

# Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number University of Vermont, Fall 2023 Assignment 04

Due: Sunday, October 1, by 11:59 pm

https://pdodds.w3.uvm.edu/teaching/courses/2023-2024pocsverse/assignments/04/

Some useful reminders:

**Deliverator:** Prof. Peter Sheridan Dodds (contact through Teams) **Assistant Deliverator:** Chris O'Neil (contact through Teams)

Office: The Ether

Office hours: See Teams calendar

Course website: https://pdodds.w3.uvm.edu/teaching/courses/2023-2024pocsverse

Overleaf: LaTeX templates and settings for all assignments are available at

https://www.overleaf.com/read/tsxfwwmwdgxj.

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you <del>conspired</del> collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The (evil) Deliverator uses (evil) Matlab.

Graduate students are requested to use  $\LaTeX$  (or related TEX variant). If you are new to  $\LaTeX$ , please endeavor to submit at least n questions per assignment in  $\LaTeX$ , where n is the assignment number.

### **Assignment submission:**

Via Brightspace or other preferred death vortex.

For Q1–5, you'll further explore the Google data set you examined earlier.

Q6 prepares for allotaxonometry.

Q7 is another piece for understanding heavy-tailed distributions.

- 1. Plot the complementary cumulative distribution function (CCDF).
- 2. Using standard linear regression, measure the exponent  $\gamma-1$  where  $\gamma$  is the exponent of the underlying distribution function. Identify and use a range of frequencies for which scaling appears consistent. Report the 95% confidence interval for your estimate.

You will find two scaling regimes—please examine them both.

3. Size-rank plots:

Using the alternate data set providing the raw word frequencies, plot word frequency as a function of rank in the manner of Zipf.

**Hint:** you will not be able to plot all points (there are close to 14 million) so think about how to plot a subsample that still shows the full form.

4. Using standard linear regression, measure  $\alpha$ , Zipf's exponent. Report the 95% confidence interval for your estimate.

Again, you will find two regimes.

- 5. For each scaling regime, write down how  $\gamma$  and  $\alpha$  are related (per lectures) and check how this expression works for your estimates here.
- 6. (3 + 3) Baby name frequencies in the US:

Note: We will use this data set again in the next assignment.

- (a) Plot the Complementary Cumulative Frequency Distributions and Size-rank plots (Zipf's law) for the following:
  - i. Baby girl names in 1952.
  - ii. Baby boy names in 1952.
  - iii. Baby girl names in 2002.
  - iv. Baby boy names in 2002.

Note that you will have counts that will make the Zipf distribution easy to plot straight away.

From these counts, you will have to create the distributions  $N_k$  and  $N_{\geq k}$ .

(b) As you did for the Google data set, fit regression lines and report values of  $\gamma$  and the Zipf exponent  $\alpha$ .

BUT: Only fit lines if fitting lines make sense!

You may only have one region of scaling or zero.

We will revisit these distributions in following assignments.

## **Download:**

Data for 1880 through 2018:

http://pdodds.w3.uvm.edu/permanent-share/pocs-babynames.zip 🗹 (8.0M)

### Files:

For each year, Zipf distribution of counts are stored in: names-girlsYYYY.txt and names-boyYYYY.txt.

For normalization to estimate rates, total number of births per year: births\_per\_year.txt. For this question, you do not need to determine rates, and this file is included for completeness.

For privacy, names with less than 5 counts are excluded.

The rare are legion and, for baby names, hidden.

## **Notes:**

You should be able to re-use scripts from previous assignments.

Data is based on names registered through Social Security within the US.

### Source:

# Baby name dataset available here:

https://catalog.data.gov/dataset?tags=baby-names 2. Separate dataset for total births available here:

https://ssa.gov/oact/babynames/numberUSbirths.html 2.

7. More on the peculiar nature of distributions of power law tails:

Consider a set of N samples, randomly chosen according to the probability distribution  $P_k = ck^{-\gamma}$  where  $k = 1, 2, 3, \ldots$ 

Estimate  $\min k_{\max}$ , the approximate minimum of the largest sample in the system, finding how it depends on N.

**Hint:** we expect that on the order of 1 of the N samples to have a value of  $\min k_{\max}$  or greater.

## Hint—Some visual help on setting this problem up:

http://www.youtube.com/watch?v=4tqlEuXA7QQ

We are just touching on the deep world of extreme value theory.  $\square$  Feel free to explore.

## Notes:

• In a later assignment, we will test this scaling by (thoughtfully) sampling from power-law size distributions.