Contagion

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Principles of Complex Systems, Vols. 1, 2, & 3D CSYS/MATH 6701, 6713, & a pretend number, 2023–2024| @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont

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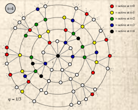
What's the Story?



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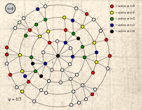

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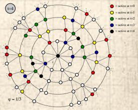

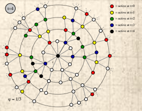

Outline


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THE UNEXPECTED TRANSFIGURATION
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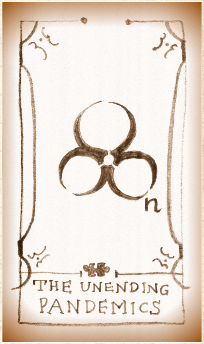
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THE UNENDING PANDEMICS


The Sun
The Golden Age of Reductionism
The Manifesto
The Well of Scales
The Laws of Decaying Power
The Unexpected Transfiguration
Ministry of Random Walks
The Rich Get Richer
The Dream of Efficient Language
The Law of Digits First
The Surprise of Being Robust Yet Fragile
The Misreading of the Book of Sand
The Many Tales of Many Tails
The Measuring
The Data Poor
The Pouring Data
The Charms of the Many Wrong Paths
The Doors That Should Not Be Opened
The Knowledge of Dead Monks
The Emergence of Structure
The Emergence of Destruction
The Emergence of Thinking
The Emergence of Stories
The Mechanics Statistical
The Thresholds of Percolation
The Networks Complex
The Elements of Webs
The Birds of a Feather
The Birds of Not a Feather
The Ark of the Random Network
The Strangeness of Friends
The Smallness of Worlds
The Theory of Six Degrees
The Networks Free of Scale
The Contagion
The Disease of Random Mixing
The Unending Pandemics
The Thresholds of the Mind
The Social Wild
The Social Contagion
The Ratio of Gain
The Mechanics of Fame
The Talk of Fame
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The John Dory
The Adjacent Narratives
The Storytellers
The Fated Astral Path
The Paths of Universality
The Dependent Paths
The Trousers of Bifurcating Realities
The Unknown Mechanism
The Landscapes of Forking Paths
The Networks of Blood
The Trees of Reality
The Orders of Streams
The Laws of Branching
The Scaling of Broken Hierarchies
The Braid or the Branch
The Law of Optimal Forks
The Church of Quarterology
The Truthicide
The Family of Pancake Cows
The Family of Spherical Cows
The Improbable Pachyderm
The Ill-Scaling of Allometric Bodies
The Supply Most Fair
The Supply to the Most
The Dark Art of Generating Functions
The Random Sum of Randomness
The Two-Sided Worlds
The Stories and the Tropes
The Truth of Diffusion
The Unveiling of Hidden Powers
The Scrying for the Shapes of Things
The Limits of Organizations
The Overcoming of Catastrophe
The Troubles Ill-Formed
The Networks of Many Ilks
The Chaos of Fettered Contagion
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The Confusion of Contagions


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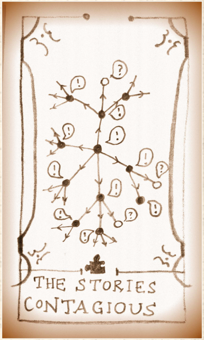

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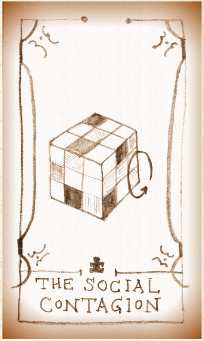
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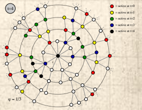
Contagion models

Some large questions concerning network contagion:



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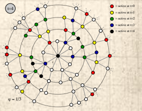

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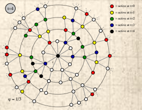

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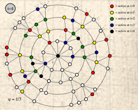

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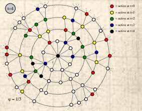

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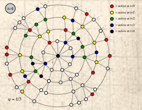

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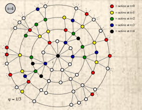
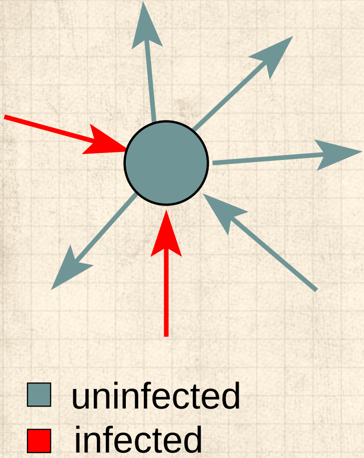
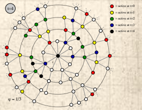
Next up: We'll look at some fundamental kinds of spreading on generalized random networks.



Spreading mechanisms



- General spreading mechanism:
  State of node i depends on history of i and i's neighbors' states.



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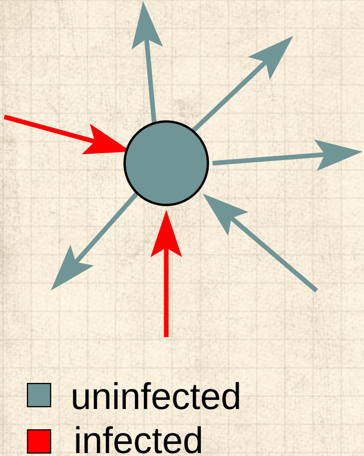
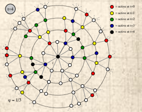


- **General spreading mechanism**: State of node i depends on history of i and i's neighbors' states.
- **Doses** of entity may be stochastic and history-dependent.



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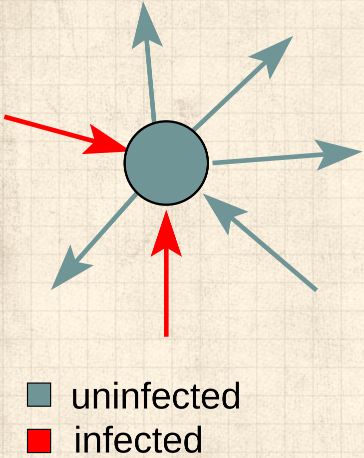
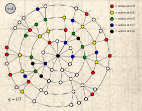
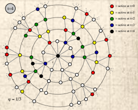


- General spreading mechanism:
  State of node i depends on history of i and i's neighbors' states.
- Doses of entity may be stochastic and history-dependent.
- May have multiple, interacting entities spreading at once.



Spreading on Random Networks

- For random networks, we know local structure is pure branching.



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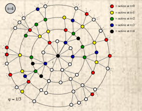
- Successful spreading is ∴ contingent on single edges infecting nodes.



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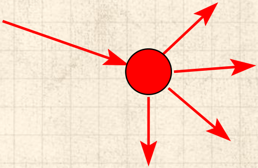
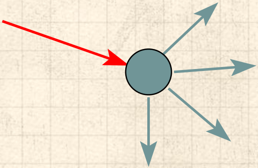
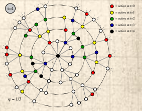


Failure:





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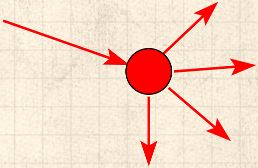

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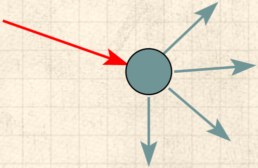

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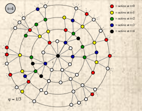

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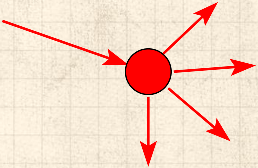

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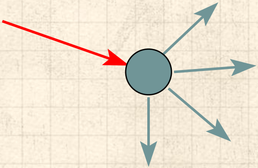
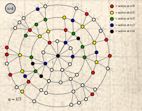
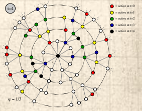


- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?



Global spreading condition

- We need to find: [5]
  R = the average # of infected edges that one random infected edge brings about.
- Call **R** the gain ratio.



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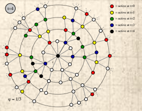

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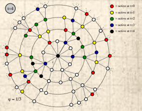

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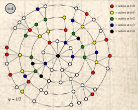

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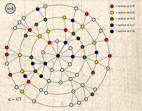

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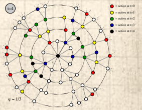

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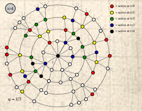

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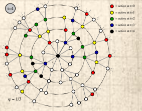

Global spreading condition

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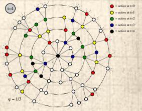
$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$



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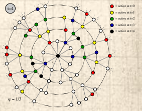

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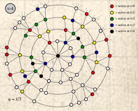

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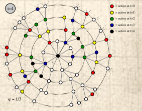

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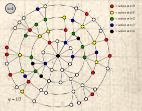
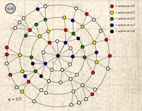
- Good: This is just our giant component condition again.



Global spreading condition

- Case 2:



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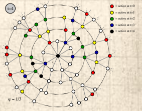
- Case 2: If $B_{k1} = \beta < 1$



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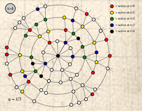

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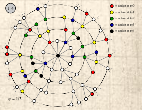

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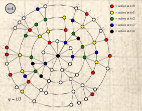

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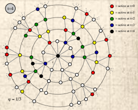

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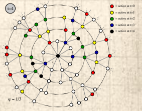
Insert assignment question



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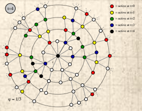
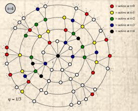
Insert assignment question

- We can show $F_{\tilde{P}}(x) = F_P(\beta x + 1 - \beta)$.



