

# Social Contagion

Last updated: 2022/08/27, 23:54:10 EDT

Principles of Complex Systems, Vols. 1, 2, & 3D  
CSYS/MATH 300, 303, & 394, 2022-2023 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

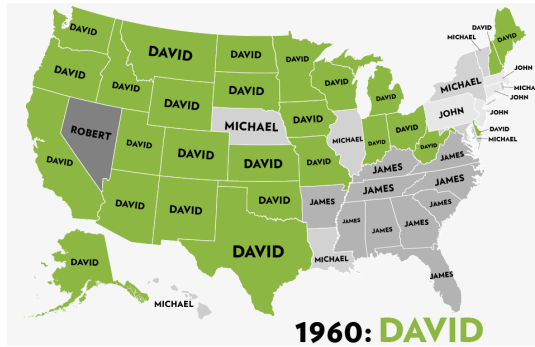
Computational Story Lab | Vermont Complex Systems Center  
Santa Fe Institute | University of Vermont



Licensed under the [Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/).

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

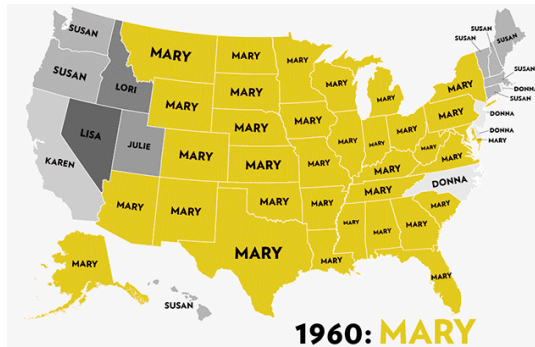


From the Atlantic

1 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



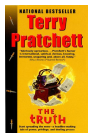
From the Atlantic

2 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

'The rumor spread through the city like wildfire which had quite often spread through Ankh-Morpork since its citizens had learned the words "fire insurance").'



"The Truth" by Terry Pratchett (2000).<sup>[22]</sup>

4 of 110

# LOL + cute + fail + wtf:

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

# Oopsie!



**BUZZFEED FELL DOWN AND WENT BOOM.**

Please try reloading this page. If the problem persists [let us know](#).

5 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

# The whole lolcats thing:



6 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

# Some things really stick:



9 of 110

# Things that spread well:

[buzzfeed.com](http://buzzfeed.com)



Dangerously self aware: 11 Elements that make a perfect viral video.

+ News ...

4 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

10 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

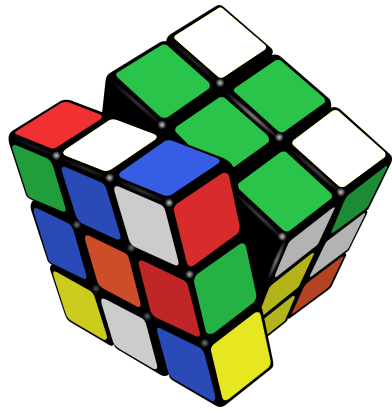
11 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

12 of 110

wtf + geeky + omg:



## Social Contagion

Examples are claimed to abound:

- Fashion
- Striking
- smoking [\[7\]](#)
- Residential segregation [\[23\]](#)
- iPhones and iThings
- obesity [\[6\]](#)
- Stupidity
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- school shootings
- yawning [\[5\]](#)
- leaving lectures

SIR and SIRS type contagion possible

- Classes of behavior versus specific behavior : **dieting, horror movies, getting married, invading countries, ...**



13 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



15 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



16 of 110

## Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking [\[7\]](#)
- The spread of spreading [\[6\]](#)
- Also: happiness [\[11\]](#), loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives [\[7\]](#)

Controversy:

- Are your friends making you fat? [\[5\]](#) (Clive Thomson, NY Times, September 10, 2009).
- Everything is contagious [\[7\]](#)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).



17 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



18 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



19 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



20 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



21 of 110

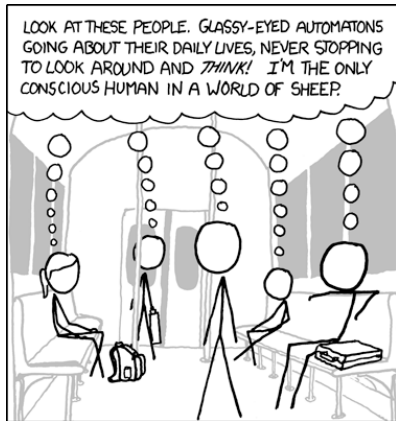
PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



22 of 110

Why social contagion works so well:



<http://xkcd.com/610/>



15 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



16 of 110

Mixed messages: Please copy, but also, don't copy ...

- Cindy Harrell appeared [\[5\]](#) in the (terrifying) music video for Ray Parker Jr.'s Ghostbusters [\[5\]](#).
- In Stranger Things 2 [\[2\]](#), Steve Harrington reveals his Fabergé secret [\[2\]](#).



15 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



16 of 110

## Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [\[12\]](#) as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [\[19\]](#)



18 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



19 of 110



21 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



22 of 110

## Social Contagion



15 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



16 of 110

Market much?

- Advertisement enjoyed during "Herstory of Dance" [\[5\]](#), Community S4E08, April 2013.



