

System Robustness

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394, 2022-2023 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont



The PoCVerse
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Robustness

HOT theory

Narrative causality

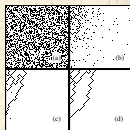
Random forests

Self-Organized Criticality

COLD theory

Network robustness

References



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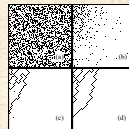


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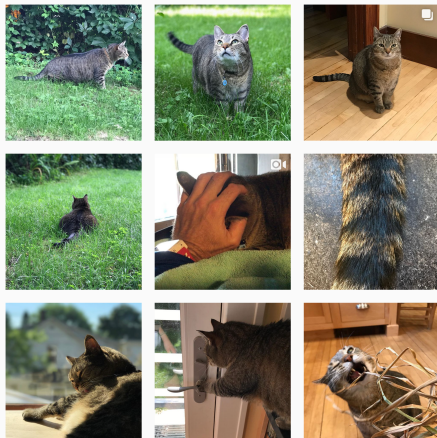
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness

References



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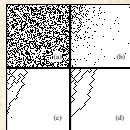




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- HOT theory
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 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 

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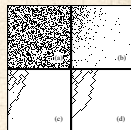
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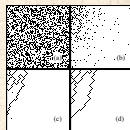
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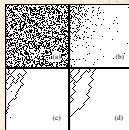
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Many complex systems are prone to cascading catastrophic failure:



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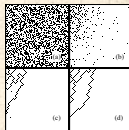
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Many complex systems are prone to cascading catastrophic failure: **exciting!!!**



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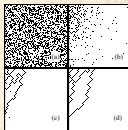
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Many complex systems are prone to cascading catastrophic failure: **exciting!!!**



Blackouts



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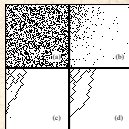
Many complex systems are prone to cascading catastrophic failure: **exciting!!!**



Blackouts



Disease outbreaks



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Many complex systems are prone to cascading catastrophic failure: **exciting!!!**



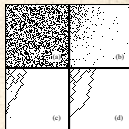
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Disease outbreaks



Wildfires



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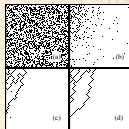
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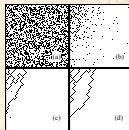
- Blackouts
- Disease outbreaks
- Wildfires
- Earthquakes





Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

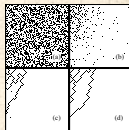
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- Disease outbreaks
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- Earthquakes
- Organisms, individuals and societies





Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

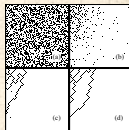
- Blackouts
- Disease outbreaks
- Wildfires
- Earthquakes
- Organisms, individuals and societies
- Ecosystems





Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

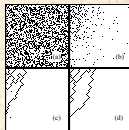
- Blackouts
- Disease outbreaks
- Wildfires
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- Organisms, individuals and societies
- Ecosystems
- Cities













Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

- Blackouts
- Disease outbreaks
- Wildfires
- Earthquakes
- Organisms, individuals and societies
- Ecosystems
- Cities
- Myths: Achilles.



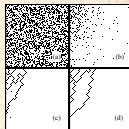


Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

-  Blackouts
-  Disease outbreaks
-  Wildfires
-  Earthquakes
-  Organisms, individuals and societies
-  Ecosystems
-  Cities
-  Myths: Achilles.



But complex systems also show persistent **robustness**



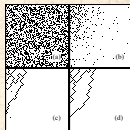



Many complex systems are prone to cascading catastrophic failure: **exciting!!!**









- Blackouts
- Disease outbreaks
- Wildfires
- Earthquakes
- Organisms, individuals and societies
- Ecosystems
- Cities
- Myths: Achilles.





But complex systems also show persistent **robustness** (not as exciting but important...)

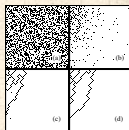


 Many complex systems are prone to cascading catastrophic failure: **exciting!!!**

-  Blackouts
-  Disease outbreaks
-  Wildfires
-  Earthquakes
-  Organisms, individuals and societies
-  Ecosystems
-  Cities
-  Myths: Achilles.

 But complex systems also show persistent **robustness** (not as exciting but important...)

 Robustness and Failure may be a power-law story...