Global spreading condition

- **Cases 3, 4, 5, ...:**



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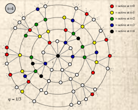

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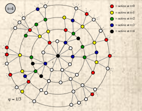

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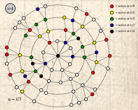

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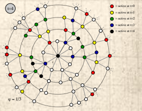

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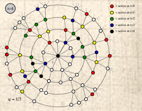

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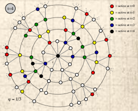

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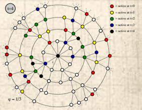

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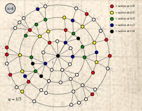

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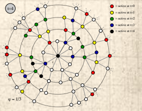
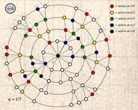
- The story:
  More well connected people are harder to influence.



Global spreading condition

- Example: $B_{k1} = 1/k$.



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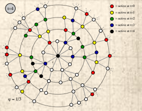

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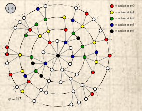

Global spreading condition

- Example: $B_{k1} = 1/k$.
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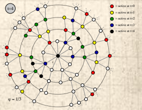

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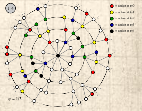

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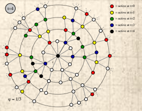

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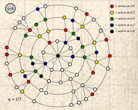

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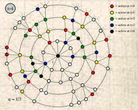

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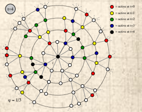
- Decay of B_{k1} is too fast.
- Result is independent of degree distribution.



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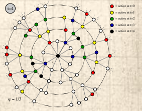

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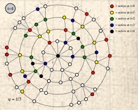

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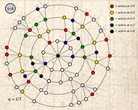

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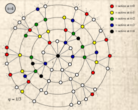

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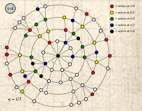

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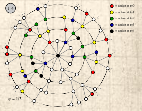
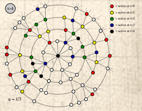
$$= \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} (k-1) \bullet \frac{kP_k}{\langle k \rangle}$$ where $\lfloor \cdot \rfloor$ means floor.



Global spreading condition

- The uniform threshold model global spreading condition:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} (k-1) \bullet \frac{kP_k}{\langle k \rangle} > 1.$$



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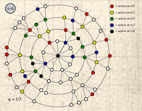

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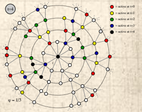

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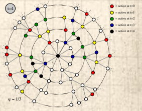

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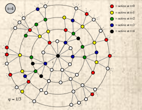
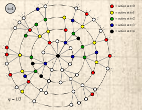
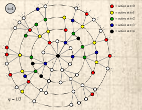
- Key: If we fix ϕ and then vary $\langle k \rangle$, we may see two phase transitions.
- Added to our standard giant component transition, we will see a cut off in spreading as nodes become more connected.



Virtual contagion: Corrupted Blood, a 2005 virtual plague in World of Warcraft:



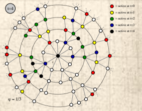
Outline



Social Contagion

Some important models (recap from CSYS 300)

- Tipping models—Schelling (1971) [11, 12, 13]



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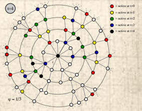

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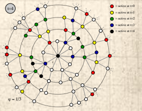

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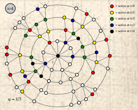

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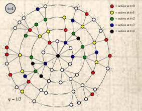

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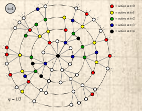
  - Social learning theory, Informational cascades,...



Threshold model on a network

Original work:


A simple model of global cascades on random networks

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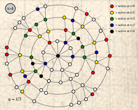
Duncan J. Watts,
Proc. Natl. Acad. Sci., **99**, 5766–5771, 2002. [15]



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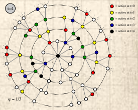
- Mean field Granovetter model $\rightarrow$ network model



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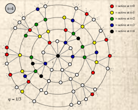
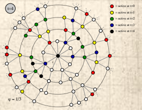
- Mean field Granovetter model $\rightarrow$ network model
- Individuals now have a limited view of the world



Threshold model on a network

- Interactions between individuals now represented by a network



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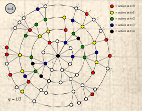

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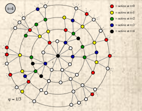

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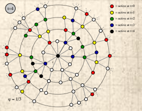

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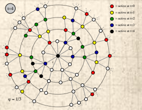

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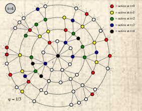

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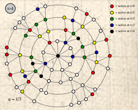

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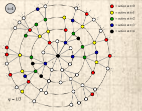

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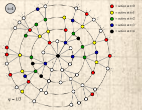

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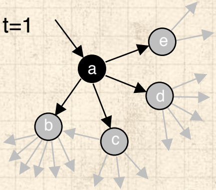

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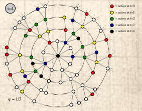

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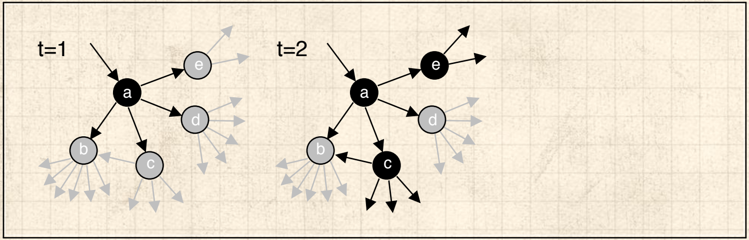

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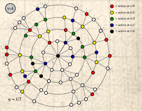

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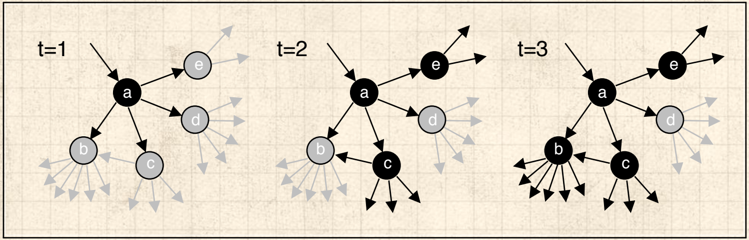

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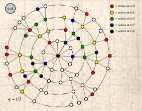
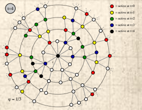


The most gullible

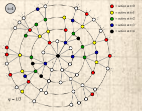
Vulnerables:



The most gullible

Vulnerables:

- Recall definition: individuals who can be activated by just one contact being active are vulnerables.



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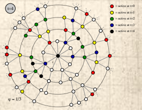
- Recall definition: individuals who can be activated by just one contact being active are vulnerables.
- The vulnerability condition for node i: $1/k_i \geq \phi_i$.



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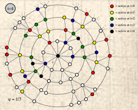

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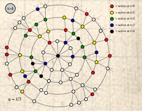
- Key: For global spreading events (cascades) on random networks, must have a *global component of vulnerables* [15]



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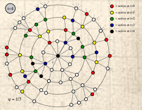
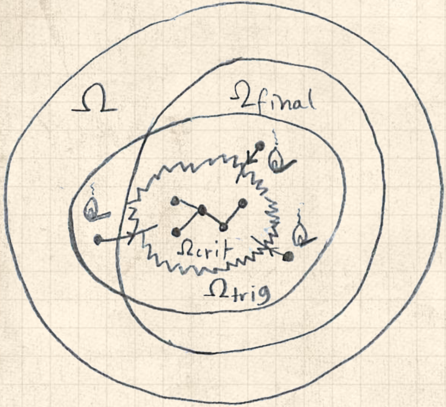
- Key: For global spreading events (cascades) on random networks, must have a *global component of vulnerables* [15]
- For a uniform threshold ϕ, our global spreading condition tells us when such a component exists:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} \frac{k P_k}{\langle k \rangle} \bullet (k-1) > 1.$$

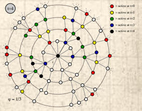


Example random network structure:

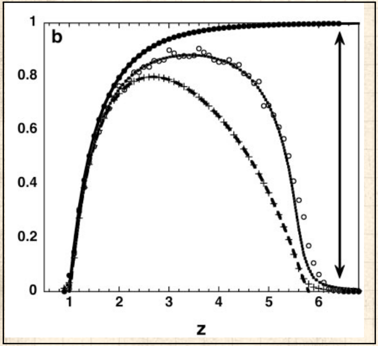


- Ω_{crit} = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

$\Omega_{\text{crit}} \subset \Omega_{\text{trig}}$; $\Omega_{\text{crit}} \subset \Omega_{\text{final}}$; and $\Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega$.

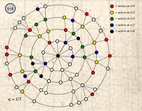


Global spreading events on random networks [15]



$z = \langle k \rangle$

- Top curve: final fraction infected if successful.



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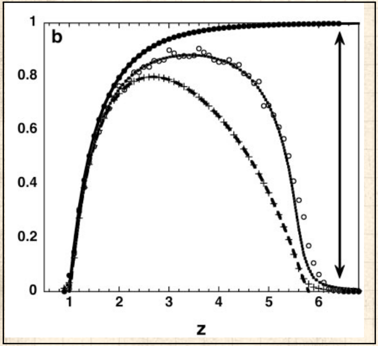

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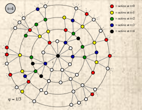
- Bottom curve: fractional size of vulnerable subcomponent. [15]



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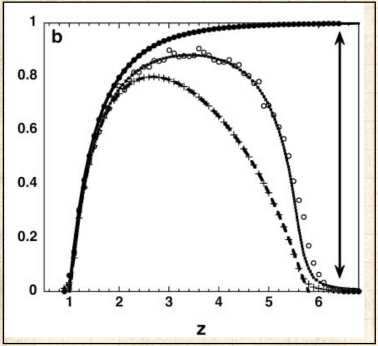

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- Middle curve: chance of starting a global spreading event (cascade).
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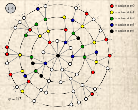

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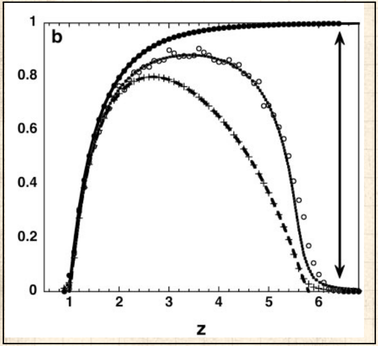

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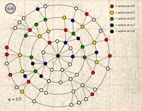
- Global spreading events occur only if size of vulnerable subcomponent > 0.



