

Random Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394, 2022-2023 | @pocsvox

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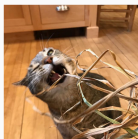
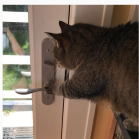
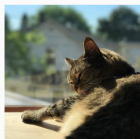
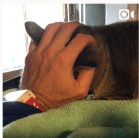
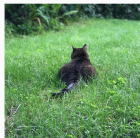
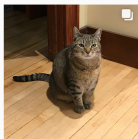
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 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 

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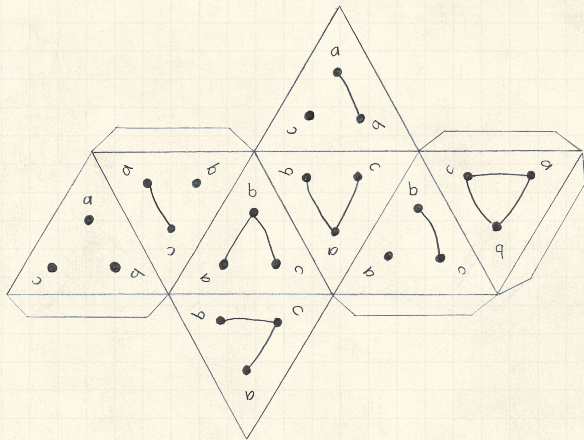
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Some important models:

1. Generalized random networks;
2. Small-world networks;
3. Generalized affiliation networks;
4. Scale-free networks;
5. Statistical generative models (p^*).



Random network generator for $N = 3$:



Get your own exciting generator [here](#)



As $N \nearrow$, polyhedral die rapidly becomes a ball...

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Pure, abstract random networks:

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Pure, abstract random networks:

 Consider set of all networks with N labelled nodes and m edges.

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Random networks

Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one **randomly chosen** network from this set.

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Pure, abstract random networks:

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 To be clear: each network is **equally** probable.



Random networks

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- Sometimes equiprobability is a good assumption, but it is always an assumption.

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Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one **randomly chosen** network from this set.
- To be clear: each network is **equally** probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or **ER graphs**.



Random networks—basic features:

 Number of possible edges:

$$0 \leq m \leq \binom{N}{2} = \frac{N(N-1)}{2}$$

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 Limit of $m = \binom{N}{2}$: complete or fully-connected graph.

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 Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2} N(N-1)}.$$

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 Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.

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 Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.

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 **Real world:** links are usually costly so real networks are almost always **sparse**.

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How to build standard random networks:

 Given N and m .

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Random networks

How to build standard random networks:

- Given N and m .
- Two probabilistic methods

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Random networks

How to build standard random networks:



Given N and m .



Two probabilistic methods (we'll see a third later on)

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Random networks

How to build standard random networks:

- Given N and m .
- Two probabilistic methods (we'll see a third later on)
 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p .

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Random networks

How to build standard random networks:

- Given N and m .
- Two probabilistic methods (we'll see a third later on)
 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p .
 2. Take N nodes and add exactly m links by selecting edges without replacement.

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Random networks

How to build standard random networks:

- Given N and m .
- Two probabilistic methods (we'll see a third later on)
 - Connect each of the $\binom{N}{2}$ pairs with appropriate probability p .
 - Useful for theoretical work.
 - Take N nodes and add exactly m links by selecting edges without replacement.

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 - 🧱 **Algorithm:** Randomly choose a pair of nodes i and j , $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.



Random networks

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- Given N and m .
- Two probabilistic methods (we'll see a third later on)
 - Connect each of the $\binom{N}{2}$ pairs with appropriate probability p .
 - Useful for theoretical work.**
 - Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm:** Randomly choose a pair of nodes i and j , $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - Best for adding relatively small numbers of links (most cases).



Random networks


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 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - 📦 **Algorithm:** Randomly choose a pair of nodes i and j , $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - 📦 Best for adding relatively small numbers of links (most cases).
 - 📦 1 and 2 are effectively equivalent for large N .



Random networks

A few more things:

 For method 1, # links is probabilistic:

$$\langle m \rangle = p \binom{N}{2}$$

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Random networks

A few more things:

 For method 1, # links is probabilistic:

$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

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$$\langle m \rangle = p \binom{N}{2} = p \frac{1}{2} N(N-1)$$

 So the expected or **average degree** is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

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 Which is what it should be...

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Which is what it should be...

If we keep $\langle k \rangle$ constant then $p \propto 1/N \rightarrow 0$ as $N \rightarrow \infty$.



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Random networks: examples

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Example realizations of random networks

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Random networks: examples

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Example realizations of random networks

 $N = 500$

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Random networks: examples

Next slides:

Example realizations of random networks

 $N = 500$

 Vary m , the number of edges from 100 to 1000.