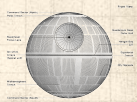
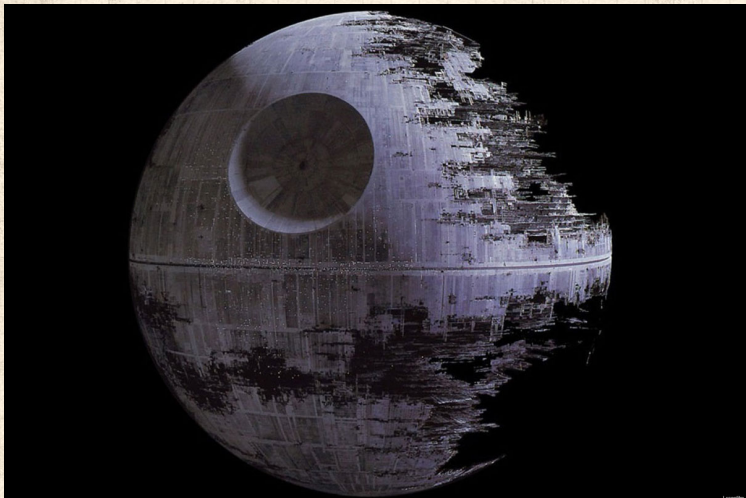
Our emblem of Robust-Yet-Fragile:

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Robustness

- HOT theory
- Narrative causality
- Random forests
- Self-Organized Criticality
- COLD theory
- Network robustness

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“Trouble ...”

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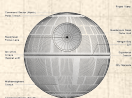
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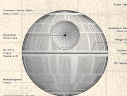
COLD theory

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System robustness may result from





System robustness may result from

1. Evolutionary processes

Robustness

HOT theory

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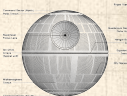
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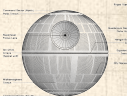
Network robustness

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System robustness may result from

1. Evolutionary processes
2. Engineering/Design



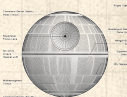


System robustness may result from

1. Evolutionary processes
2. Engineering/Design



Idea: Explore systems optimized to perform under uncertain conditions.





System robustness may result from

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2. Engineering/Design

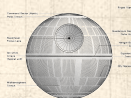


Idea: Explore systems optimized to perform under uncertain conditions.



The handle:

'Highly Optimized Tolerance' (HOT) [4, 5, 6, 10]



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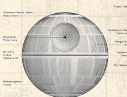


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The catchphrase: Robust yet Fragile





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The people: Jean Carlson and John Doyle 

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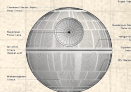
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 System robustness may result from


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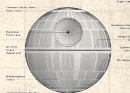
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 The people: Jean Carlson and John Doyle 

 Great abstracts of the world #73: "There aren't any." [7]



Features of HOT systems: [5, 6]

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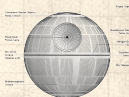
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Features of HOT systems: [5, 6]

 High performance and robustness

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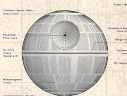
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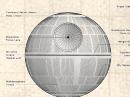
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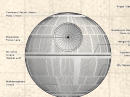
Features of HOT systems: [5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability







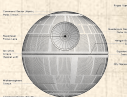
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- High performance and robustness
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- Fragile** in the face of unpredicted environmental signals



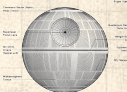
Features of HOT systems: [5, 6]

-  High performance and robustness
-  Designed/evolved to handle known stochastic environmental variability
-  **Fragile** in the face of unpredicted environmental signals
-  Highly specialized, low entropy configurations




Features of HOT systems: [5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability
- Fragile** in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)



Robustness

HOT combines things we've seen:

 Variable transformation

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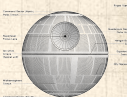
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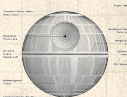
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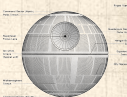
 Constrained optimization



HOT combines things we've seen:

- Variable transformation
- Constrained optimization

Need power law transformation between variables: $(Y = X^{-\alpha})$

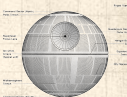


HOT combines things we've seen:

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- Constrained optimization

Need power law transformation between variables: $(Y = X^{-\alpha})$

Recall PLIPL0 is bad...



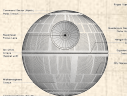
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Recall PLIPL0 is bad...