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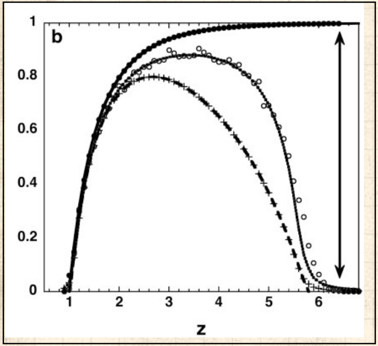

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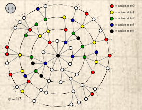
- System is robust-yet-fragile just below upper boundary [3, 4, 14]



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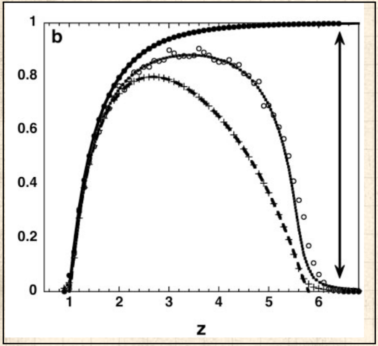

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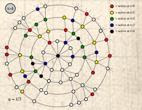
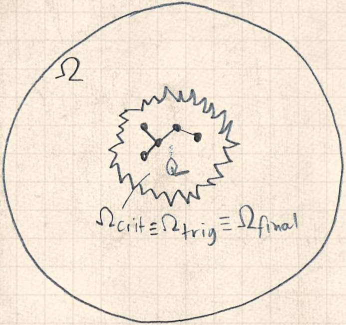
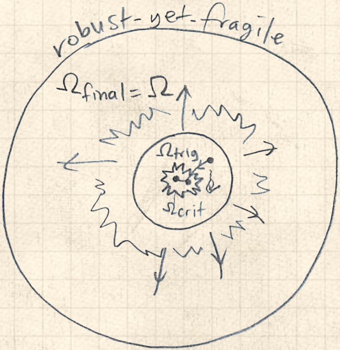
- 'Ignorance' facilitates spreading.

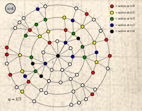


Cascades on random networks





- Above lower phase transition
- Just below upper phase transition



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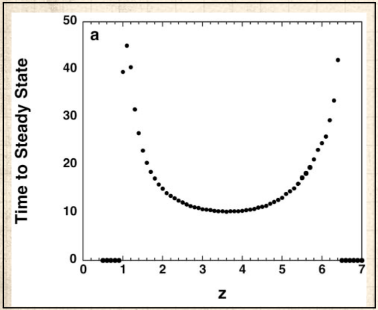
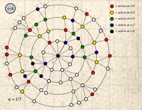


(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]



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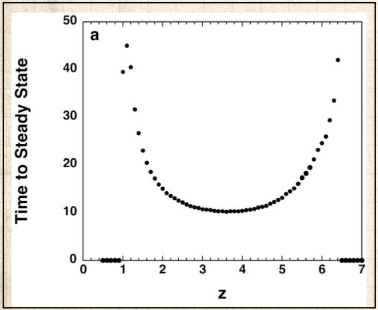
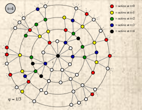
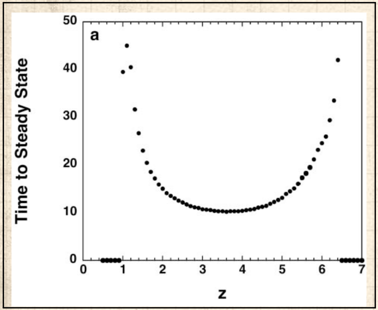


(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]
- Two phase transitions.



Cascades on random networks



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- Time taken for cascade to spread through network. [15]
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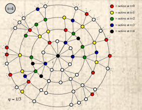
- Largest vulnerable component = critical mass.



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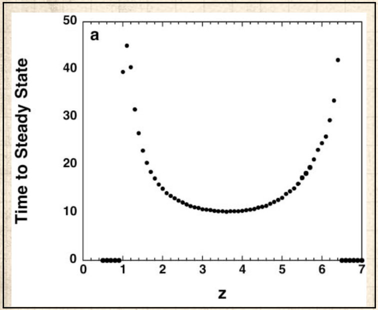
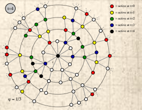


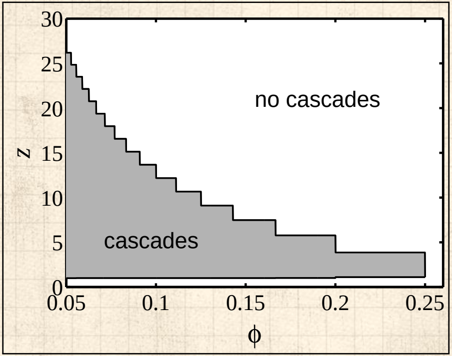
(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]
- Two phase transitions.

- Largest vulnerable component = critical mass.
- Now have endogenous mechanism for spreading from an individual to the critical mass and then beyond.

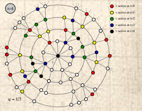


Cascade window for random networks



(n.b., $z = \langle k \rangle$)

- Outline of cascade window for random networks.



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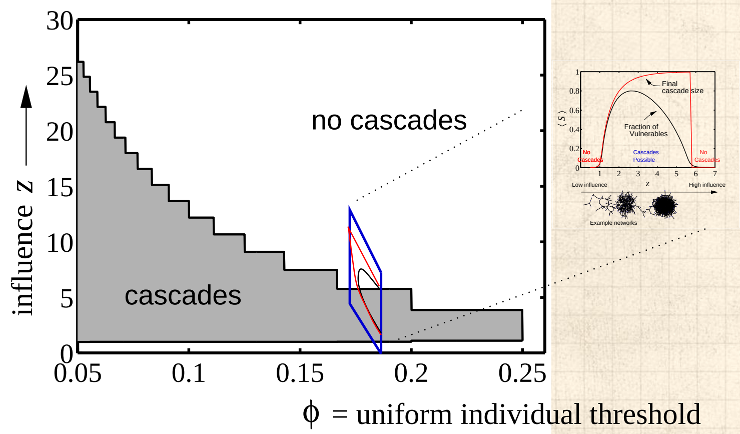
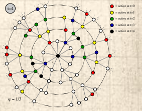
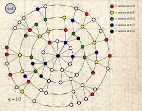
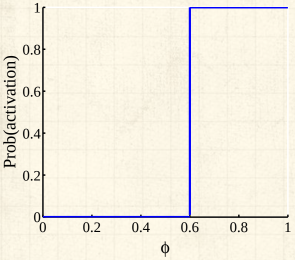




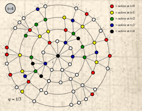
Outline



Granovetter's Threshold model—recap

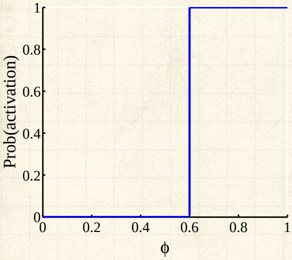


- Assumes deterministic response functions

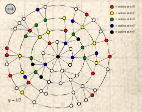


Social Contagion

Granovetter's Threshold model—recap



- Assumes deterministic response functions
- ϕ_* = threshold of an individual.



Social Contagion

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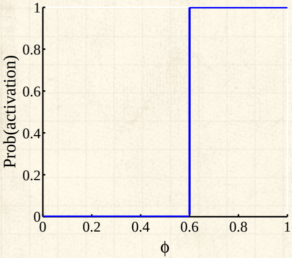

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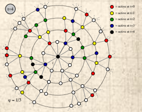
- ϕ_* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.



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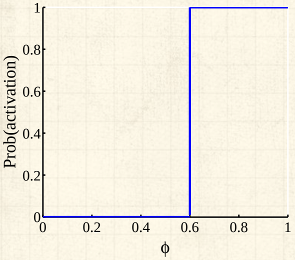
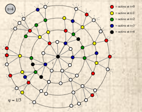


- Assumes deterministic response functions
- ϕ_* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$



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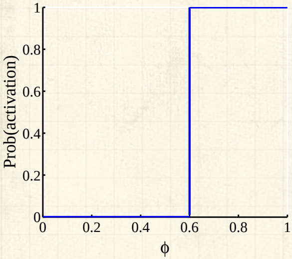

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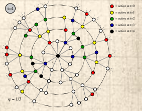
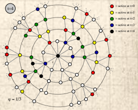
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.



Social Sciences—Threshold models

- At time $t+1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.



Social Sciences—Threshold models

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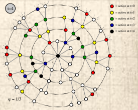
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*) \mathsf{d}\phi_* = F(\phi_*)\Big|_0^{\phi_t} = F(\phi_t)$$



Social Sciences—Threshold models

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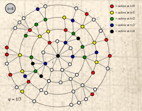
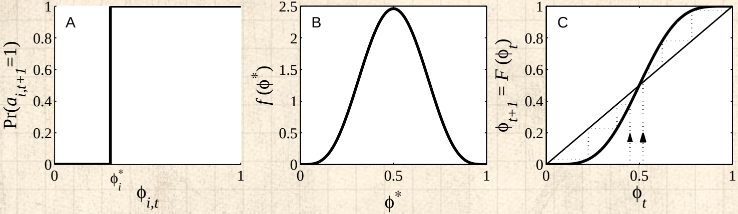
-
$$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)\mathrm{d}\phi_* = F(\phi_*)\big|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0,1]$.

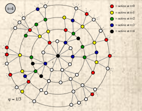


Social Sciences—Threshold models

Action based on perceived behavior of others.



