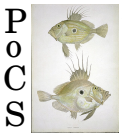

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number
University of Vermont, Fall 2024
"Baby, you've got a stew going."
Assignment 11

Tobias Funke, , Marta Complex, S1E12.
Episode links: Wikipedia, IMDB, Fandom, TV Tropes.

Due: Not due
https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse/assignments/11/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python, R, and/or Julia. **Please do not use any kind of AI thing.** The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via Brightspace (which is not to be confused with the death vortex of the same name).

Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace four days after the due date for this assignment (normally a Friday). For teams, please list all team member names clearly at the start.

This is a do-not-do assignment!

The assignment is included for some degree of completeness.

Please just read through to put the questions into your mind.

Please focus on your projects for the rest of the semester (see Assignment 12).

1. For a uniformly distributed population, to minimize the average distance between individuals and their nearest facility, we've made a claim that facilities would be placed at the centres of the tiles on a hexagonal lattice (or the vertices of a triangular lattice). Why is this?

2. In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the $2/3$ exponent ([1]; also see Supply Networks lecture notes).

   Repeat this argument for the d-dimensional case and find the general form of the exponent μ in $D \propto \rho^{\mu}$.

3. Following Um et al.'s approach [2], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the cost function:

   $$c_i = n_i \langle r_i \rangle^{\beta} \text{ with } 0 \leq \beta \leq 1,$$

   where, respectively, n_i and $\langle r_i \rangle$ are population and the average 'source to sink' distance for the population of the ith Voronoi cell (which surrounds the ith facility).

   Note that $\beta = 0$ corresponds to purely commercial facilities, and $\beta = 1$ to strongly social ones.

References

[1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. Phys. Rev. E, 74:016117, 2006. pdf

[2] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. Proc. Natl. Acad. Sci., 106:14236–14240, 2009. pdf