15 of 110

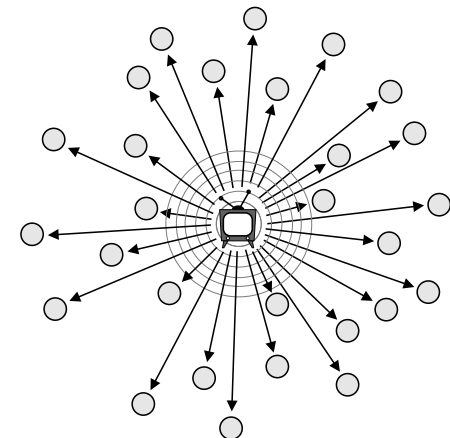
PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



16 of 110

## The hypodermic model of influence



18 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



19 of 110



21 of 110

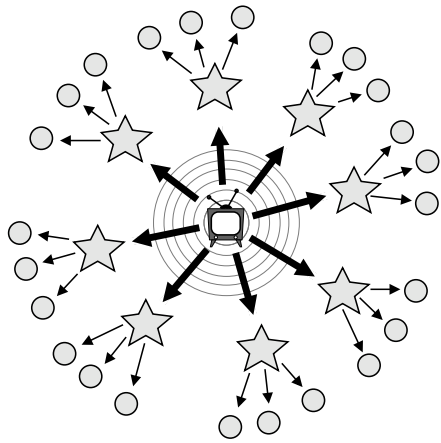
PoCS @pocsvox Social Contagion

Social Contagion Models Background Granovetter's model Network version Final size Spreading success Groups References



22 of 110

## The two step model of influence [19]

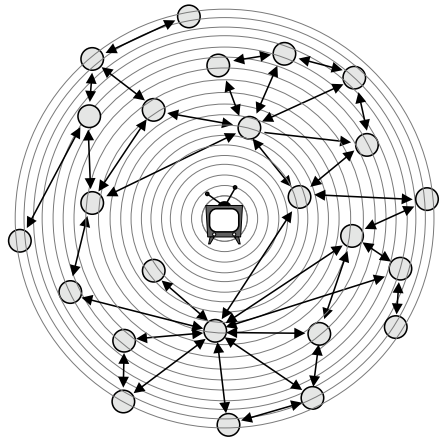


PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

23 of 110

## The general model of influence: the Social Wild



PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

24 of 110

## Why do things spread socially?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers: *homo narrativus*.
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

26 of 110

## The Mona Lisa



- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

27 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

27 of 110

## The dismal predictive powers of editors...



PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

28 of 110



## From a 2013 Believer Magazine interview with Maurice Sendak:

BLVR: Did the success of Where the Wild Things Are ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention Moby-Dick. They're all going to talk about my first book, about \*\*\*ing maidens in Tahiti." He was right. No mention of Moby-Dick then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came Moby-Dick and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. Moby-Dick didn't get famous until 1930.

- Sendak named his dog Herman.
- The essential Colbert interview: [Pt. 1](#) and [Pt. 2](#).

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

29 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

30 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

31 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

32 of 110

## 'Tattooed Guy' Was Pivotal in Armstrong Case [nytimes]



- "... Leogrande's doping sparked a series of events ..."

24 of 110

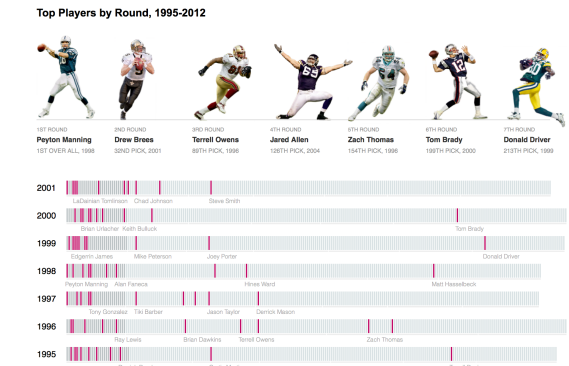
## The completely unpredicted fall of Eastern Europe:



Timunr Kuran: [20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"

26 of 110

## Drafting success in the NFL:



29 of 110

# Social Contagion

## Messaging with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent
  - Harnessing of BzzAgents to directly market through social ties.
  - Generally: BzzAgents did not reveal their BzzAgent status and did not want to be paid.
  - NYT, 2004-12-05: "The Hidden (in Plain Sight) Persuaders"
- One of Facebook's early advertising attempts: Beacon
- All of Facebook's advertising attempts.
- Seriously, Facebook. What could go wrong?

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

33 of 110

# Getting others to do things for you

A very good book: 'Influence' [8] by Robert Cialdini

## Six modes of influence:

- Reciprocation:** *The Old Give and Take... and Take;* e.g., Free samples, Hare Krishnas.
- Commitment and Consistency:** *Hobgoblins of the Mind;* e.g., Hazing.
- Social Proof:** *Truths Are Us;* e.g., Jonestown, Kitty Genovese (contested).
- Liking:** *The Friendly Thief;* e.g., Separation into groups is enough to cause problems.
- Authority:** *Directed Deference;* e.g., Milgram's obedience to authority experiment.
- Scarcity:** *The Rule of the Few;* e.g., Prohibition.