- Two states: S and I
- Recover now possible (SIS)
- ϕ = fraction of contacts 'on' (e.g., rioting)



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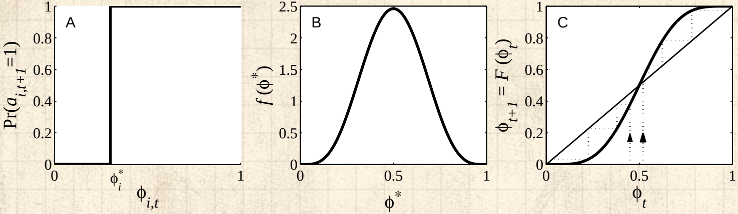

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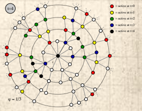
- Discrete time, synchronous update (strong assumption!)



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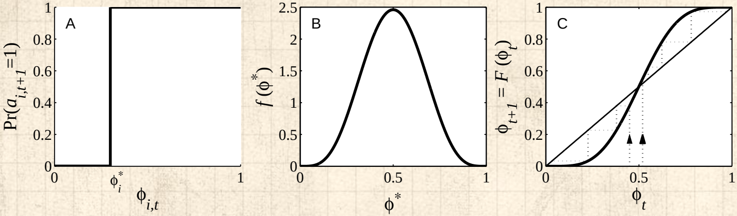
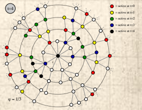
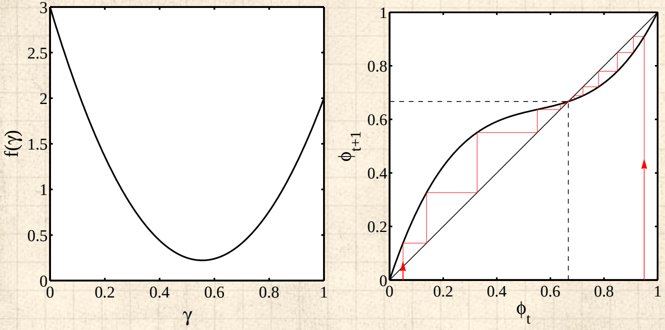


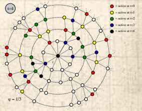
- Two states: S and I
- Recover now possible (SIS)
- ϕ = fraction of contacts ‘on’ (e.g., rioting)
- Discrete time, synchronous update (strong assumption!)
- This is a Critical mass model



Social Sciences—Threshold models

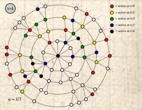


- Example of single stable state model



Social Sciences—Threshold models

Implications for collective action theory:



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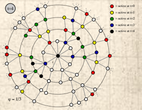
Implications for collective action theory:

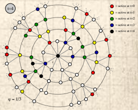
1. Collective uniformity $\not\Rightarrow$ individual uniformity



Social Sciences—Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes



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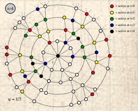

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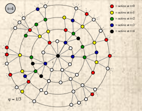
- Connect mean-field model to network model.



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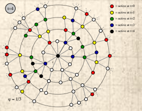

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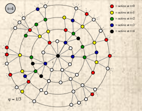
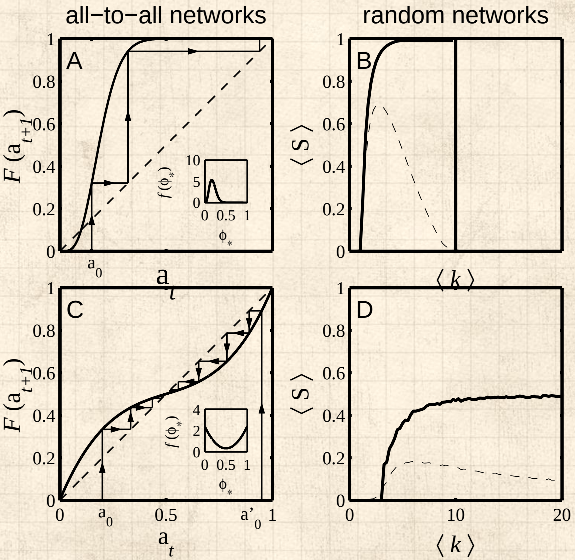
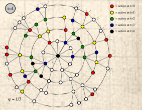
- Connect mean-field model to network model.
- Single seed for network model: $1/N \rightarrow 0$.
- Comparison between network and mean-field model sensible for vanishing seed size for the latter.



All-to-all versus random networks



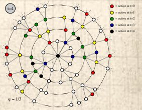


Spreadworthiness: Cat videos

Bowling with Ragdolls:

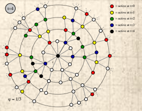
https://www.youtube.com/watch?v=XX-g2nmqL9Q?rel=0

- Organic extreme outlier?
- Success did not spread to other videos.



Threshold contagion on random networks

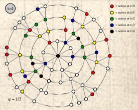
Three key pieces to describe analytically:



Threshold contagion on random networks

Three key pieces to describe analytically:

1. The fractional size of the largest subcomponent of vulnerable nodes, S_{vuln}.



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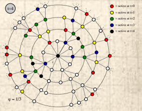

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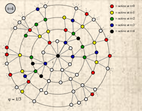

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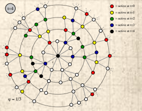
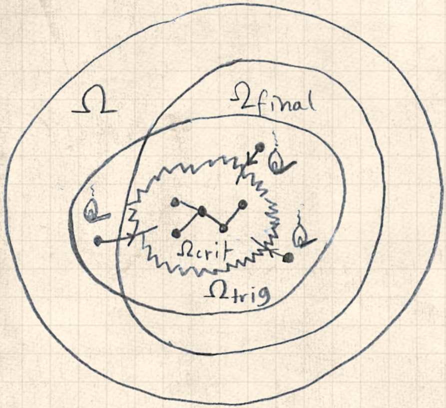
   - n.b., the distribution of S is almost always bimodal.

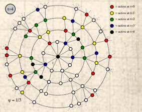


Example random network structure:

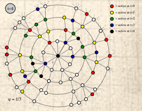


- $\Omega_{\text{crit}} = \Omega_{\text{vuln}}$ = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

$$\Omega_{\text{crit}} \subset \Omega_{\text{trig}};\ \Omega_{\text{crit}} \subset \Omega_{\text{final}};\ \text{and } \Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega.$$

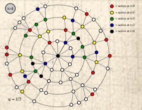


Outline



Threshold contagion on random networks

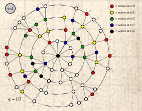
- **First goal:** Find the largest component of vulnerable nodes.



Threshold contagion on random networks

- First goal: Find the largest component of vulnerable nodes.
- Recall that for finding the giant component's size, we had to solve:

$$F_\pi(x) = xF_P\left(F_\rho(x)\right) \text{ and } F_\rho(x) = xF_R\left(F_\rho(x)\right)$$



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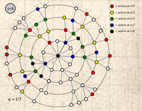

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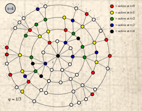
- This is a node-based percolation problem.



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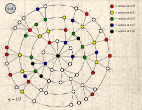
- This is a node-based percolation problem.
- For a general monotonic threshold distribution $f(\phi)$, a degree k node is vulnerable with probability

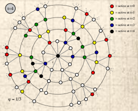
$$B_{k1} = \int_0^{1/k} f(\phi)\mathrm{d}\phi\,.$$



Threshold contagion on random networks

- We now have a generating function for the probability that a randomly chosen node is vulnerable and has degree k:

$$F_P^{(\mathrm{vuln})}(x) = \sum_{k=0}^{\infty} P_k B_{k1} x^k.$$



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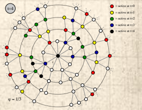

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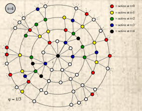

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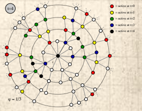

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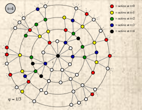
- Detail: We still have the underlying degree distribution involved in the denominator.



Threshold contagion on random networks

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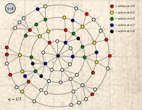

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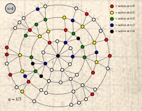

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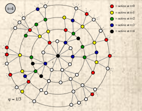

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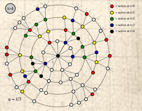

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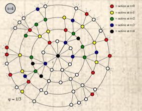

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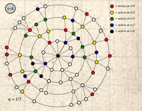
- Can now solve as before to find

$$S_{\text{vuln}} = 1 - F_{\pi}^{(\text{vuln})}(1).$$



Outline

Basic Contagion Models

Global spreading condition

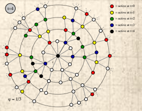
Social Contagion Models

- Network version
- All-to-all networks

Theory

- Spreading possibility
- **Spreading probability**
- Physical explanation
- Final size

References



Threshold contagion on random networks

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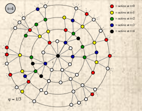

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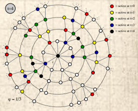

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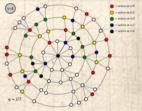

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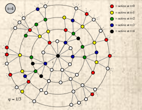
- Solve as before to find $P_{\text{trig}} = S_{\text{trig}} = 1 - F_\pi^{(\text{trig})}(1)$.



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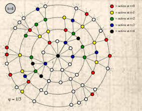
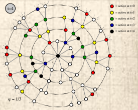


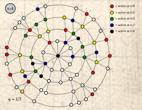
Physical derivation of possibility and probability of global spreading:

- Possibility: binary indicator of phase. Global spreading events are either possible or can never happen.



Physical derivation of possibility and probability of global spreading:

- Possibility: binary indicator of phase. Global spreading events are either possible or can never happen.
- For random networks, global spreading possibility is understood as meaning a giant component of vulnerable nodes exists.