MIWO is good



HOT combines things we've seen:



Variable transformation



Constrained optimization



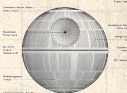
Need power law transformation between variables: $(Y = X^{-\alpha})$



Recall PLIPLD is bad...



MIWO is good: Mild In, Wild Out



HOT combines things we've seen:



Variable transformation



Constrained optimization



Need power law transformation between variables: $(Y = X^{-\alpha})$



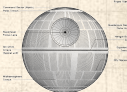
Recall PLIPLO is bad...



MIWO is good: Mild In, Wild Out



X has a characteristic size but Y does not



Robustness

Forest fire example: [5]

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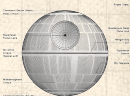
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Forest fire example: ^[5]

 Square $N \times N$ grid

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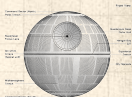
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Forest fire example: [5]



Square $N \times N$ grid



Sites contain a tree with probability $\rho = \text{density}$

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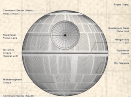
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Forest fire example: [5]

- 🧱 Square $N \times N$ grid
- 🧱 Sites contain a tree with probability $\rho = \text{density}$
- 🧱 Sites are empty with probability $1 - \rho$

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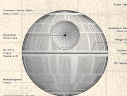
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Forest fire example: [5]

- ☰ Square $N \times N$ grid
- ☰ Sites contain a tree with probability $\rho = \text{density}$
- ☰ Sites are empty with probability $1 - \rho$
- ☰ Fires start at location (i, j) according to some distribution P_{ij}

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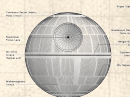
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- ☰ Fires spread from tree to tree (nearest neighbor only)

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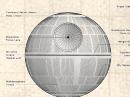
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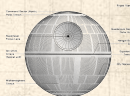
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- 🧱 Fires start at location (i, j) according to some distribution P_{ij}
- 🧱 Fires spread from tree to tree (nearest neighbor only)
- 🧱 Connected clusters of trees burn completely
- 🧱 Empty sites block fire

Robustness

HOT theory

Narrative causality

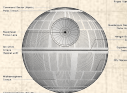
Random forests

Self-Organized Criticality

COLD theory

Network robustness

References



Forest fire example: [5]

- 🧱 Square $N \times N$ grid
- 🧱 Sites contain a tree with probability $\rho =$ density
- 🧱 Sites are empty with probability $1 - \rho$
- 🧱 Fires start at location (i, j) according to some distribution P_{ij}
- 🧱 Fires spread from tree to tree (nearest neighbor only)
- 🧱 Connected clusters of trees burn completely
- 🧱 Empty sites block fire
- 🧱 **Best case scenario:**
Build firebreaks to maximize average # trees left intact given one spark

Robustness

HOT theory

Narrative causality

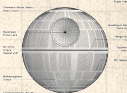
Random forests

Self-Organized Criticality

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Robustness

Forest fire example: [5]

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HOT theory

Narrative causality

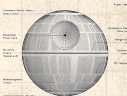
Random forests

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Robustness

Forest fire example: [5]

 Build a forest by adding one tree at a time

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Narrative causality

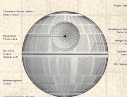
Random forests

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References



Robustness

Forest fire example: ^[5]

- 🧱 Build a forest by adding one tree at a time
- 🧱 Test D ways of adding one tree

Robustness

HOT theory

Narrative causality

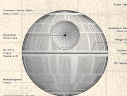
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Forest fire example: ^[5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- $D =$ design parameter

Robustness

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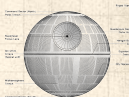
Random forests

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Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- $D =$ design parameter
- Average over $P_{i,j}$ = spark probability

Robustness

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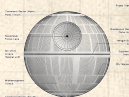
Random forests

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Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- $D =$ design parameter
- Average over $P_{i,j}$ = spark probability
- $D = 1$: random addition

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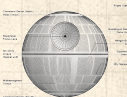
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- $D =$ design parameter
- Average over $P_{i,j}$ = spark probability
- $D = 1$: random addition
- $D = N^2$: test all possibilities

Robustness

HOT theory

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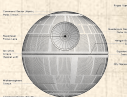
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- $D = N^2$: test all possibilities

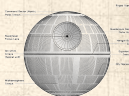
Measure average area of forest left untouched

