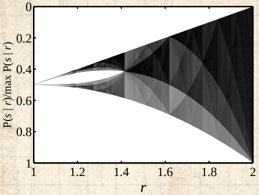


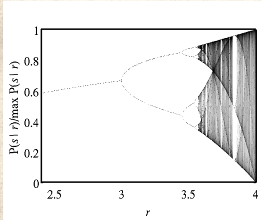



Trying out higher values of $\langle k \rangle$...

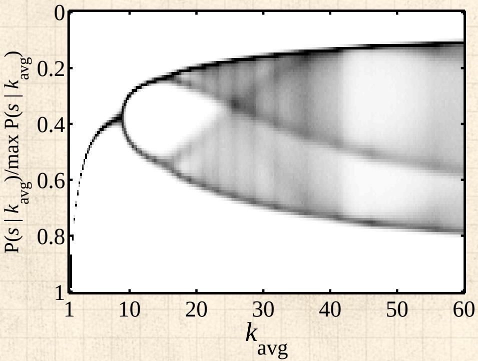
# Connectivity leads to chaos:





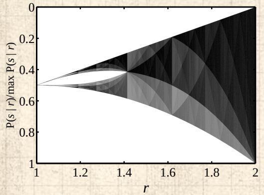



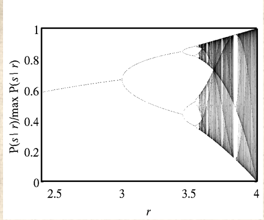

## Stochastic response functions:

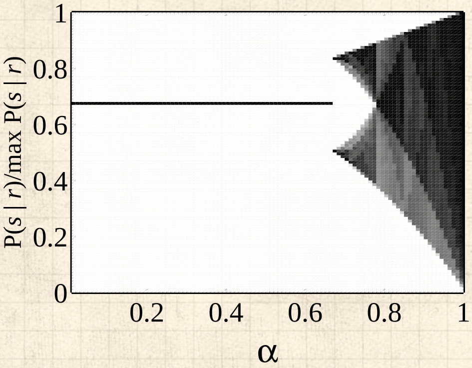


# Bifurcation diagram: Asynchronous updating





# Bifurcation diagram: Asynchronous updating



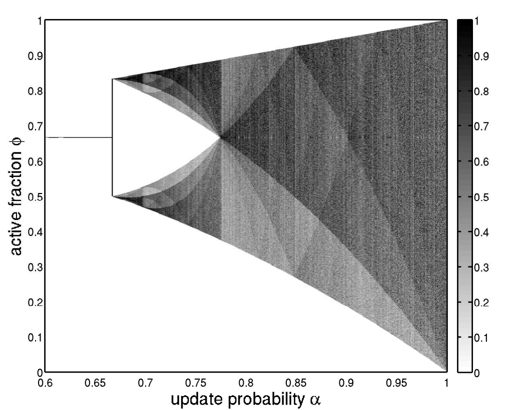



FIG. 3. Bifurcation diagram for the dense map $\Phi(\phi;\alpha)$, Eqn. (18). This was generated by iterating the map at 1000 $\alpha$ values between 0 and 1. The iteration was carried out with 3 random initial conditions for 10000 time steps each, discarding the first 1000. The $\phi$-axis contains 1000 bins and the invariant density, shown by the grayscale value, is normalized by the maximum for each $\alpha$. With $\alpha < 2/3$, all trajectories go to the fixed point at $\phi = 2/3$.

https://www.youtube.com/watch?v=7JHrZyyq870?rel=0

How the bifurcation diagram changes with increasing average degree $\langle k \rangle$ as a function of the synchronicity parameter $\alpha$ for the stochastic response (tent map) case.

https://www.youtube.com/watch?v=_zwK6poIBvc?rel=0

How the bifurcation diagram changes with increasing $\alpha$, the synchronicity parameter as a function of average degree $\langle k \rangle$ for the stochastic response (tent map) case.

https://www.youtube.com/watch?v=3bo4fzp4Snw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 6, update synchronicity parameter $\alpha$ = 1. The macroscopic behavior is period-1, plus noisy fluctuations.

https://www.youtube.com/watch?v=7UCuIa_ktmw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 11, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is period-2, plus noisy fluctuations.

https://www.youtube.com/watch?v=oWKt8Zj1Ccw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. $\langle k \rangle = 30$, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is chaotic.