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

34 of 110

# Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

## Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

36 of 110

# Social Contagion

## Some important models:

- Tipping models—Schelling (1971) [23, 24, 25]
  - Simulation on checker boards
  - Idea of thresholds
  - Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)
- Threshold models—Granovetter (1978) [15]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
  - Social learning theory, Informational cascades,...

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

38 of 110

# Social contagion models

## Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

39 of 110

# Social Contagion

## Some possible origins of thresholds:

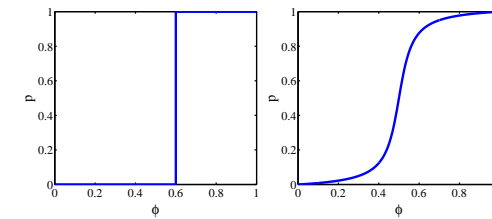
- Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [1]
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
  - Externalities = Effects on others not directly involved in a transaction
  - Examples: telephones, fax machine, Facebook, operating systems
  - An individual's utility increases with the adoption level among peers and the population in general

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

40 of 110

# Threshold models—response functions



- Example threshold influence response functions: deterministic and stochastic
- $\phi$  = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

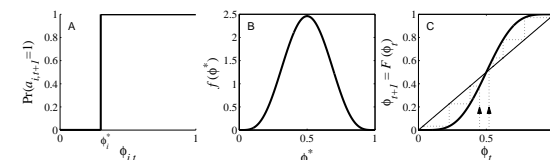
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

42 of 110

# Threshold models

## Action based on perceived behavior of others:



- Two states: S and I.
- $\phi$  = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

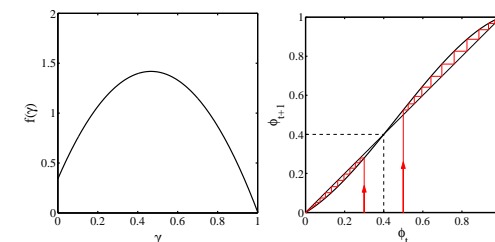
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

43 of 110

# Threshold models

## Another example of critical mass model:



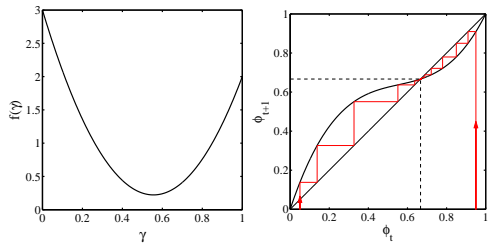
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

44 of 110

# Threshold models

## Example of single stable state model:



PoCS  
@pocsvox  
Social Contagion

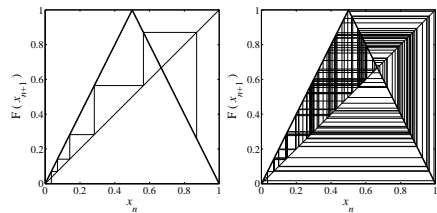
Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

45 of 110

PoCS  
@pocsvox  
Social Contagion

# Threshold models

## Chaotic behavior possible [17, 16, 9, 18]



Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

46 of 110

PoCS  
@pocsvox  
Social Contagion

# Threshold models—Nutshell

## Implications for collective action theory:

- Collective uniformity  $\nRightarrow$  individual uniformity
- Small individual changes  $\Rightarrow$  large global changes
- The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
- System stories live in left null space of our stories—we can't even see them.
- But we happily impose simplistic, individual-centric stories—we can't help ourselves.

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

47 of 110

PoCS  
@pocsvox  
Social Contagion

# Many years after Granovetter and Soong's work:

- "A simple model of global cascades on random networks"  
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [27]
- Mean field model  $\rightarrow$  network model
- Individuals now have a limited view of the world

## We'll also explore:

- "Seed size strongly affects cascades on random networks" [14]  
Gleeson and Cahalane, Phys. Rev. E, 2007.
- "Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks" [10]  
Dodds, Harris, and Payne, Phys. Rev. E, 2011
- "Influentials, Networks, and Public Opinion Formation" [28]  
Watts and Dodds, J. Cons. Res., 2007.

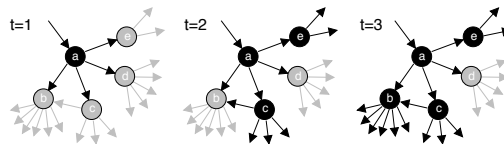
PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

49 of 110

PoCS  
@pocsvox  
Social Contagion

# Threshold model on a network



All nodes have threshold  $\phi = 0.2$ .

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

50 of 110

PoCS  
@pocsvox  
Social Contagion

# Threshold model on a network

- Interactions between individuals now represented by a network.
- Network is **sparse**.
- Individual  $i$  has  $k_i$  contacts.
- Influence on each link is **reciprocal** and of **unit weight**.
- Each individual  $i$  has a fixed threshold  $\phi_i$ .
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individual  $i$  becomes active when fraction of active contacts  $\frac{a_i}{k_i} \geq \phi_i$ .
- Individuals remain active when switched (no recovery = SI model).