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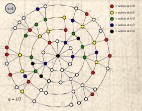

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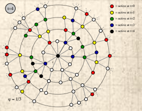

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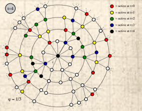

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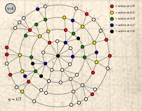

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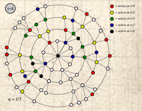
- Later: Generalize to more complex networks involving assortativity of all kinds.



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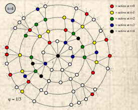

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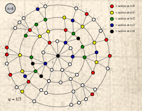

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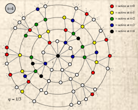

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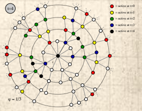

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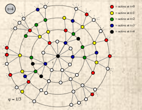

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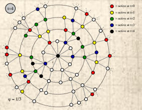
- Put everything together and solve for Q_{trig}:

$$Q_{\text{trig}} = \sum_k \frac{kP_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - (1 - Q_{\text{trig}})^{k-1}\right].$$



Good things about our equation for Q_{trig}:

$$Q_{\text{trig}} = \sum_k \frac{kP_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - (1 - Q_{\text{trig}})^{k-1}\right] = f(Q_{\text{trig}}; P_k, B_{k1})$$

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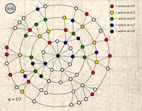

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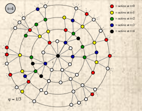

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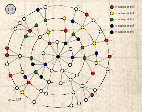

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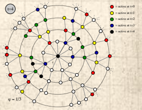

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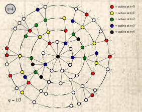

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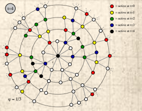
- Start with a suitably small seed $Q_{\text{trig}}^{(1)} > 0$ and iterate while rubbing hands together.



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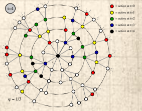

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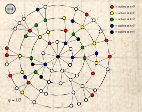

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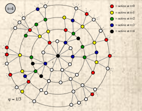

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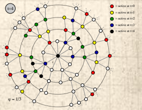

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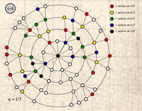

Connection to generating function results:

- We found that $F_{\rho}^{(\text{vuln})}(1)$—the probability that a random edge leads to a finite vulnerable component—satisfies

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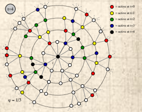

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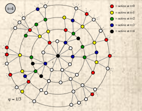

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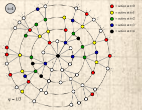

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- Some breathless algebra it all matches:

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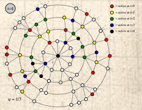

Fractional size of the largest vulnerable component:

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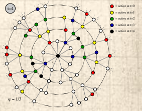

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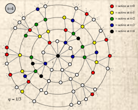

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- Excited scrabbling about gives us, as before:

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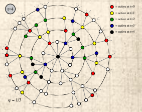

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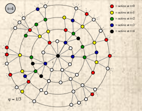

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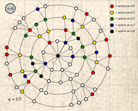

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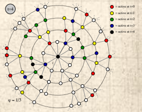

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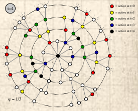
$$1 - S_{\text{trig}} = 1 + \sum_{k=0}^{\infty} P_k \left(1 - Q_{\text{trig}}\right)^k.$$

- More scruffing around brings happiness:

$$S_{\text{trig}} = \sum_{k=0}^{\infty} P_k \left[1 - \left(1 - Q_{\text{trig}}\right)^k\right].$$



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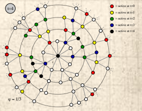

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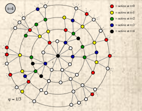

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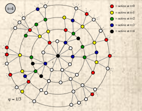

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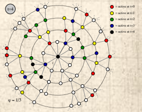

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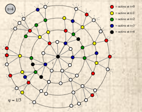

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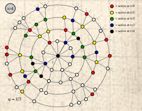
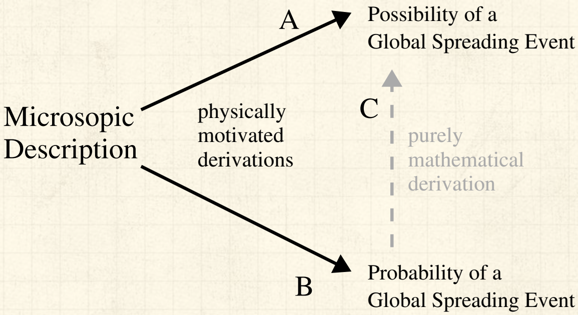
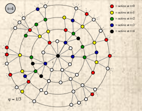
- We need to find out what happens as $Q_{\text{trig}} \to 0$. [9]



What we’re doing:





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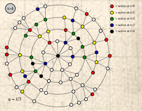

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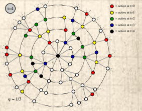

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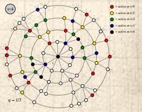

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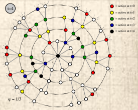

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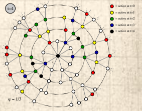

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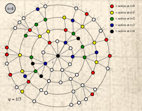
- Inequality?



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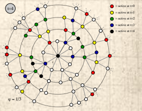
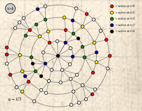
- From this connection, we don't know anything about a gain ratio **R** or how to arrange the pieces.

Outline



Threshold contagion on random networks

- Third goal: Find expected fractional size of spread.



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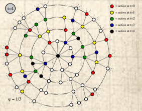

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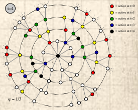

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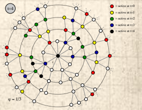

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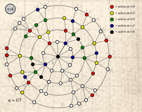
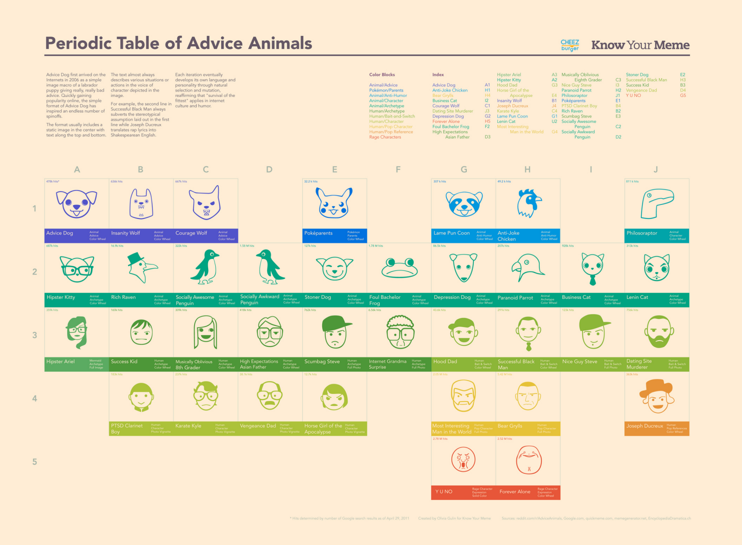
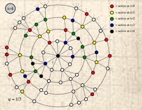
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [6]



Meme species:



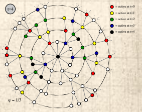


More here at http://knowyourmeme.com

Expected size of spread

Idea:

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$



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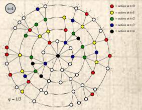

Expected size of spread

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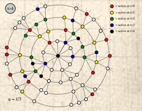

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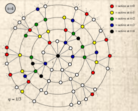

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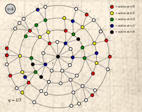

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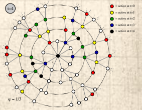

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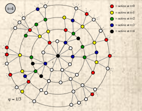
  - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i.



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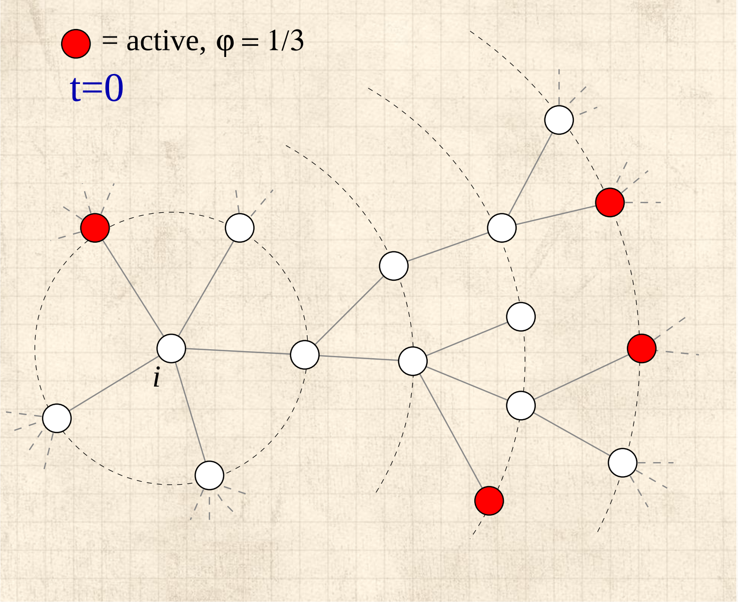

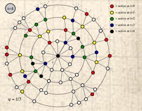

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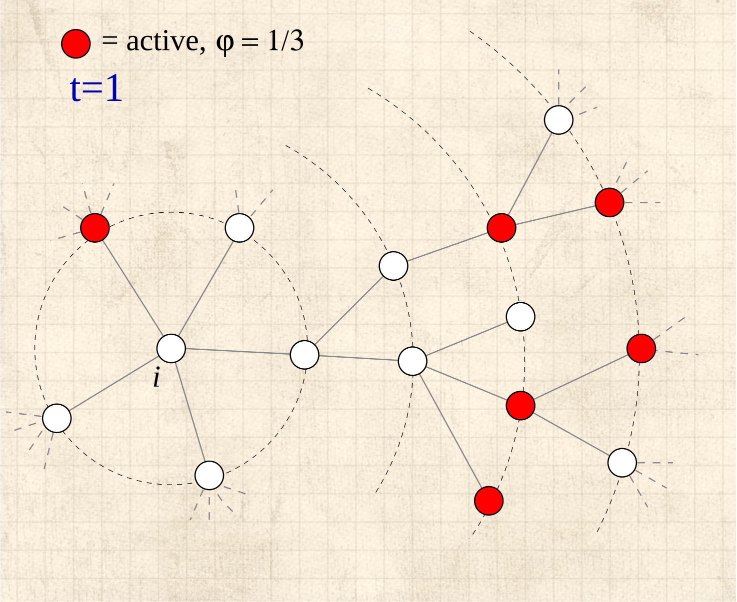