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Example realizations of random networks

 $N = 500$

 Vary m , the number of edges from 100 to 1000.

 Average degree $\langle k \rangle$ runs from 0.4 to 4.



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Next slides:

Example realizations of random networks

 $N = 500$

 Vary m , the number of edges from 100 to 1000.

 Average degree $\langle k \rangle$ runs from 0.4 to 4.

 Look at full network plus the largest component.



Random networks: examples for $N=500$

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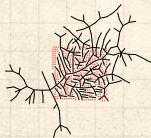
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$m = 100$
 $\langle k \rangle = 0.4$



$m = 200$
 $\langle k \rangle = 0.8$



$m = 230$
 $\langle k \rangle = 0.92$



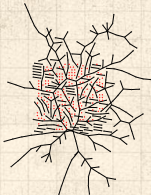
$m = 240$
 $\langle k \rangle = 0.96$



$m = 250$
 $\langle k \rangle = 1$



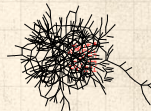
$m = 260$
 $\langle k \rangle = 1.04$



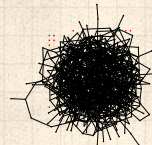
$m = 280$
 $\langle k \rangle = 1.12$



$m = 300$
 $\langle k \rangle = 1.2$



$m = 500$
 $\langle k \rangle = 2$



$m = 1000$
 $\langle k \rangle = 4$

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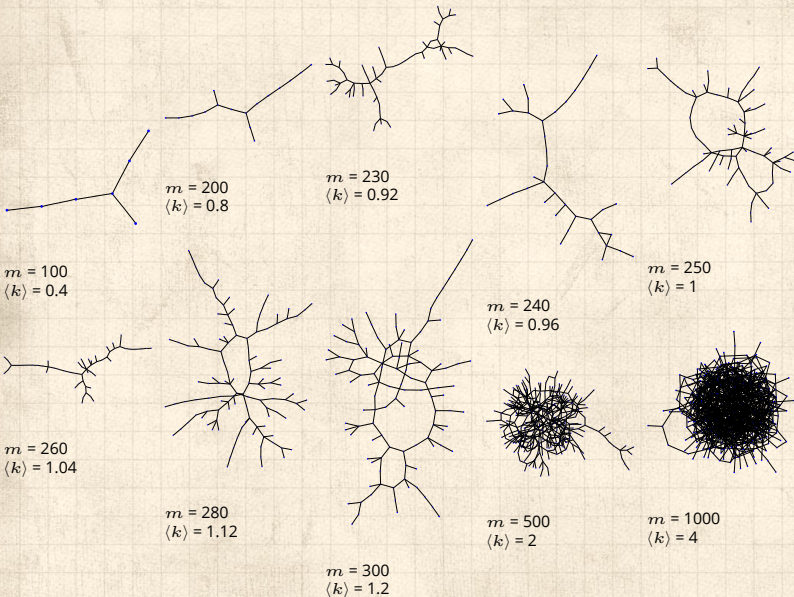
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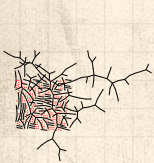
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$m = 250$
 $\langle k \rangle = 1$



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 $\langle k \rangle = 1$



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 $\langle k \rangle = 1$



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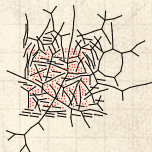
$m = 250$
 $\langle k \rangle = 1$



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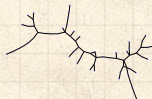
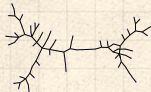
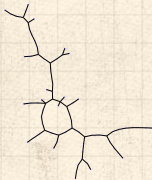
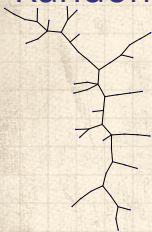
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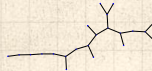
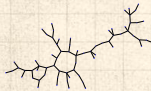
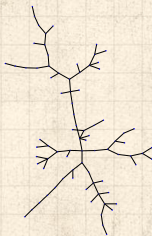
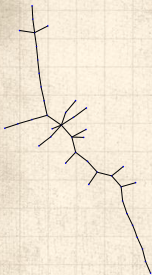
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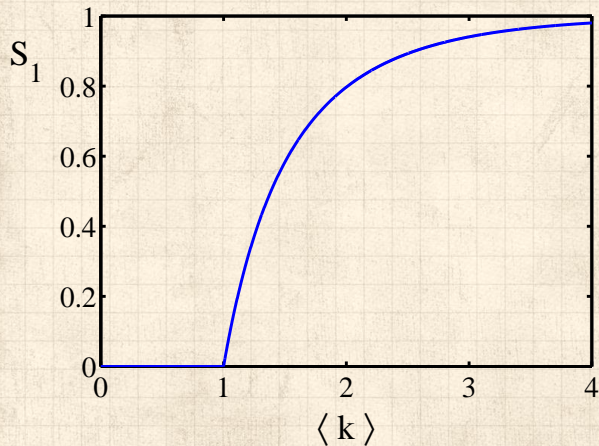
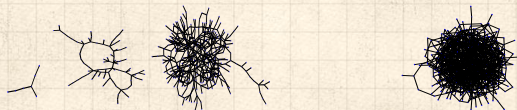
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Clustering in random networks:



For construction method 1, what is the clustering coefficient for a finite network?

