PoCS What's The Story?

Principles of Complex Systems, Vols. 1 and 2
CSYS/MATH 6701, 6713
University of Vermont, Fall 2025
"With all my heart."
Assignment 06

It's Always Sunny in Philadelphia: Flowers for Charlie, S9E08
Episode links: IMDB, Fandom, TV Tropes.

Due: Friday, October 10, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse/assignments/06/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2025-2026pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python. And it's going to be a no for the catachrestic Excel. Please do not use any kind of AI thing unless directed. The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via **Brightspace** (which is not to be confused with the death vortex of the same name, just a weird coincidence). Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace.

Special instruction: Pencil and paper only! None of this LaTeX madness. Please submit a PDF of your handwritten work.

1. (3 points)

   Everyday random walks and the Central Limit Theorem:

   Show that the observation that the number of discrete random walks of duration $t = 2n$ starting at $x_0 = 0$ and ending at displacement $x_{2n} = 2k$ where $k \in \{0, \pm 1, \pm 2, \ldots, \pm n\}$ is

   $$N(0, 2k, 2n) = \binom{2n}{n+k} = \binom{2n}{n-k}$$

   leads to a Gaussian distribution for large $t = 2n$:

   $$\mathbf{Pr}(x_t \equiv x) \simeq \frac{1}{\sqrt{2\pi t}} e^{-\frac{x^2}{2t}}.$$

   Please note that $k \ll n$.

   Stirling's sterling approximation will prove most helpful.

   Hint: You should be able to reach this form:

   $$\frac{\text{Some stuff not involving spotted quokkas}}{\text{Some other quokka-free stuff} \times (1 - k^2/n^2)^{n+1/2}(1 + k/n)^k (1 - k/n)^{-k}}.$$

   Lots of sneakiness here. You'll want to examine the natural log of the piece shown above, and see how it behaves for large n.

   You may very well need to use the Taylor expansion $\ln(1 + z) \simeq z$.

   Exponentiate and carry on.

   Tip: If at any point quokkas appear in your expression, you're in real trouble. Get some fresh air and start again.

2. (3 points)

   From lectures, show that the number of distinct 1-d random walk that start at $x = i$ and end at $x = j$ after t time steps is

   $$N(i, j, t) = \binom{t}{(t + j - i)/2}.$$

   Assume that j is reachable from i after t time steps.

   Hint—Counting random walks:
   http://www.youtube.com/watch?v=daSIYz-0U3E

3. (3 + 3)

   Discrete random walks:

   In class, we argued that the number of random walks returning to the origin for the first time after $2n$ time steps is given by

$$N_{\mathrm{fr}}(2n) = 2 \times N_{\mathrm{fr}}^{+}(2n) = 2\left(N(1,1,2n-2) - N(-1,1,2n-2)\right)$$

   where

$$N(i,j,t) = \binom{t}{(t+j-i)/2}.$$

   Find the leading order term for $N_{\mathrm{fr}}(2n)$ as $n \to \infty$.

   Two-step approach:

   (a) Combine the terms to form a single fraction,
   (b) and then again use Stirling's approximation.

   If you enjoy this sort of thing, you may like to explore the same problem for random walks in higher dimensions. Seek out George Pólya.

   And we are connecting to much other good stuff in combinatorics; more to come in the solutions.