Robustness

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- $D = N^2$: test all possibilities

Measure average area of forest left untouched

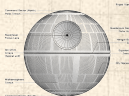
- $f(c) =$ distribution of fire sizes c (= cost)

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Forest fire example: [5]

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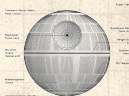
- $f(c)$ = distribution of fire sizes c (= cost)
- Yield = $Y = \rho - \langle c \rangle$

Robustness

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Specifics:



$$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

where

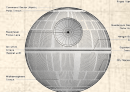
$$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$



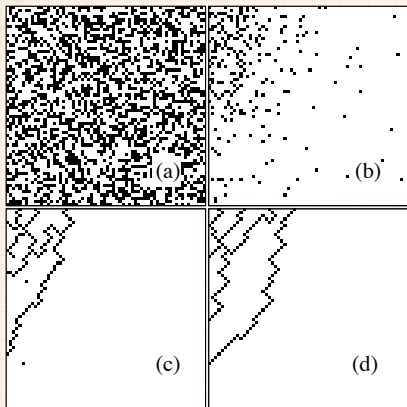
In the original work, $b_y > b_x$



Distribution has more width in y direction.



HOT Forests



$$N = 64$$

$$(a) D = 1$$

$$(b) D = 2$$

$$(c) D = N$$

$$(d) D = N^2$$

P_{ij} has a
Gaussian decay

[5]

Robustness

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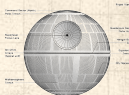
Random forests

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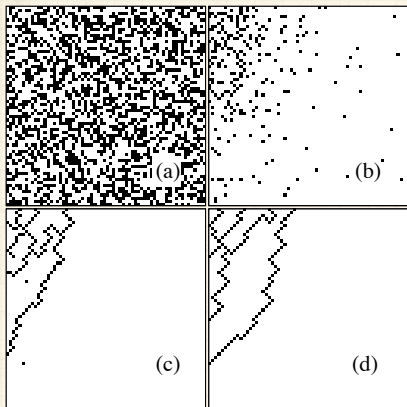
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Gaussian decay

[5]

 Optimized forests do well on average

Robustness

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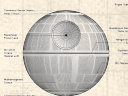
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Self-Organized Criticality

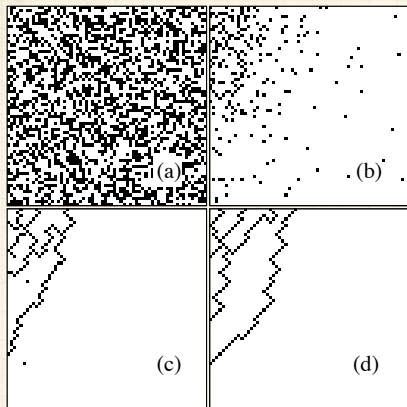
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HOT Forests



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P_{ij} has a
Gaussian decay

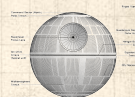
[5]

- 🧱 Optimized forests do well on average
- 🧱 But rare extreme events occur

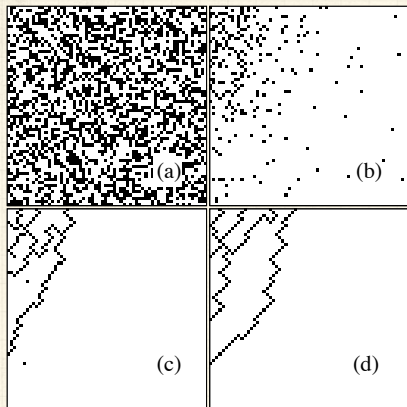
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HOT Forests



$$N = 64$$

$$(a) D = 1$$

$$(b) D = 2$$

$$(c) D = N$$

$$(d) D = N^2$$

P_{ij} has a
Gaussian decay

[5]

- 🧱 Optimized forests do well on average (**robustness**)
- 🧱 But rare extreme events occur