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

51 of 110

PoCS  
@pocsvox  
Social Contagion

# Snowballing

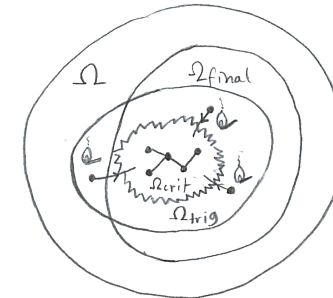
## First study random networks:

- Start with  $N$  nodes with a degree distribution  $P_k$
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a **cascade condition**

## The Cascade Condition:

- If one individual is initially activated, what is the probability that an activation will spread over a network?
- What features of a network determine whether a cascade will occur or not?

# Example random network structure:



- $\Omega_{crit} = \Omega_{vuln} =$  critical mass = global vulnerable component
- $\Omega_{trig} =$  triggering component
- $\Omega_{final} =$  potential extent of spread
- $\Omega =$  entire network

$$\Omega_{crit} \subset \Omega_{trig}; \Omega_{crit} \subset \Omega_{final}; \text{ and } \Omega_{trig}, \Omega_{final} \subset \Omega.$$

# Snowballing

## Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- We need to understand which nodes can be activated when only one of their neighbors becomes active.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

52 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

53 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

54 of 110

PoCS  
@pocsvox  
Social Contagion

# The most gullible

## Vulnerables:

- We call individuals who can be activated by just one contact being active **vulnerables**
- The vulnerability condition for node  $i$ :

$$1/k_i \geq \phi_i$$

- Which means # contacts  $k_i \leq \lceil 1/\phi_i \rceil$
- For global cascades on random networks, must have a *global cluster of vulnerables* [27]
- Cluster of vulnerables = critical mass**
- Network story: 1 node  $\rightarrow$  critical mass  $\rightarrow$  everyone.

# Cascade condition

## Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \underbrace{\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1 - \beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

# Cascade condition

So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

- $\beta_k$  = probability a degree  $k$  node is vulnerable.
- $P_k$  = probability a node has degree  $k$ .

# Cascade condition

## Two special cases:

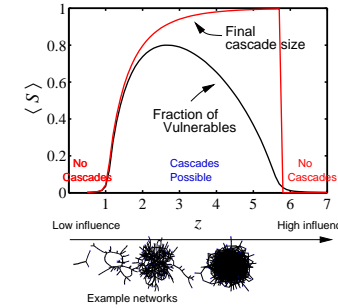
- (1) Simple disease-like spreading succeeds:  $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

- (2) Giant component exists:  $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

# Cascades on random networks



- Cascades occur only if size of max vulnerable cluster  $> 0$ .
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

# Cascade condition

## Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree  $k$  node with probability  $\propto kP_k$ .
- Follows from there being  $k$  ways to connect to a node with degree  $k$ .
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

# Cascade condition

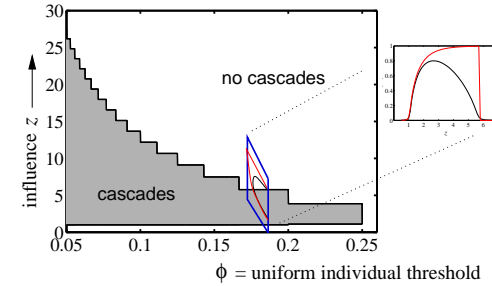
## Next: Vulnerability of linked node

- Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) d\phi'_*$$

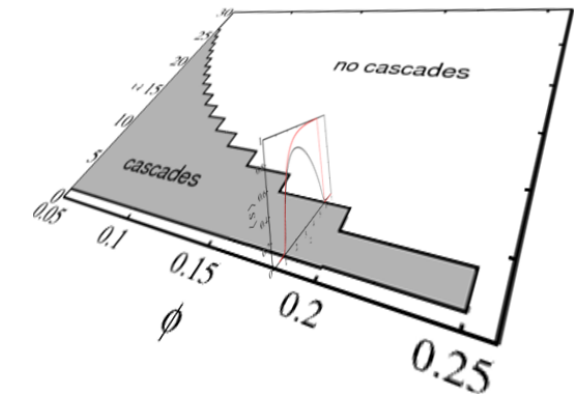
- If linked node is **vulnerable**, it produces  $k-1$  new outgoing active links
- If linked node is **not vulnerable**, it produces **no** active links.

# Cascade window for random networks

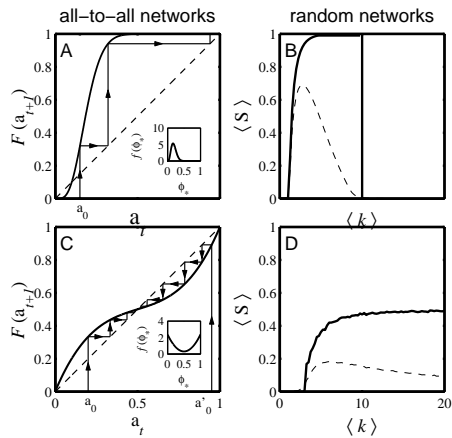


- 'Cascade window' widens as threshold  $\phi$  decreases.
- Lower thresholds enable spreading.