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Clustering in random networks:

- For construction method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: [7]

$$C_2 = \frac{3 \times \#triangles}{\#triples}$$

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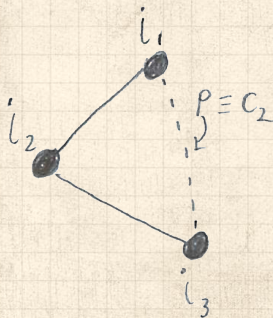
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For construction method 1, what is the clustering coefficient for a finite network?

Consider triangle/triple clustering coefficient: [7]

$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

Recall: C_2 = probability that two friends of a node are also friends.



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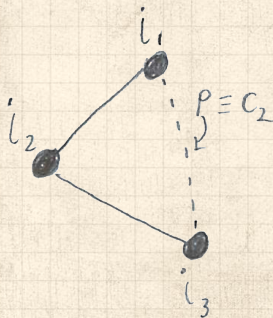
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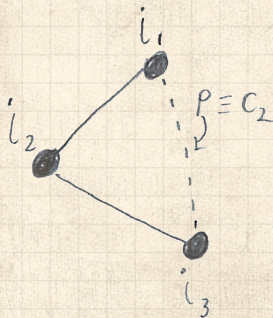
$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

Recall: C_2 = probability that two friends of a node are also friends.

Or: C_2 = probability that a triple is part of a triangle.

For standard random networks, we have simply that

$$C_2 = p.$$



Clustering in random networks:



So for large random networks ($N \rightarrow \infty$), clustering drops to zero.

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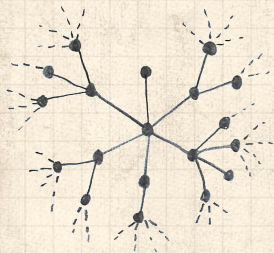
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Clustering in random networks:



So for large random networks ($N \rightarrow \infty$), clustering drops to zero.



Key structural feature of random networks is that they locally look like pure branching networks

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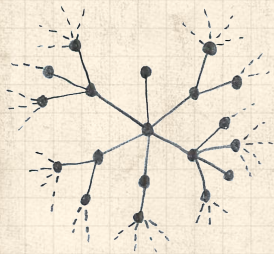
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Clustering in random networks:



So for large random networks ($N \rightarrow \infty$), clustering drops to zero.



Key structural feature of random networks is that they locally look like **pure branching networks**



No small loops.



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Degree distribution:

 Recall P_k = probability that a randomly selected node has degree k .

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Degree distribution:

- Recall P_k = probability that a randomly selected node has degree k .
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- Each connection occurs with probability p , each non-connection with probability $(1 - p)$.
- Therefore have a binomial distribution 

$$P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$



Limiting form of $P(k; p, N)$:

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Limiting form of $P(k; p, N)$:



Our degree distribution:

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What happens as $N \rightarrow \infty$?

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We must end up with the normal distribution right?



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So examine limit of $P(k; p, N)$ when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k \rangle = p(N-1) = \text{constant}$.

$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \rightarrow \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$



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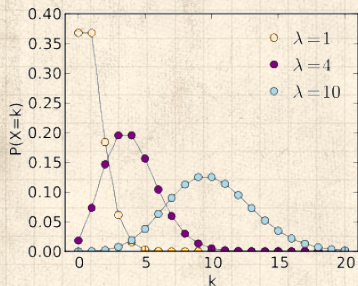


This is a Poisson distribution  with mean $\langle k \rangle$.



Poisson basics:

$$P(k; \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$



$\lambda > 0$



$k = 0, 1, 2, 3, \dots$



Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.



e.g.:
phone calls/minute,
horse-kick deaths.



'Law of small numbers'



Poisson basics:



Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

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 Checking:

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Poisson basics:

Mean degree: we must have

$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k; \langle k \rangle).$$

Checking:

$$\begin{aligned} \sum_{k=0}^{\infty} k P(k; \langle k \rangle) &= \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \\ &= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!} \\ &= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!} \\ &= \langle k \rangle e^{-\langle k \rangle} \sum_{i=0}^{\infty} \frac{\langle k \rangle^i}{i!} = \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} = \langle k \rangle \end{aligned}$$

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In CocoNuTs, we find a different, crazier way of doing this...



