PoCS What's The Story?

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number
University of Vermont, Fall 2024
"That was a freebie"
Assignment 08

Lucille Bluth, Arrested Development, The Cabin Show, S3E01.
Episode links: Wikipedia, IMDB, Fandom, TV Tropes.

Due: Mondday, October 21, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse/assignments/08/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Office: The Ether and/or Innovation, fourth floor
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2024-2025pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at https://www.overleaf.com/read/tsxfwwmwdgxj.

Some guidelines:

1. Each student should submit their own assignment.
2. All parts are worth 3 points unless marked otherwise.
3. Please show all your work/workings/workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.
4. We recommend that you write up your assignments in LaTeX (using the Overleaf template). However, if you are new to LaTeX or it is all proving too much, you may submit handwritten versions. Whatever you do, please only submit single PDFs.
5. For coding, we recommend you improve your skills with Python, R, and/or Julia. **Please do not use any kind of AI thing.** The (evil) Deliverator uses (evil) Matlab.
6. There is no need to include your code but you can if you are feeling especially proud.

Assignment submission:

Via Brightspace (which is not to be confused with the death vortex of the same name).

Again: One PDF document per assignment only.

Please submit your project's current draft in pdf format via Brightspace four days after the due date for this assignment (normally a Friday). For teams, please list all team member names clearly at the start.

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1. The 1-d theoretical percolation problem:

   Consider an infinite 1-d lattice forest with a tree present at any site with probability p.

   (a) Find the distribution of forest sizes as a function of p. Do this by moving along the 1-d world and figuring out the probability that any forest you enter will extend for a total length ℓ.

   (b) Find p_c, the critical probability for which a giant component exists.

   Hint: One way to find critical points is to determine when certain average quantities explode. Compute $\langle l \rangle$ and find p such that this expression goes boom (if it does).

2. Show analytically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.

   Hint—Real-space renormalization gets it done.:
   http://www.youtube.com/watch?v=JlkbU5U7QqU

3. (3 + 3)

   Coding, it's what's for breakfast:

   (a) Percolation in two dimensions (2-d) on a simple square lattice provides a classic, nutritious example of a phase transition.

   Your mission, whether or not you choose to accept it, is to code up and analyse the L by L square lattice percolation model for varying L.

   Take L = 20, 50, 100, 200, 500, and 1000.

   (Go higher if you feel L = 1000 is for mere mortals.)

   (Go lower if your code explodes.)

   Let's continue with the tree obsession. A site has a tree with probability p, and a sheep grazing on what's left of a tree with probability $1 - p$.

   Forests are defined as any connected component of trees bordered by sheep, where connections are possible with a site's four nearest neighbors on a lattice.

The overall square lattice is to be considered as a landscape on which forests and sheep co-exist.

Do not bagelize (or doughnutize) the landscape (no periodic boundary conditions—boundaries are boundaries).

Note: the construction here up is called site percolation. Bond percolation is the alternate case when all links between neighboring sites exist with probability p.

Steps:

i. For each L, run N_{tests}=100 tests for occupation probability p moving from 0 to 1 in increments of 10^{-2}. (As for L, you may use a smaller or larger increment depending on how things go.)

ii. Determine the fractional size of the largest connected forest for each of the N_{tests}, and find the average of these, S_{avg}.

iii. On a single figure, for each L, plot the average S_{avg} as a function of p.

(b) Comment on how $S_{\text{avg}}(p; N)$ changes as a function of L and estimate the critical probability p_c (the percolation threshold).

For the few Matlabbers, a helpful reuse of code (intended for black and white image analysis): You can use Matlab's bwconncomp to find the sizes of components. Very nice.

4. (3 + 3)

(a) Using your model from the previous question and your estimate of p_c, plot the distribution of forest sizes (meaning cluster sizes) for $p \simeq p_c$ for the largest L your code and psychological makeup can withstand. (You can average the distribution over separate simulations.)

Comment on what kind of distribution you find.

(b) Repeat the above for $p = p_c/2$ and $p = p_c + (1 - p_c)/2$, i.e., well below and well above p_c.

Produce plots for both cases, and again, comment on what you find.

5. Present a plan for your project in a short video.[1]

Details:

You will collectively generate a mini-conference of short talks, with each PoCS team member presenting for two to three minutes (max 3:00, 180 seconds).

[1] Please do not request to make a diorama instead of a video. Because of the [redacted] in Professor [redacted]'s 2017 course on [redacted] [redacted], dioramas are now strictly forbidden.

Here's what you need to know and do:

First: If you have not already done so, please firm up your team and project topic.

Per earlier guidance: Feel free to talk in Teams about possible projects. Pitch your idea and recruit people to your team.

Teams of 2 to 3 are strongly encouraged (4 is maybe too many, 1 is totally okay, 5 is right out).

Below are instructions for the talks and how to submit a video of your talk to Microsoft Streams along with your slides within our Teams environment.

There should be one video and one set of slides per team.

Videos will be private to the course.

These talks always prove to be interesting, diverse, and fun.

Okay, here's the plan for these first talks:

(a) Talks will be capped at 3:00 minutes per person.

(b) Your mission, which you have accepted, is to:

    i. Clearly state the problem/area you're going to investigate;

    ii. Why it's interesting;
    and

    iii. What you plan to do for the remainder of the semester.

    iv. Please also quickly introduce yourself at the start (name + your field).

(c) Talks should absolutely be PG and respectful of others.

(d) If you are part of a group, you will need to speak for 3:00 minutes each. Please coordinate your talk with your fellow group members.

(e) Talks that are longer than $n \times$3:00 will be removed and you will be asked to resubmit.

(f) Slides: Mandatory. The number should be 1 to 3 per speaker. More can work but certainly not, say, 20, unless flipping through them rapidly helps with your presentation. Your assessment will in part be based on your slides.

(g) Practice before recording! These are short talks so you can run through them a number of times to straighten everything out.

(h) **Please submit all three artifacts of (a) slides, (b) talk video, and (c) the current draft of your project to both this week's project submission box on Brightspace and in the Teams Team for PoCS.**

    The channel's name will be non-cryptically named with something like "Please post your project videos here!"

(i) All students will be requested to watch all talks. Providing helpful comments and feedback via Teams is encouraged.

(j) If you have not done so, please set up your project in Overleaf.