P o C S What's The Story?

Principles of Complex Systems, Vols. 1 and 2
CSYS/MATH 6701, 6713
University of Vermont, Fall 2025
"Not an easy situation indeed but handled with grace and aplomb."
Assignment 11

: Wolf Cola: A Public Relations Nightmare, S12E04
Episode links: IMDB, Fandom, TV Tropes.

Due: Monday, November 24
https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse/assignments/11/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

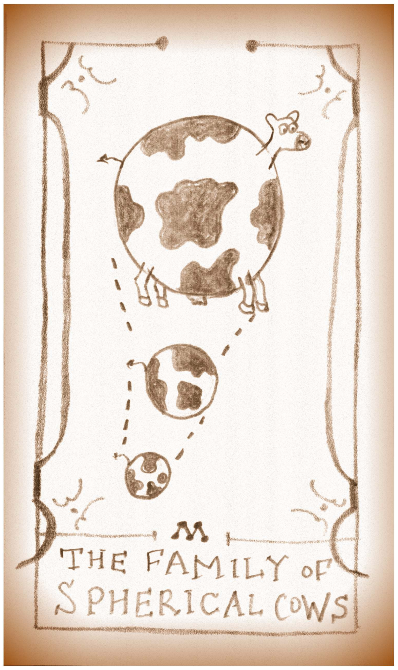
1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python. And it's going to be a no for the catachrestic Excel. Please do not use any kind of AI thing unless directed. The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

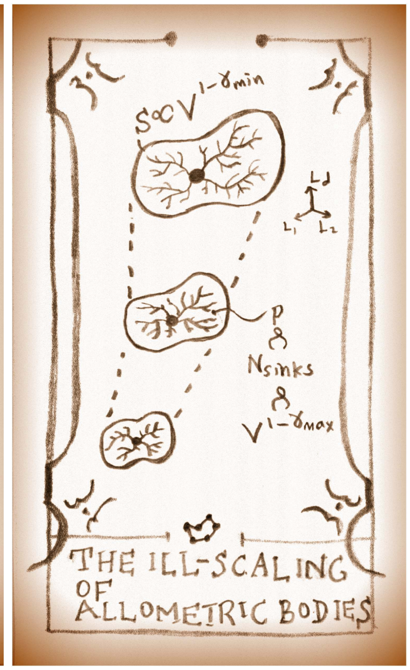
Assignment submission:

Via **Brightspace** (which is not to be confused with the death vortex of the same name, just a weird coincidence). Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace.

Relevant tarot cards, for your consideration:





1. Show that for an allometrically growing family of d dimensional objects in D dimensions, the volume of the minimal supply network $\min V_{\text{net}}$ scales with object volume V as:

$$\min V_{\text{net}} \propto \int_{\Omega_{d,D}(V)} \rho \, ||\vec{x}|| \, \mathrm{d}\vec{x} \sim \rho V^{1+\gamma_{\max}}.$$

   Ingredients:

   We defined $L_i = c_i^{-1} V^{\gamma_i}$ where $\gamma_1 + \gamma_2 + \ldots + \gamma_d = 1$,
   $\gamma_1 = \gamma_{\max} \geq \gamma_2 \geq \ldots \geq \gamma_d$., and $c = \prod_i c_i \leq 1$ is a shape factor.

   Assume the first k lengths scale in the same way with $\gamma_1 = \ldots = \gamma_k = \gamma_{\max}$, and write $||\vec{x}|| = (x_1^2 + x_2^2 + \ldots + x_d^2)^{1/2}$.

2. Surface area of allometrically growing ~~Love~~Minecraftian organisms:

   Let's consider animals as parallelepipeds (e.g., the well known box cow), with dimensions L_1, L_2, and L_3 and volume $V = L_1 \times L_2 \times L_3$.

   As for questions 1, let's assume length L_i scales with volume as $L_i = c_i^{-1} V^{\gamma_i}$ where the exponents satisfy $\gamma_1 + \gamma_2 + \gamma_3 = 1$ and the c_i are prefactors such that $c_1 \times c_2 \times c_3 = 1$. Let's again arrange our organisms so that $\gamma_1 \geq \gamma_2 \geq \gamma_3$.

(a) Show that the scalings $L_i = c_i^{-1} V^{\gamma_i}$ mean that indeed $L_1 \times L_2 \times L_3 = V$.

(b) Write down the γ_i corresponding to isometric scaling.

(c) Calculate the surface area S of our imaginary blockular beings for general allometric scaling of the sides.

(d) Show how S behaves as V becomes large (i.e., which term(s) dominate).

(e) Which sets of γ_i give the fastest and slowest possible scaling of S as a function of V?

3. For biological organisms in $D = d = 3$:

   Given the scalings for the minimal supply network and surface area you have determined above, what must follow?