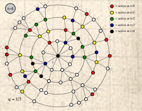

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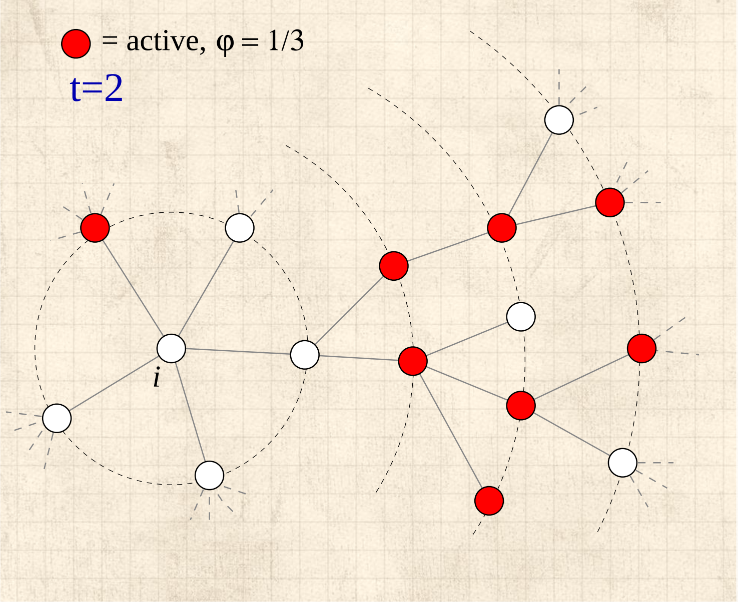

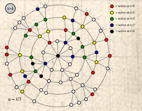

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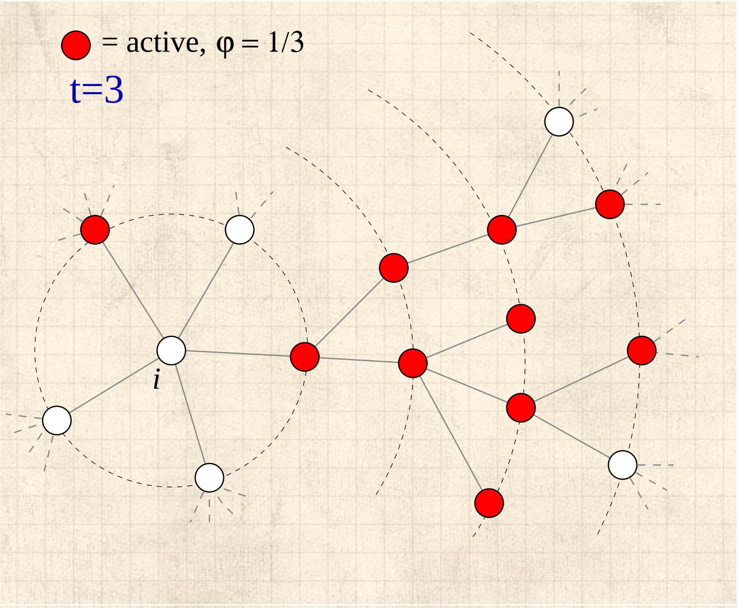

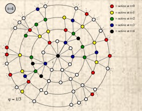

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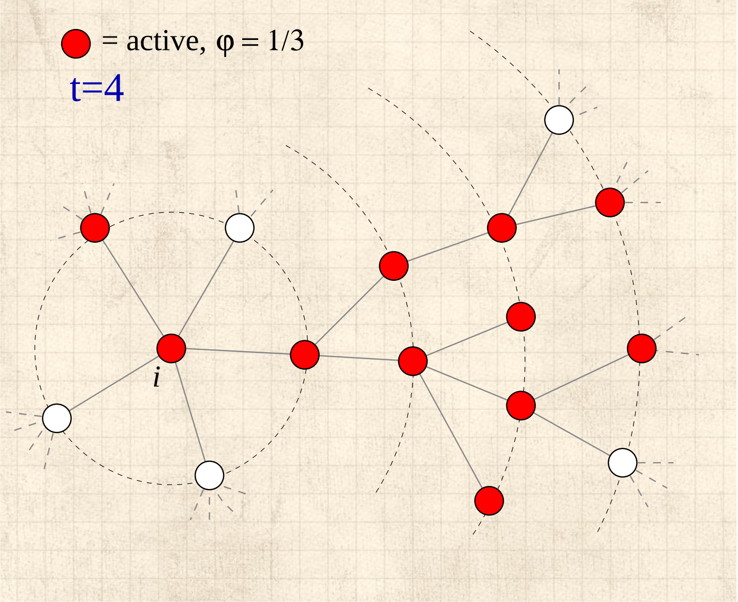

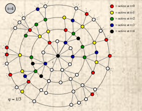

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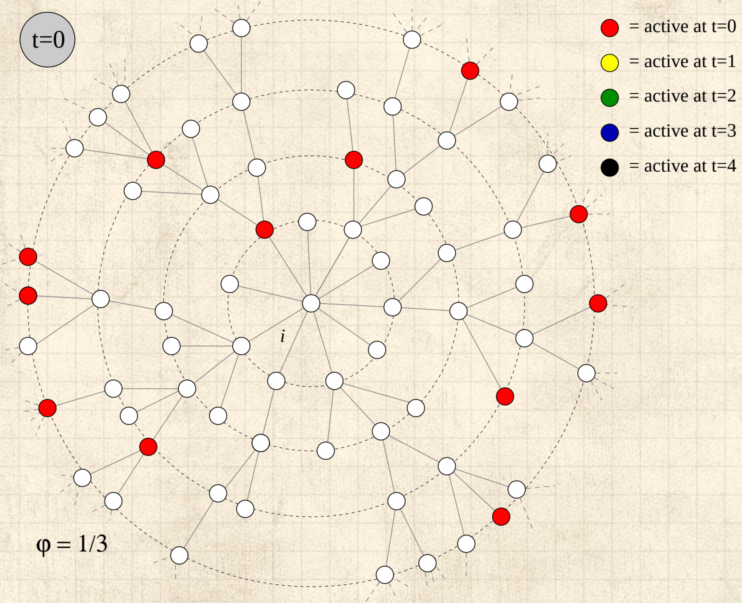

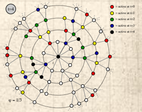

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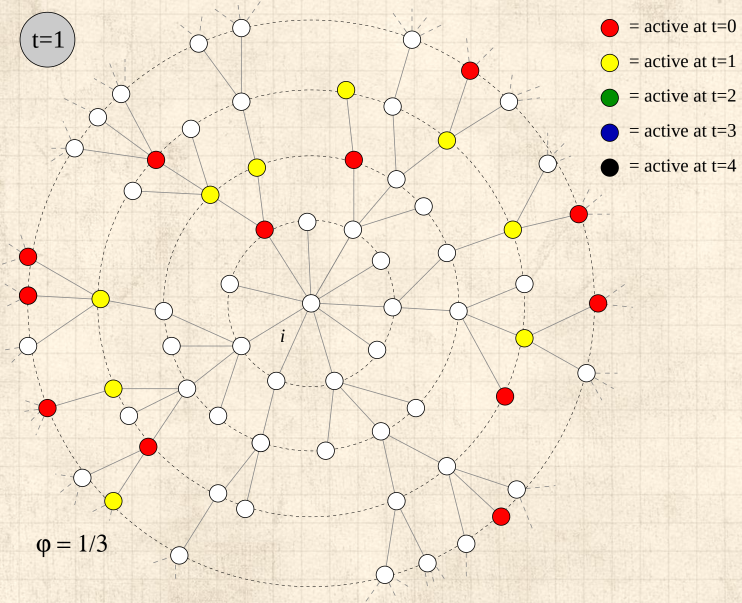

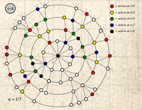

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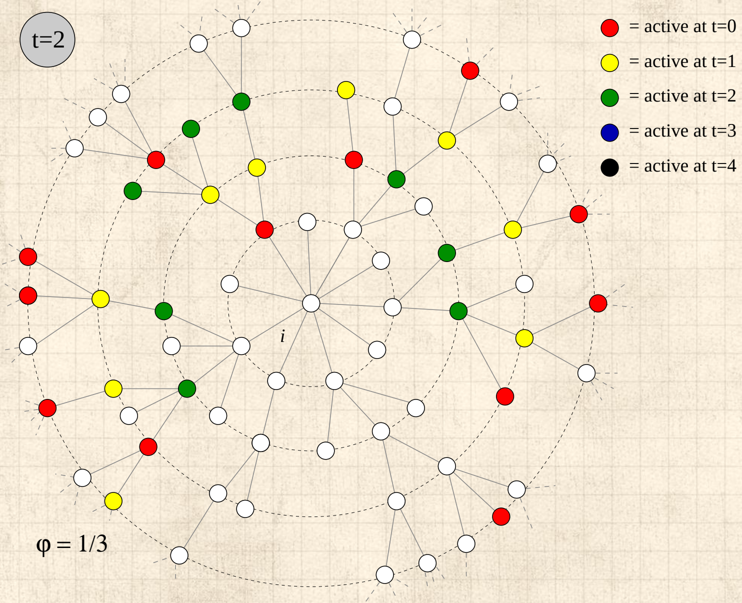