# Cascade window for random networks



## All-to-all versus random networks



PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

64 of 110

## Determining expected size of spread:

- Randomly turn on a fraction  $\phi_0$  of nodes at time  $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node  $i$  to become active at time  $t$ :
  - $t = 0$ :  $i$  is one of the seeds (prob =  $\phi_0$ )
  - $t = 1$ :  $i$  was not a seed but enough of  $i$ 's friends switched on at time  $t = 0$  so that  $i$ 's threshold is now exceeded.
  - $t = 2$ : enough of  $i$ 's friends and friends-of-friends switched on at time  $t = 0$  so that  $i$ 's threshold is now exceeded.
  - $t = n$ : enough nodes within  $n$  hops of  $i$  switched on at  $t = 0$  and their effects have propagated to reach  $i$ .

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

68 of 110

## Expected size of spread

### Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine  $\Pr(\text{node of degree } k \text{ switching on at time } t)$ .
- Asynchronous updating can be handled too.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

71 of 110

## Cascade window—summary

### For our simple model of a uniform threshold:

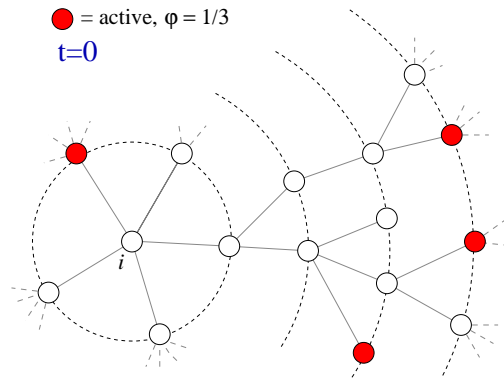
- Low  $\langle k \rangle$ : No cascades in poorly connected networks. No global clusters of any kind.
- High  $\langle k \rangle$ : Giant component exists but not enough vulnerables.
- Intermediate  $\langle k \rangle$ : Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

65 of 110

## Expected size of spread



PoCS  
@pocsvox  
Social Contagion

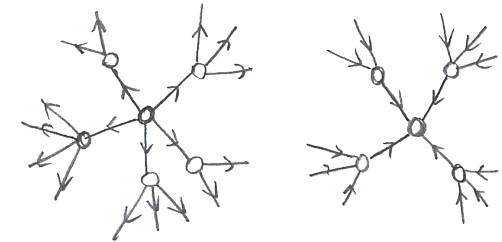
Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

69 of 110

## Expected size of spread

### Pleasantness:

- Taking off from a single seed story is about **expansion** away from a node.
- Extent of spreading story is about **contraction** at a node.



PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

72 of 110

## Threshold contagion on random networks

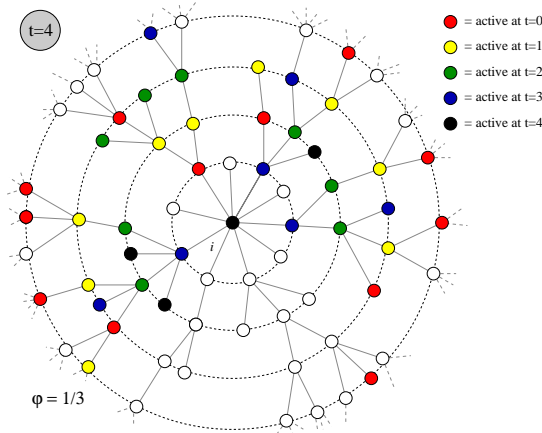
- Next:** Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need  $\geq 2$  hits switch on.
- Problem **beautifully solved** for infinite seed case by Gleeson and Cahalane: "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. [14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [13]

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

67 of 110

## Expected size of spread



PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

70 of 110

## Expected size of spread

- Notation:**  $\phi_{k,t} = \Pr(\text{a degree } k \text{ node is active at time } t)$ .
- Notation:**  $B_{kj} = \Pr(\text{a degree } k \text{ node becomes active if } j \text{ neighbors are active})$ .
- Our starting point:  $\phi_{k,0} = \phi_0$ .
- $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \Pr(j \text{ of a degree } k \text{ node's neighbors were seeded at time } t = 0)$ .
- Probability a degree  $k$  node was a seed at  $t = 0$  is  $\phi_0$  (as above).
- Probability a degree  $k$  node was not a seed at  $t = 0$  is  $(1 - \phi_0)$ .
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}$$

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

73 of 110

For general  $t$ , we need to know the probability an edge coming into a degree  $k$  node at time  $t$  is active.

**Notation:** call this probability  $\theta_t$ .

We already know  $\theta_0 = \phi_0$ .

Story analogous to  $t = 1$  case. For node  $i$ :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i,j}$$

Average over all nodes to obtain expression for  $\phi_{t+1}$ :

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k,j}$$

So we need to compute  $\theta_t$ ... massive excitement...



74 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Expected size of spread

Iterative map for  $\theta_t$  is key:

$$\theta_{t+1} = \underbrace{\phi_0 + (1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{k,j}}_{\text{social effects}} = G(\theta_t; \phi_0)$$



74 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Expected size of spread:

Retrieve cascade condition for spreading from a single seed in limit  $\phi_0 \rightarrow 0$ .

Depends on map  $\theta_{t+1} = G(\theta_t; \phi_0)$ .

First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k,0} > 0.$$

meaning  $B_{k,0} > 0$  for at least one value of  $k \geq 1$ .

