PoCS What's The Story?

Principles of Complex Systems, Vols. 1 and 2
CSYS/MATH 6701, 6713
University of Vermont, Fall 2025
"Not one dry heave, nothing!"
Assignment 10

: The Gang Broke Dee, S9E01
Episode links: IMDB, Fandom, TV Tropes.

Due: Friday, November 14, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse/assignments/10/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python. And it's going to be a no for the catachrestic Excel. Please do not use any kind of AI thing unless directed. The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via **Brightspace** (which is not to be confused with the death vortex of the same name, just a weird coincidence). Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace.

This week, please focus on your projects.

This is a do-not-do assignment!

The assignment is included for some degree of completeness.

Please just read through to put the questions into your mind.

The geographies of fairness, greed, justice, fun, belief, ...:

1. For a uniformly distributed population, to minimize the average distance between individuals and their nearest facility, we've made a claim that facilities would be placed at the centres of the tiles on a hexagonal lattice (or the vertices of a triangular lattice). Why is this?

2. In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent ([1]; also see Supply Networks lecture notes).

   Repeat this argument for the d-dimensional case and find the general form of the exponent μ in $D \propto \rho^{\mu}$.

3. Following Um et al.'s approach [2], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the cost function:

   $$c_i = n_i \langle r_i \rangle^{\beta} \text{ with } 0 \leq \beta \leq 1,$$

   where, respectively, n_i and $\langle r_i \rangle$ are population and the average 'source to sink' distance for the population of the ith Voronoi cell (which surrounds the ith facility).

   Note that $\beta = 0$ corresponds to purely commercial facilities, and $\beta = 1$ to strongly social ones.

References

[1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. Phys. Rev. E, 74:016117, 2006. pdf

[2] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. Proc. Natl. Acad. Sci., 106:14236–14240, 2009. pdf