Robustness

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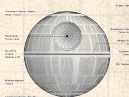
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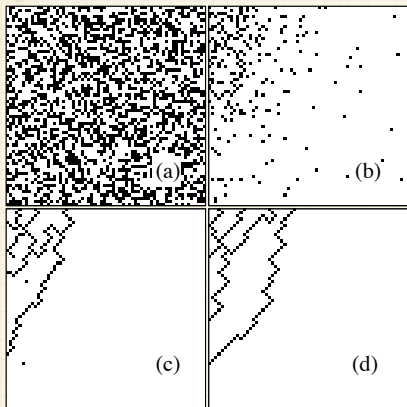
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HOT Forests



$$N = 64$$

$$(a) D = 1$$


$$(b) D = 2$$


$$(c) D = N$$

$$(d) D = N^2$$

P_{ij} has a
Gaussian decay

[5]

 Optimized forests do well on average (**robustness**)

 But rare extreme events occur (**fragility**)

Robustness

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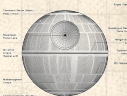
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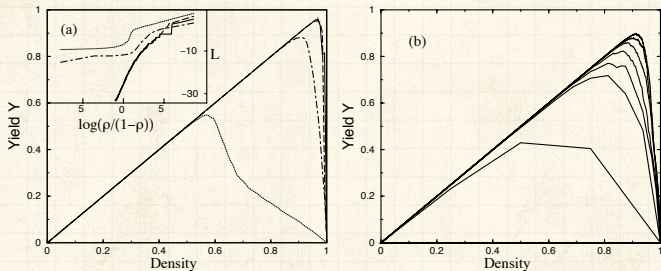
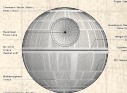


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters $D = 1$ (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with $N = 64$, and (b) for $D = 2$ and $N = 2, 2^2, \dots, 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f \rangle / (1 - \langle f \rangle)]$, on a scale which more clearly differentiates between the curves.

[5]




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 Y = 'the average density of trees left unburned in a configuration after a single spark hits.' [5]

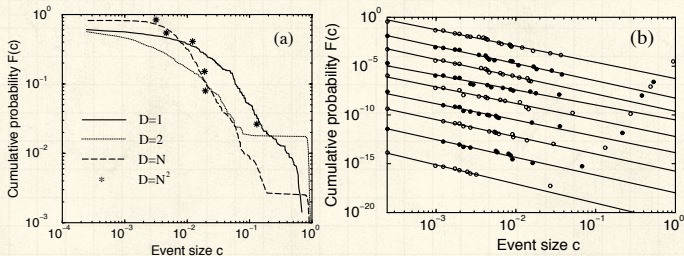
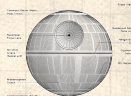


FIG. 3. Cumulative distributions of events $F(c)$: (a) at peak yield for $D = 1, 2, N$, and N^2 with $N = 64$, and (b) for $D = N^2$, and $N = 64$ at equal density increments of 0.1, ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).



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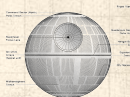
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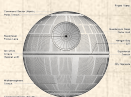
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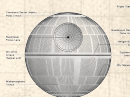
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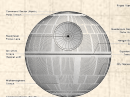
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$D = 1$: Random forests = Percolation ^[11]

 Randomly add trees.



Random Forests

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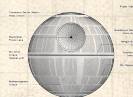
Network robustness

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$D = 1$: Random forests = Percolation ^[11]




 Randomly add trees.

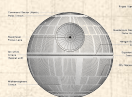
 Below critical density ρ_c , no fires take off.



Random Forests

$D = 1$: Random forests = Percolation ^[11]

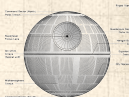
-  Randomly add trees.
-  Below critical density ρ_c , no fires take off.
-  Above critical density ρ_c , percolating cluster of trees burns.



Random Forests

$D = 1$: Random forests = Percolation ^[11]

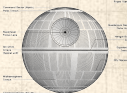
- ☎ Randomly add trees.
- ☎ Below critical density ρ_c , no fires take off.
- ☎ Above critical density ρ_c , percolating cluster of trees burns.
- ☎ Only at ρ_c , the critical density, is there a power-law distribution of tree cluster sizes.



Random Forests

$D = 1$: Random forests = Percolation ^[11]

- ☰ Randomly add trees.
- ☰ Below critical density ρ_c , no fires take off.
- ☰ Above critical density ρ_c , percolating cluster of trees burns.
- ☰ Only at ρ_c , the critical density, is there a power-law distribution of tree cluster sizes.
- ☰ Forest is random and featureless.



HOT forests nutshell:

- Highly structured
- Power law distribution of tree cluster sizes for a broad range of ρ , including below ρ_c .