If  $\theta = 0$  is a fixed point of  $G$  (i.e.,  $G(0; \phi_0) = 0$ ) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k,1} > 1.$$



75 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Expected size of spread:

In words:

If  $G(0; \phi_0) > 0$ , spreading must occur because some nodes turn on for free.

If  $G$  has an **unstable fixed point** at  $\theta = 0$ , then cascades are also always possible.

Non-vanishing seed case:

Cascade condition is more complicated for  $\phi_0 > 0$ .

If  $G$  has a **stable fixed point** at  $\theta = 0$ , and an **unstable fixed point** for some  $0 < \theta_* < 1$ , then for  $\theta_0 > \theta_*$ , spreading takes off.

Tricky point:  $G$  depends on  $\phi_0$ , so as we change  $\phi_0$ , we also change  $G$ .

A version of a critical mass model again.



76 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Expected size of spread

First connect  $\theta_0$  to  $\theta_1$ :

$\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{k,j}$$

$\frac{k P_k}{\langle k \rangle} = R_k = \mathbf{Pr}$  (edge connects to a degree  $k$  node).

$\sum_{j=0}^{k-1}$  piece gives  $\mathbf{Pr}$ (degree node  $k$  activates) of its neighbors  $k-1$  incoming neighbors are active.

$\phi_0$  and  $(1 - \phi_0)$  terms account for state of node at time  $t = 0$ .

See this all generalizes to give  $\theta_{t+1}$  in terms of  $\theta_t$ ...



75 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Expected size of spread

Two pieces: edges first, and then nodes

1.  $\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$

$$+ (1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{k,j}}_{\text{social effects}}$$

with  $\theta_0 = \phi_0$ .

2.  $\phi_{t+1} =$

$$\underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k,j}}_{\text{social effects}}$$

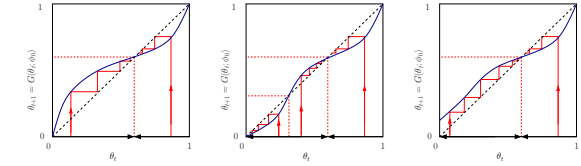


76 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## General fixed point story:



Given  $\theta_0 (= \phi_0)$ ,  $\theta_{\infty}$  will be the nearest stable fixed point, either above or below.

n.b., adjacent fixed points must have opposite stability types.

**Important:** Actual form of  $G$  depends on  $\phi_0$ .

So choice of  $\phi_0$  dictates both  $G$  and starting point—can't start anywhere for a given  $G$ .

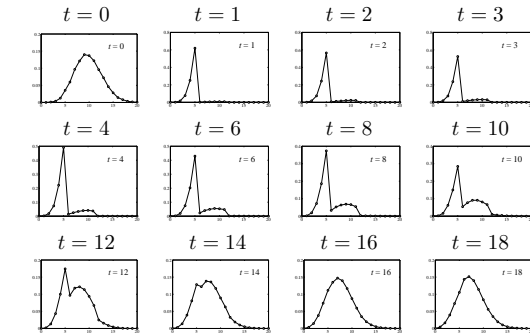


77 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

## Early adopters—degree distributions



$P_{k,t}$  versus  $k$

Unpublished?



78 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

"Influentials, Networks, and Public Opinion Formation" Watts and Dodds, J. Consum. Res., **34**, 441–458, 2007. [28]

Exploration of threshold model of social contagion on various networks.

"Influentials" are limited in power.

Connected groups of weakly influential-vulnerable individuals are key.

Average individuals can have more power than well connected ones.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



80 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



82 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



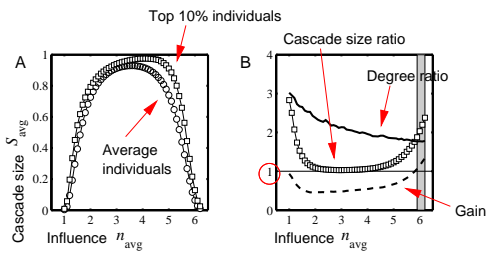
83 of 110

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



## The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.



84 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



85 of 110

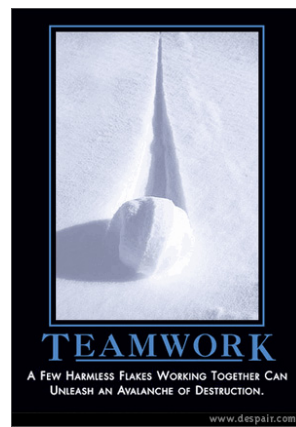
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



86 of 110

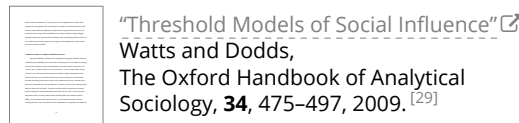
## The power of groups...



despair.com

"A few harmless flakes working together can unleash an avalanche of destruction."

## Extensions



- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: **group structure**



85 of 110

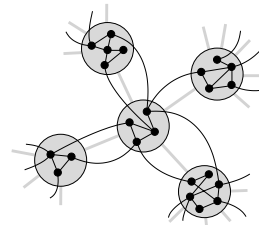
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



86 of 110

## Group structure—Ramified random networks



$p$  = intergroup connection probability  
 $q$  = intragroup connection probability.

