P o C S What's The Story?

Principles of Complex Systems, Vols. 1 & 2, CSYS/MATH 300 and 303
University of Vermont, Fall 2021
Assignment 13
code name: Séance and Sensibility

Due: Friday, December 3, by 11:59 pm, 2021.
Relevant clips, episodes, and slides are listed on the assignment's page:
https://pdodds.w3.uvm.edu//teaching/courses/2021-2022principles-of-complex-systems//assignments/13/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Assistant Deliverator: Michael Arnold (contact through Teams)
Office: The Ether
Office hours: TBD
Course website:
https://pdodds.w3.uvm.edu//teaching/courses/2021-2022principles-of-complex-systems

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All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The Deliverator uses Matlab.

Graduate students are requested to use LaTeX (or related TeX variant). If you are new to LaTeX, please endeavor to submit at least n questions per assignment in LaTeX, where n is the assignment number.

Assignment submission: Via Blackboard.

Please submit your project's current draft in pdf format via Blackboard by the same time specified for this assignment. For teams, please list all team member names clearly at the start.

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1. (3 + 3 + 3 + 3 + 3)

   We take a look at the 80/20 rule, 1 per centers, and similar concepts.

   Take x to be the wealth held by an individual in a population of n people, and the number of individuals with wealth between x and $x + \mathrm{d}x$ to be approximately $N(x)\mathrm{d}x$.

   Given a power-law size frequency distribution $N(x) = cx^{-\gamma}$ where $x_{\min} \ll x \ll \infty$, determine the value of γ for which the so-called 80/20 rule holds.

In other words, find γ for which the bottom 4/5 of the population holds 1/5 of the overall wealth, and the top 1/5 holds the remaining 4/5.

Note that inherent in our construction of the wealth frequency distribution is that the population is ordered by increasing wealth.

Assume the mean is finite, i.e., $\gamma > 2$.

(a) Determine the total wealth W in the system given $\int_{x_{\min}}^{\infty} \mathrm{d}x N(x) = n$.

(b) Imagine that the bottom $100\,\theta_{\mathrm{pop}}$ percent of the population holds $100\,\theta_{\mathrm{wealth}}$ percent of the wealth.

Show γ depends on θ_{pop} and θ_{wealth} as

$$\gamma = 1 + \frac{\ln \frac{1}{(1-\theta_{\mathrm{pop}})}}{\ln \frac{1}{(1-\theta_{\mathrm{pop}})} - \ln \frac{1}{(1-\theta_{\mathrm{wealth}})}}. \tag{1}$$

(c) Given the above, is every pairing of θ_{pop} and θ_{wealth} possible?

(d) Find γ for the 80/20 requirement ($\theta_{\mathrm{pop}} = 4/5$ and $\theta_{\mathrm{wealth}} = 1/5$).

(e) For the "80/20" γ you find, determine the fraction of wealth θ_{wealth} that the bottom fraction θ_{pop} of the population possesses as a function of θ_{pop} and plot the result.

2. Show that the Gini coefficient G for our idealized power-law size distribution of wealth is:

$$G = \begin{cases} 1 & \text{if } 1 < \gamma \leq 2, \\ \frac{1}{1+2(\gamma-2)} & \text{if } \gamma > 2. \end{cases} \tag{2}$$

Having developed a sense of what values of γ mean, and because of the simplicity of the relationship between G and γ, we can convert a real-world wealth distribution's value of G to γ for the equivalent idealized power-law size distribution:

$$\begin{aligned} \gamma \leq 2 \quad & \text{if } G = 1, \\ \gamma = \tfrac{1}{2}\left(\tfrac{1}{G} + 3\right) \quad & \text{if } G < 1. \end{aligned} \tag{3}$$

For example, what does a Gini coefficient of 1/2 mean for an idealized power law?

Eq. 3 gives $\gamma = 5/2$, which we recognized as coming from the Bad Place of finite mean and infinite variance.