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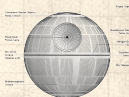
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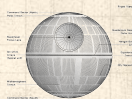
HOT forests nutshell:

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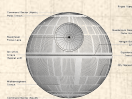
HOT forests nutshell:

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- Forest states are **tolerant**

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HOT forests nutshell:

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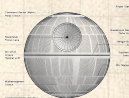
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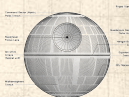
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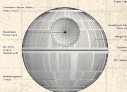
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- If P_{ij} is characterized poorly or changes too fast, failure becomes **highly likely**



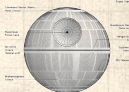
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- Growth is key to toy model which is both algorithmic and physical.



HOT forests nutshell:

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- Uncertainty is okay if well characterized
- If P_{ij} is characterized poorly or changes too fast, failure becomes **highly likely**
- Growth is key to toy model which is both algorithmic and physical.
- HOT theory is more general than just this toy model.



HOT forests—Real data:

“Complexity and Robustness,” Carlson & Dolye [6]

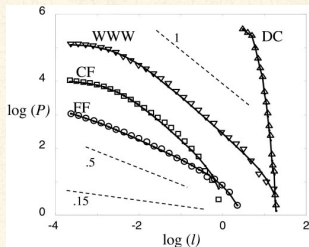


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5, 1$ (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l_i)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the $>10,000$ largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km² (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.



These are CCDFs
(Eek: $P, \mathcal{P}(l \geq l_i)$)



PLR = probability-loss-resource.



Minimize cost subject to resource (barrier) constraints:

$$C = \sum_i p_i l_i$$

given

$$l_i = f(r_i) \text{ and } \sum r_i \leq R.$$



DC = Data Compression.

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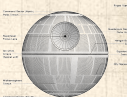
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“Complexity and Robustness,” Carlson & Dolye [6]

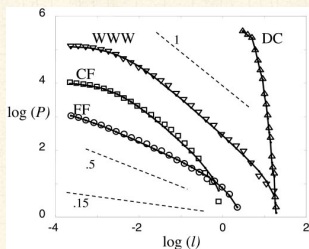


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5, 1$ (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the $>10,000$ largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km² (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.

These are CCDFs
(Eek: $P, \mathcal{P}(l \geq l_i)$)



PLR = probability-loss-resource.



Minimize cost subject to resource (barrier) constraints:

$$C = \sum_i p_i l_i$$

given

$$l_i = f(r_i) \text{ and } \sum r_i \leq R.$$



DC = Data Compression.



Horror: log. Screaming:
“The base! What is the base!
? You monsters!”

Robustness

HOT theory

Narrative causality

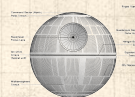
Random forests

Self-Organized Criticality

COLD theory

Network robustness

References



HOT theory:

The abstract story, using figurative forest fires:

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Robustness

HOT theory

Narrative causality

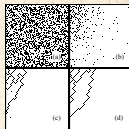
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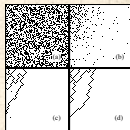


HOT theory:

The abstract story, using figurative forest fires:

 Given some measure of failure size y_i and correlated resource size x_i with relationship

$$y_i = x_i^{-\alpha}, i = 1, \dots, N_{\text{sites}}.$$



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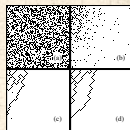
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 Minimize cost:

$$C = \sum_{i=1}^{N_{\text{sites}}} \mathbf{Pr}(y_i) y_i$$



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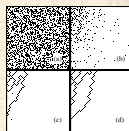
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Subject to $\sum_{i=1}^{N_{\text{sites}}} x_i = \text{constant}.$



1. Cost: Expected size of fire:

$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i.$$

a_i = area of i th site's region, and p_i = avg. prob. of fire at i th site over some time frame.

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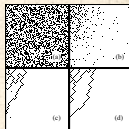
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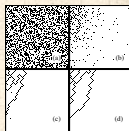
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2. Constraint: building and maintaining firewalls.