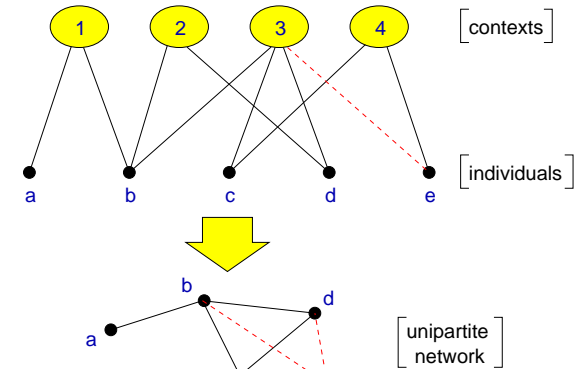


86 of 110

## Bipartite networks

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



88 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



89 of 110

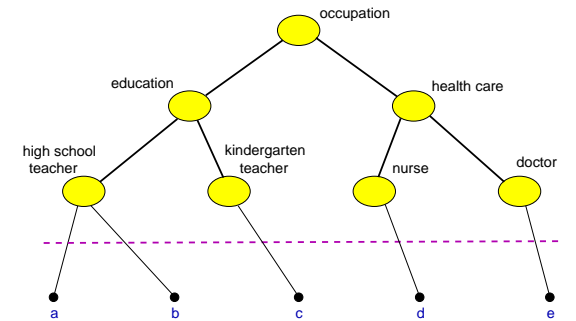
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



90 of 110

## Context distance



89 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



90 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



92 of 110

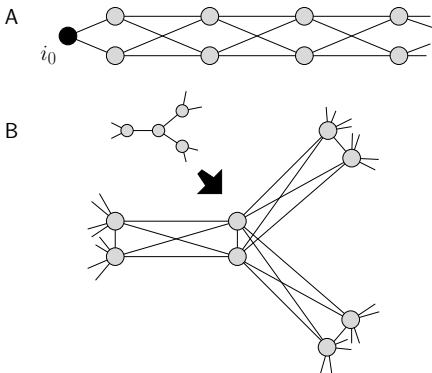
PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



93 of 110

## Special subnetworks can act as triggers



$\phi = 1/3$  for all nodes



85 of 110

PoCS @pocsvox Social Contagion

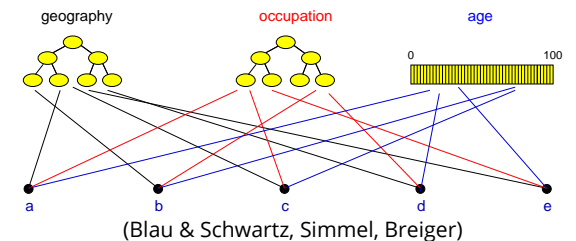
Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References



86 of 110

## Group structure—Ramified random networks

## Generalized affiliation model



(Blau & Schwartz, Simmel, Breiger)



90 of 110

PoCS @pocsvox Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups  
References

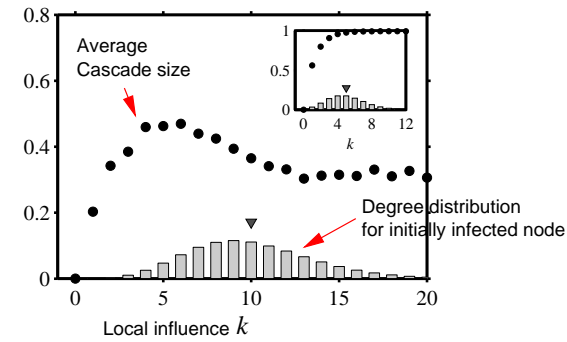


93 of 110

# Generalized affiliation model networks with triadic closure

- Connect nodes with probability  $\propto e^{-\alpha d}$  where
  - $\alpha$  = homophily parameter and
  - $d$  = distance between nodes (height of lowest common ancestor)
- $\tau_1$  = intergroup probability of friend-of-friend connection
- $\tau_2$  = intragroup probability of friend-of-friend connection

# Assortativity in group-based networks

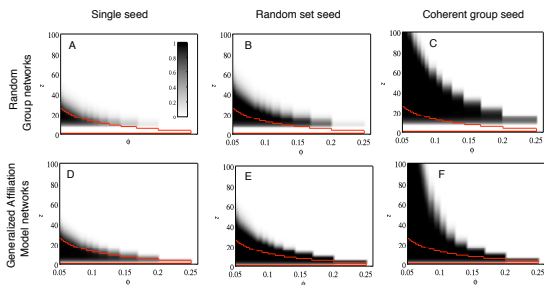


- The most connected nodes aren't always the most 'influential.'
- Degree assortativity** is the reason.