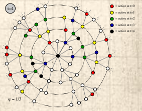

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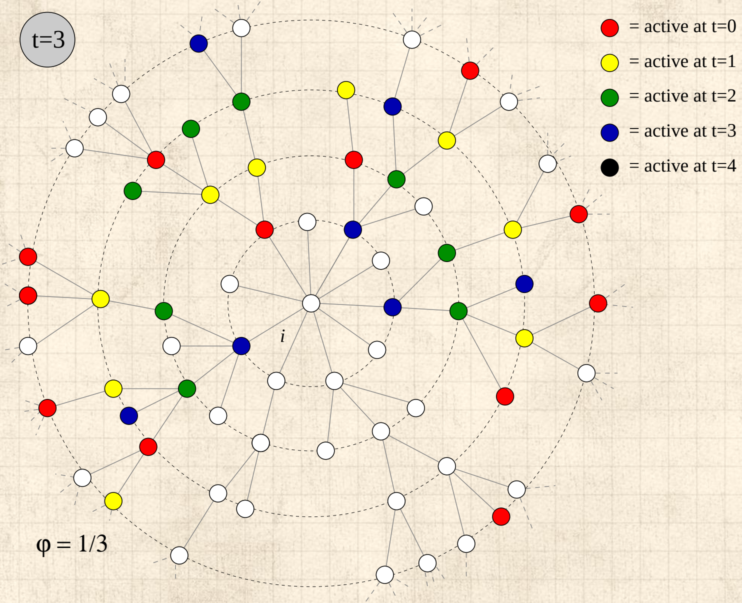

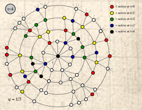

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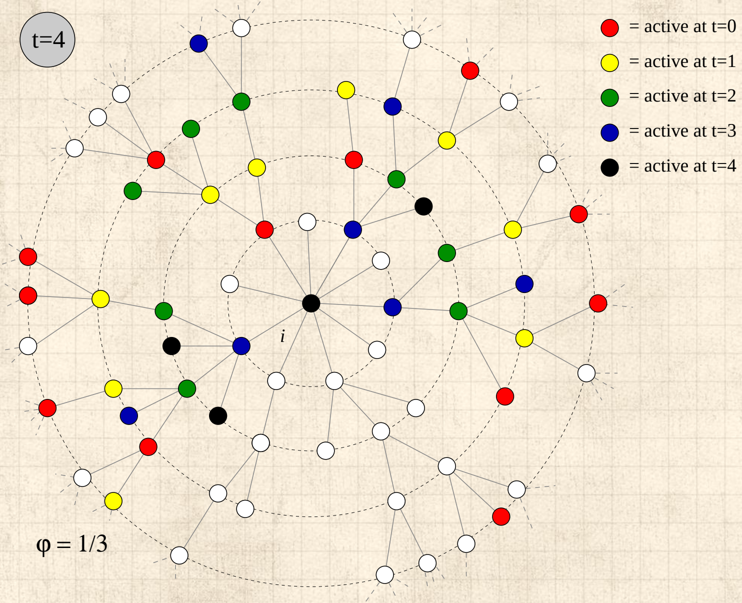

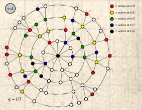

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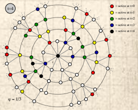

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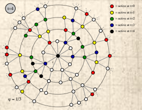

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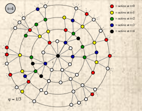

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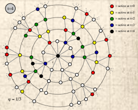

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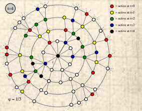

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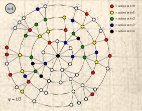
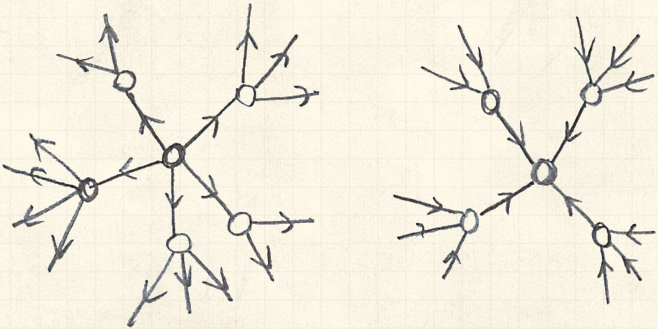
- Even more, we can compute: **Pr**(specific node i switches on at time t).
- Asynchronous updating can be handled too.

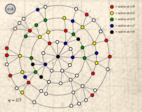


Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.





Expected size of spread

Pleasantness:

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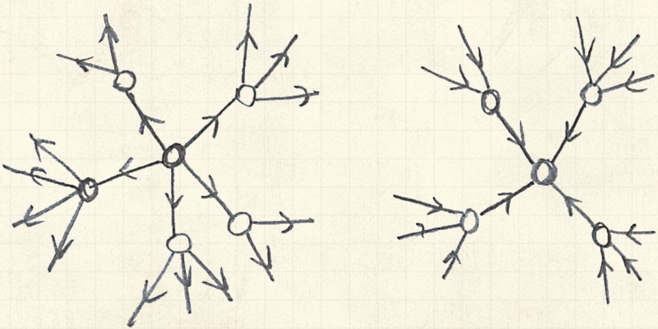
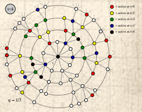
- Extent of spreading story is about contraction at a node.





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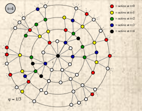
- Notation:
  $\phi_{k,t} =$ **Pr**(a degree k node is active at time t).



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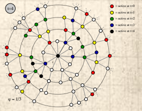

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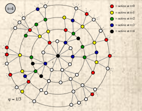

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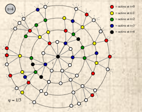

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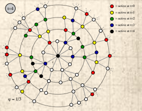

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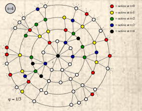

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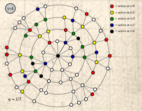
- Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1-\phi_0)\sum_{j=0}^{k}\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}B_{kj}.$$



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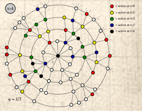

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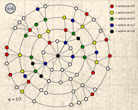

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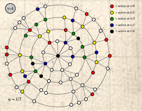

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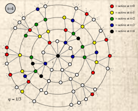

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- Average over all nodes with degree k to obtain expression for ϕ_{t+1}:

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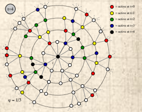

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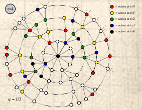
- So we need to compute θ_t...



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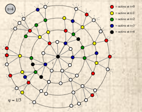


- So we need to compute θ_t... massive excitement...

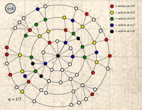
Expected size of spread

First connect θ_0 to θ_1:

- $\theta_1 = \phi_0 +$

$$(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k \rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_0^{\,j}(1-\theta_0)^{k-1-j}B_{kj}$$

- $\frac{kP_k}{\langle k \rangle} = Q_k$ = **Pr** (edge connects to a degree k node).
- $\sum_{j=0}^{k-1}$ piece gives **Pr** (degree node k activates if j of its $k-1$ incoming neighbors are active).
- ϕ_0 and $(1-\phi_0)$ terms account for state of node at time $t=0$.



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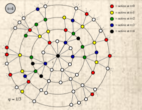
- See this all generalizes to give θ_{t+1} in terms of θ_t...



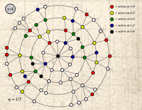
Expected size of spread

Two pieces: edges first, and then nodes

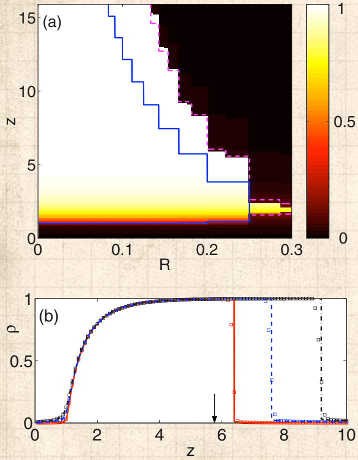
1. $$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1-\phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^{\,j} (1-\theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

   with $\theta_0 = \phi_0$.

2. $$\phi_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1-\phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^{\,j} (1-\theta_t)^{k-j} B_{kj}}_{\text{social effects}}.$$

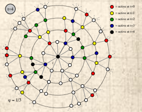


Comparison between theory and simulations



From Gleeson and Cahalane [7]

- Pure random networks with simple threshold responses
- R = uniform threshold (our ϕ_*); z = average degree; $\rho = \phi$; $q = \theta$; $N = 10^5$.
- $\phi_0 = 10^{-3}$, 0.5×10^{-2}, and 10^{-2}.
- Cascade window is for $\phi_0 = 10^{-2}$ case.
- Sensible expansion of cascade window as ϕ_0 increases.



Notes:

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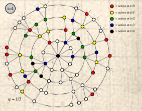

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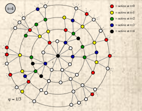

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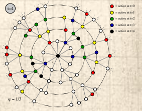
$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.



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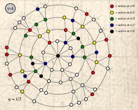
  meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.
- If $\theta = 0$ is a fixed point of G (i.e., $G(0;\phi_0) = 0$) then spreading occurs for a small seed if

$$G'(0;\phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

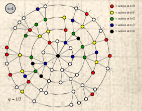
Insert assignment question



Notes:

In words:

- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.



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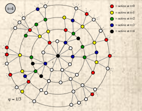
- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an unstable fixed point at $\theta = 0$, then cascades are also always possible.



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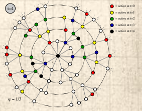

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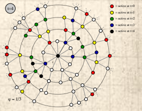

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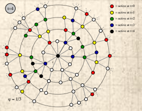
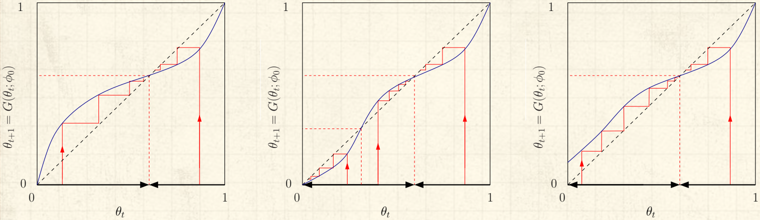
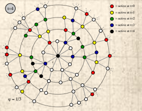
- Tricky point: G depends on ϕ_0, so as we change ϕ_0, we also change G.