Per unit area, and over same time frame:

$$C_{\text{firewalls}} \propto \sum_{i=1}^{N_{\text{sites}}} a_i^{1/2} a_i^{-1}.$$



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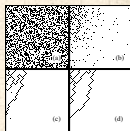
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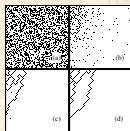
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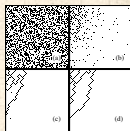
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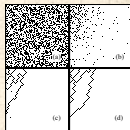
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3. Insert question from assignment 7 to find:

$$\Pr(a_i) \propto a_i^{-\gamma}.$$

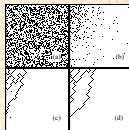


Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x})p(\vec{x})d\vec{x}$$

where C is some cost to be evaluated at each point in space \vec{x} (e.g., $V(\vec{x})^\alpha$),

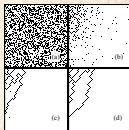


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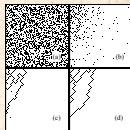
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$$\int R(\vec{x})d\vec{x} = c$$

where c is a constant.



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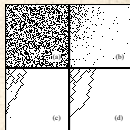
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Claim/observation is that typically [4]

$$V(\vec{x}) \sim R^{-\beta}(\vec{x})$$



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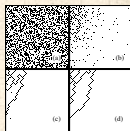
$$\int R(\vec{x})d\vec{x} = c$$

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 Claim/observation is that typically ^[4]

$$V(\vec{x}) \sim R^{-\beta}(\vec{x})$$

 For spatial systems with barriers: $\beta = d$.



The Emperor's Robust-Yet-Fragileness:

Robustness

HOT theory

Narrative causality

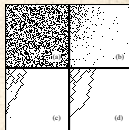
Random forests

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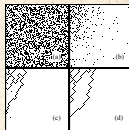
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SOC theory

SOC = Self-Organized Criticality

 Idea: natural dissipative systems exist at 'critical states';

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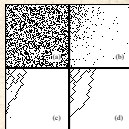
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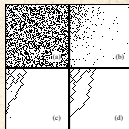
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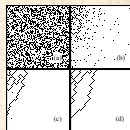
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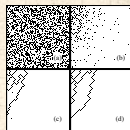
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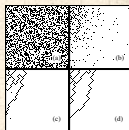
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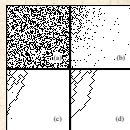
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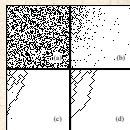
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- 🧱 Much criticism and arguing...



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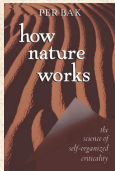
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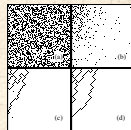
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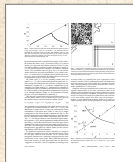
References



“How Nature Works: the Science of Self-Organized Criticality” [a](#) [↗](#)
by Per Bak (1997). [2]

Avalanches of Sand and Rice ...






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Carlson and Doyle,
Proc. Natl. Acad. Sci., **99**, 2538–2545,
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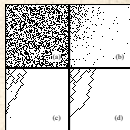
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HOT versus SOC

 Both produce power laws

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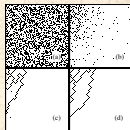
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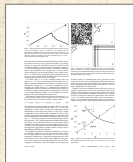
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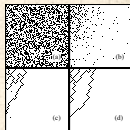
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HOT versus SOC

-  Both produce power laws
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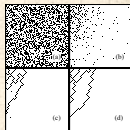
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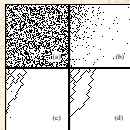


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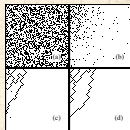


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HOT versus SOC

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-  SOC systems have one special density
-  HOT systems produce specialized structures
-  SOC systems produce generic structures



HOT theory—Summary of designed tolerance ^[6]

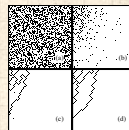
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Table 1. Characteristics of SOC, HOT, and data

	Property	SOC	HOT and Data
1	Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2	Robustness	Generic	Robust, yet fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for power laws	Critical internal fluctuations	Robust performance
7	Exponent α	Small	Large
8	α vs. dimension d	$\alpha \approx (d - 1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large (∞)
10	Increase model resolution	No change	New structures, new sensitivities
11	Response to forcing	Homogeneous	Variable



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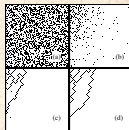
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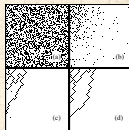
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Avoidance of large-scale failures

 Constrained Optimization with Limited Deviations ^[9]





Robustness

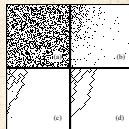
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COLD theory
Network robustness

References

Avoidance of large-scale failures

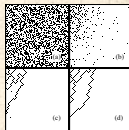
 Constrained Optimization with Limited Deviations ^[9]

 Weight cost of larges losses more strongly



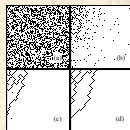
Avoidance of large-scale failures

- ❏ Constrained Optimization with Limited Deviations ^[9]
- ❏ Weight cost of larges losses more strongly
- ❏ Increases average cluster size of burned trees...