# References I

- A. Bentley, M. Earls, and M. J. O'Brien. I'll Have What She's Having: Mapping Social Behavior. MIT Press, Cambridge, MA, 2011.
- S. Bikhchandani, D. Hirshleifer, and I. Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. J. Polit. Econ., 100:992-1026, 1992.
- S. Bikhchandani, D. Hirshleifer, and I. Welch. Learning from the behavior of others: Conformity, fads, and informational cascades. J. Econ. Perspect., 12(3):151-170, 1998. [pdf](#)

# Cascade windows for group-based networks



# Social contagion

"Without followers, evil cannot spread." -Leonard Nimoy

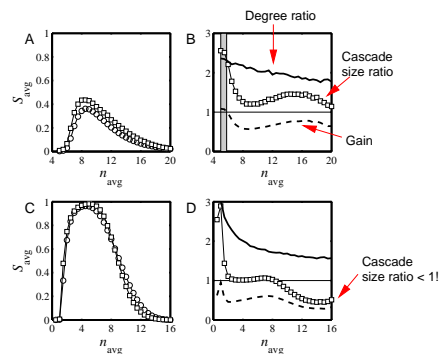
## Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

# References II

- J. M. Carlson and J. Doyle. Highly optimized tolerance: A mechanism for power laws in designed systems. Phys. Rev. E, 60(2):1412-1427, 1999. [pdf](#)
- J. M. Carlson and J. Doyle. Highly optimized tolerance: Robustness and design in complex systems. Phys. Rev. Lett., 84(11):2529-2532, 2000. [pdf](#)
- N. A. Christakis and J. H. Fowler. The spread of obesity in a large social network over 32 years. New England Journal of Medicine, 357:370-379, 2007. [pdf](#)

# Multiplier effect for group-based networks:



Multiplier almost always below 1.

# Social contagion

## Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

# References III

- N. A. Christakis and J. H. Fowler. The collective dynamics of smoking in a large social network. New England Journal of Medicine, 358:2249-2258, 2008. [pdf](#)
- R. B. Cialdini. Influence: Science and Practice. Allyn and Bacon, Boston, MA, 4th edition, 2000.
- P. S. Dodds, K. D. Harris, and C. M. Danforth. Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability. Phys. Rev. Lett., 110:158701, 2013. [pdf](#)

## References IV

- [10] P. S. Dodds, K. D. Harris, and J. L. Payne. Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks. [Phys. Rev. E](#), 83:056122, 2011. [pdf](#)
- [11] J. H. Fowler and N. A. Christakis. Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study. [BMJ](#), 337:article #2338, 2008. [pdf](#)
- [12] M. Gladwell. [The Tipping Point](#). Little, Brown and Company, New York, 2000.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References V

- [13] J. P. Gleeson. Cascades on correlated and modular random networks. [Phys. Rev. E](#), 77:046117, 2008. [pdf](#)
- [14] J. P. Gleeson and D. J. Cahalane. Seed size strongly affects cascades on random networks. [Phys. Rev. E](#), 75:056103, 2007. [pdf](#)
- [15] M. Granovetter. Threshold models of collective behavior. [Am. J. Sociol.](#), 83(6):1420–1443, 1978. [pdf](#)

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References VI

- [16] M. Granovetter and R. Soong. Threshold models of diversity: Chinese restaurants, residential segregation, and the spiral of silence. [Sociological Methodology](#), 18:69–104, 1988. [pdf](#)
- [17] M. S. Granovetter and R. Soong. Threshold models of interpersonal effects in consumer demand. [J. Econ. Behav. Organ.](#), 7:83–99, 1986. [pdf](#)
- [18] K. D. Harris, C. M. Danforth, and P. S. Dodds. Dynamical influence processes on networks: General theory and applications to social contagion. [Phys. Rev. E](#), 88:022816, 2013. [pdf](#)

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References VII

- [19] E. Katz and P. F. Lazarsfeld. [Personal Influence](#). The Free Press, New York, 1955.
- [20] T. Kuran. Now out of never: The element of surprise in the east european revolution of 1989. [World Politics](#), 44:7–48, 1991. [pdf](#)
- [21] T. Kuran. [Private Truths, Public Lies: The Social Consequences of Preference Falsification](#). Harvard University Press, Cambridge, MA, Reprint edition, 1997.
- [22] T. Pratchett. [The Truth](#). HarperCollins, 2000.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References VIII

- [23] T. C. Schelling. Dynamic models of segregation. [J. Math. Sociol.](#), 1:143–186, 1971. [pdf](#)
- [24] T. C. Schelling. Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities. [J. Conflict Resolut.](#), 17:381–428, 1973. [pdf](#)
- [25] T. C. Schelling. [Micromotives and Macrobehavior](#). Norton, New York, 1978.
- [26] D. Sornette. [Critical Phenomena in Natural Sciences](#). Springer-Verlag, Berlin, 1st edition, 2003.

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References IX

- [27] D. J. Watts. A simple model of global cascades on random networks. [Proc. Natl. Acad. Sci.](#), 99(9):5766–5771, 2002. [pdf](#)
- [28] D. J. Watts and P. S. Dodds. Influentials, networks, and public opinion formation. [Journal of Consumer Research](#), 34:441–458, 2007. [pdf](#)
- [29] D. J. Watts and P. S. Dodds. Threshold models of social influence. In P. Hedström and P. Bearman, editors, [The Oxford Handbook of Analytical Sociology](#), chapter 20, pages 475–497. Oxford University Press, Oxford, UK, 2009. [pdf](#)

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References



## References X

PoCS  
@pocsvox  
Social Contagion

Social Contagion Models  
Background  
Granovetter's model  
Network version  
Final size  
Spreading success  
Groups

References