https://www.youtube.com/watch?v=AfhUlkIOiOU?rel=0

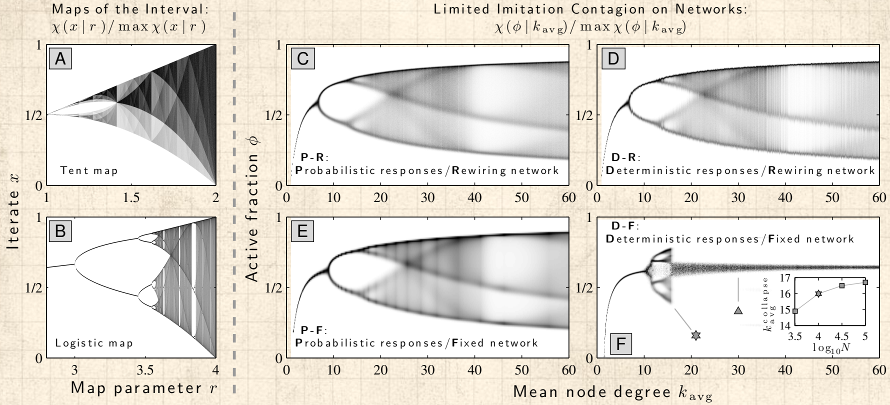
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter $\alpha$ = 1. Shown are nodes which continue changing (703/1000) after the transient chaotic behavior has "collapsed."

https://www.youtube.com/watch?v=ZwY0hTstJ2M?rel=0

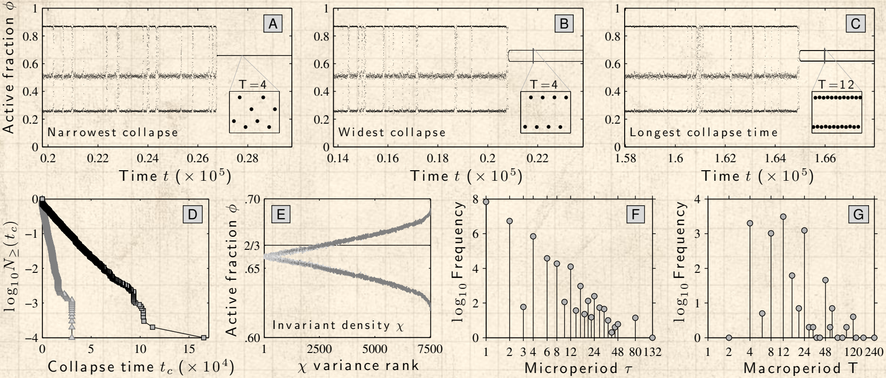
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter $\alpha$ = 1. The dynamics exhibit transient chaotic behavior before collapsing to a fixed point.

https://www.youtube.com/watch?v=YDhjmFyBSn4?rel=0

LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 17, update synchronicity parameter $\alpha$ = 1. The dynamics exhibit transient chaotic behavior before collapsing to a period-4 orbit.



Maps of the Interval:
$\chi(x \mid r)/\max\chi(x \mid r)$
Limited Imitation Contagion on Networks:
$\chi(\phi \mid k_{\rm avg})/\max\chi(\phi \mid k_{\rm avg})$
A
Tent map
B
Logistic map
Iterate $x$
Map parameter $r$
C
P-R:
Probabilistic responses/Rewiring network
D
D-R:
Deterministic responses/Rewiring network
E
P-F:
Probabilistic responses/Fixed network
F
D-F:
Deterministic responses/Fixed network
$k_{\rm avg}^{\rm collapse}$
$\log_{10}N$
Active fraction $\phi$
Mean node degree $k_{\rm avg}$


A
B
C
D
E
F
G
Narrowest collapse
Widest collapse
Longest collapse time
T = 4
T = 4
T = 12
Active fraction $\phi$
Time $t$ ($\times 10^5$)
$\log_{10} N_{\geq}(t_c)$
Collapse time $t_c$ ($\times 10^4$)
Invariant density $\chi$
$\chi$ variance rank
$\log_{10}$ Frequency
Microperiod $\tau$
Macroperiod T

# References I

[1] P. S. Dodds, K. D. Harris, and C. M. Danforth.
Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability.
Phys. Rev. Lett., 110:158701, 2013. pdf

[2] K. D. Harris, C. M. Danforth, and P. S. Dodds.
Dynamical influence processes on networks: General theory and applications to social contagion.
Phys. Rev. E, 88:022816, 2013. pdf