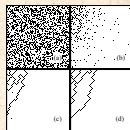
Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations ^[9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe



Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations ^[9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated




Cutoffs

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness

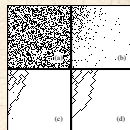
References

Observed:

 Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.



Observed:

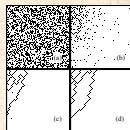
- Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.

- May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-ax^{-\gamma+1}}$$



Outline

Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness

References

The PoCSverse
System
Robustness
38 of 44

Robustness

HOT theory

Narrative causality

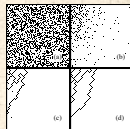
Random forests

Self-Organized Criticality

COLD theory

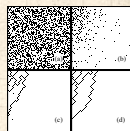
Network robustness

References



We'll return to this later on:

- 🧱 Network robustness.
- 🧱 Albert et al., Nature, 2000:
"Error and attack tolerance of complex networks" ^[1]
- 🧱 General contagion processes acting on complex networks. ^[13, 12]
- 🧱 Similar robust-yet-fragile stories ...

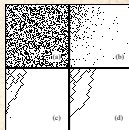


The Emperor's Robust-Yet-Fragileness:




Robustness

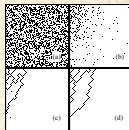
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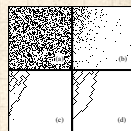


References I

- [1] R. Albert, H. Jeong, and A.-L. Barabási.
Error and attack tolerance of complex networks.
[Nature](#), 406:378–382, 2000. [pdf](#) 
- [2] P. Bak.
How Nature Works: the Science of Self-Organized
Criticality.
Springer-Verlag, New York, 1997.
- [3] P. Bak, C. Tang, and K. Wiesenfeld.
Self-organized criticality - an explanation of $1/f$
noise.
[Phys. Rev. Lett.](#), 59(4):381–384, 1987. [pdf](#) 
- [4] J. M. Carlson and J. Doyle.
Highly optimized tolerance: A mechanism for
power laws in designed systems.
[Phys. Rev. E](#), 60(2):1412–1427, 1999. [pdf](#) 

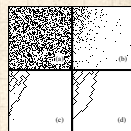


- [5] J. M. Carlson and J. Doyle.
Highly optimized tolerance: Robustness and design in complex systems.
[Phys. Rev. Lett., 84\(11\):2529–2532, 2000. pdf](#) ↗
- [6] J. M. Carlson and J. Doyle.
Complexity and robustness.
[Proc. Natl. Acad. Sci., 99:2538–2545, 2002. pdf](#) ↗
- [7] J. Doyle.
Guaranteed margins for LQG regulators.
[IEEE Transactions on Automatic Control, 23:756–757, 1978. pdf](#) ↗



References III

- [8] H. J. Jensen.
Self-Organized Criticality: Emergent Complex Behavior in Physical and Biological Systems.
Cambridge Lecture Notes in Physics. Cambridge University Press, Cambridge, UK, 1998.
- [9] M. E. J. Newman, M. Girvan, and J. D. Farmer.
Optimal design, robustness, and risk aversion.
Phys. Rev. Lett., 89:028301, 2002.
- [10] D. Sornette.
Critical Phenomena in Natural Sciences.
Springer-Verlag, Berlin, 1st edition, 2003.
- [11] D. Stauffer and A. Aharony.
Introduction to Percolation Theory.
Taylor & Francis, Washington, D.C., Second edition, 1992.



- [12] D. J. Watts and P. S. Dodds.
Influentials, networks, and public opinion
formation.
[Journal of Consumer Research](#), 34:441–458, 2007.

pdf 

- [13] D. J. Watts, P. S. Dodds, and M. E. J. Newman.
Identity and search in social networks.
[Science](#), 296:1302–1305, 2002. pdf 