General fixed point story:



- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.



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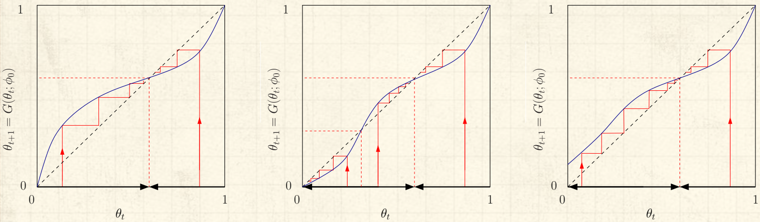

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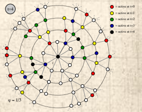
- n.b., adjacent fixed points must have opposite stability types.



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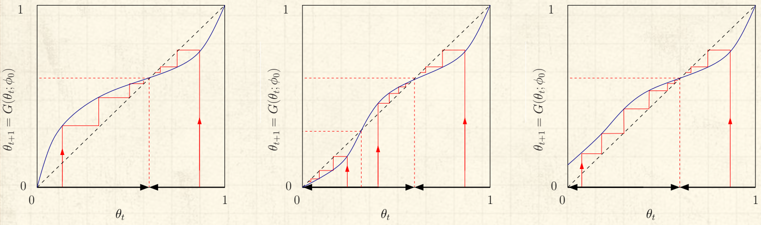

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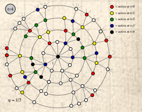

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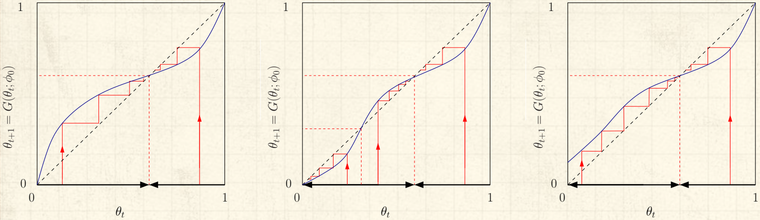

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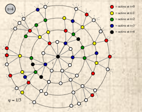

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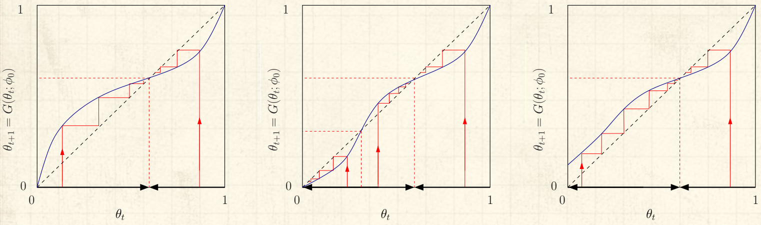

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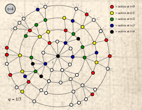

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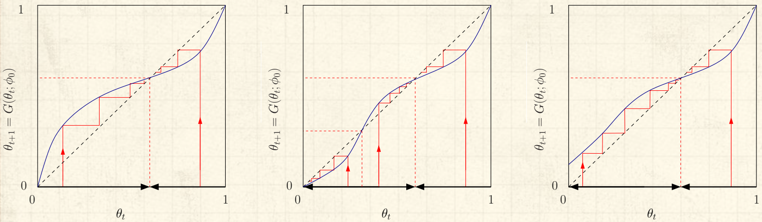

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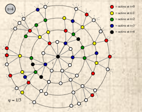
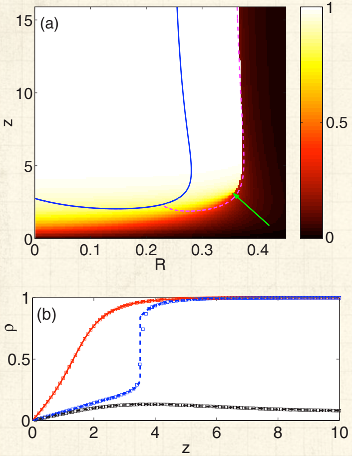
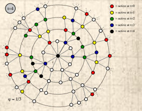


Interesting behavior:



- Now allow thresholds to be distributed according to a Gaussian with mean R.
- R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.

From Gleeson and Cahalane [7]



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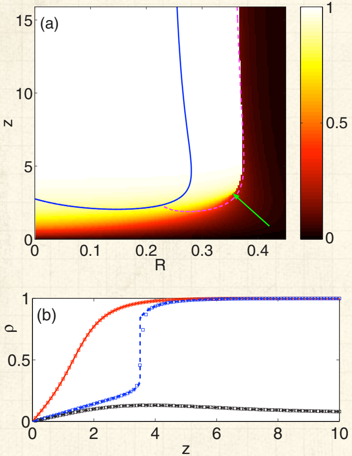

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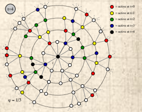

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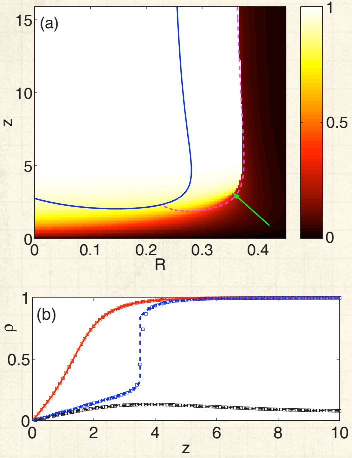

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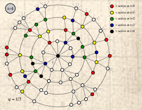
- R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.
- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.
- Now see a (nasty) discontinuous phase transition for low $\langle k \rangle$.



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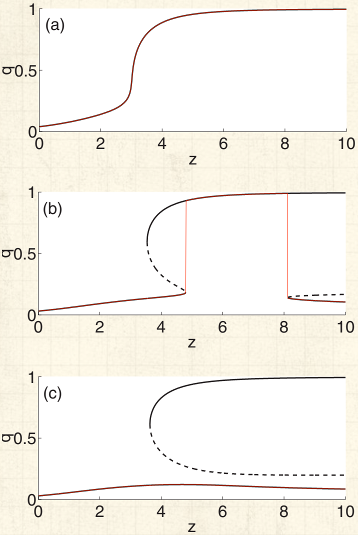
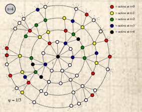
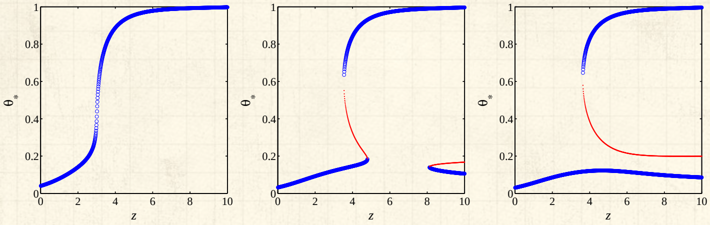


- Plots of stability points for $\theta_{t+1} = G(\theta_t; \phi_0)$.
- n.b.: 0 is not a fixed point here: $\theta_0 = 0$ always takes off.
- Top to bottom: R = 0.35, 0.371, and 0.375.
- Saddle node bifurcations appear and merge (b and c).

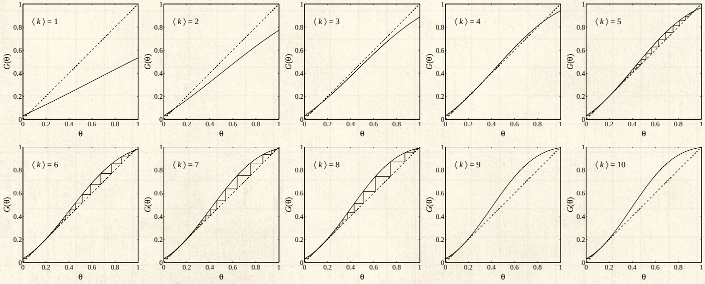
From Gleeson and Cahalane [7]

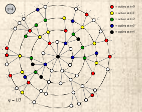


What's happening:



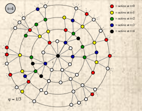
- Fixed points slip above and below the $\theta_{t+1} = \theta_t$ line:





Time-dependent solutions

Synchronous update



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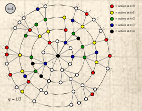
- Done: Evolution of ϕ_t and θ_t given exactly by the maps we have derived.



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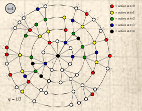
- Update nodes with probability α.



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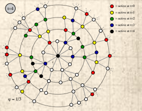

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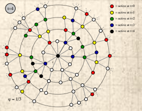
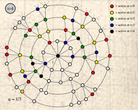
- Now can talk about $\phi(t)$ and $\theta(t)$.



Nutshell:

- Solid dive into understanding contagion on generalized random networks.



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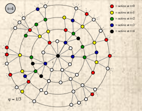

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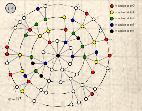

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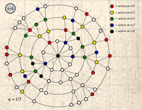

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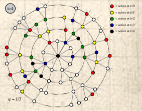

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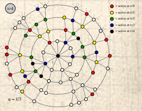

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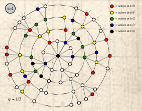
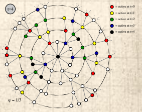
- Many connections to other kinds of models: Voter models, Ising models, …



Neural reboot (NR):

Pangolin happiness:

https://www.youtube.com/watch?v=LMiYjkG4onM?rel=0



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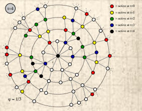

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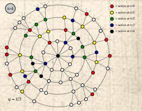

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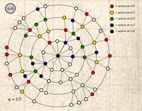

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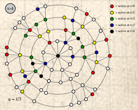

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