Poisson basics:

 The **variance** of degree distributions for random networks turns out to be **very important**.

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Poisson basics:

 The **variance** of degree distributions for random networks turns out to be **very important**.

 Using calculation similar to one for finding $\langle k \rangle$ we find the **second moment** to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



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🧱 So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

🧱 Note: This is a special property of Poisson distribution and can trip us up...



Neural reboot (NR):

Unrelated: Feline elevation

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General random networks



So... standard random networks have a Poisson degree distribution

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General random networks

- So... standard random networks have a Poisson degree distribution
- Generalize to arbitrary degree distribution P_k .

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General random networks

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$$P(\text{link between } i \text{ and } j) \propto w_i w_j.$$



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 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.



General random networks

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- But we'll be more interested in
 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
 2. Examining mechanisms that lead to networks with certain degree distributions.



Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

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Random networks: examples

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Example realizations of random networks with power law degree distributions:

 $N = 1000$.

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Random networks: examples

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Example realizations of random networks with power law degree distributions:

 $N = 1000$.

 $P_k \propto k^{-\gamma}$ for $k \geq 1$.

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 Set $P_0 = 0$ (no isolated nodes).

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Random networks: examples

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Example realizations of random networks with power law degree distributions:

-  $N = 1000$.
-  $P_k \propto k^{-\gamma}$ for $k \geq 1$.
-  Set $P_0 = 0$ (no isolated nodes).
-  Vary exponent γ between 2.10 and 2.91.

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-  Again, look at full network plus the largest component.

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-  Vary exponent γ between 2.10 and 2.91.
-  Again, look at full network plus the largest component.
-  Apart from degree distribution, wiring is random.

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Random networks: examples for $N=1000$

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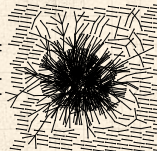
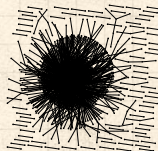
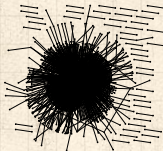
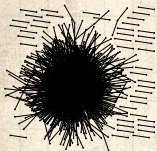
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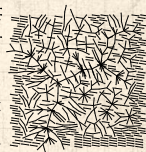
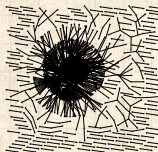
$\gamma = 2.1$
 $\langle k \rangle = 3.448$

$\gamma = 2.19$
 $\langle k \rangle = 2.986$

$\gamma = 2.28$
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$\gamma = 2.46$
 $\langle k \rangle = 1.856$



$\gamma = 2.55$
 $\langle k \rangle = 1.712$

$\gamma = 2.64$
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$\gamma = 2.73$
 $\langle k \rangle = 1.862$

$\gamma = 2.82$
 $\langle k \rangle = 1.386$

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 $\langle k \rangle = 1.49$

Random networks: largest components

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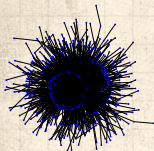
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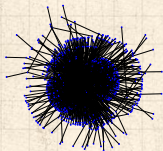
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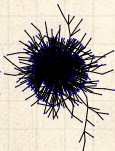
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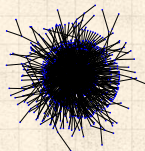
$\gamma = 2.1$
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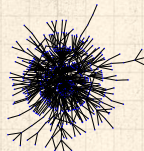
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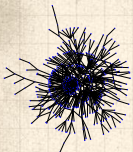
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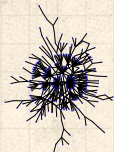
$\gamma = 2.37$
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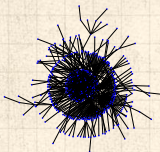
$\gamma = 2.46$
 $\langle k \rangle = 1.856$



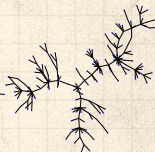
$\gamma = 2.55$
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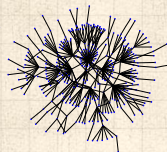
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Generalized random networks:

 Arbitrary degree distribution P_k .

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Generalized random networks:

- Arbitrary degree distribution P_k .
- Create (unconnected) nodes with degrees sampled from P_k .

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Generalized random networks:

-  Arbitrary degree distribution P_k .
-  Create (unconnected) nodes with degrees sampled from P_k .
-  Wire nodes together randomly.




Generalized random networks:

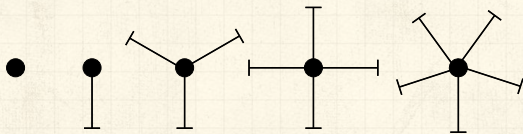
-  Arbitrary degree distribution P_k .
-  Create (unconnected) nodes with degrees sampled from P_k .
-  Wire nodes together randomly.
-  Create ensemble to test deviations from randomness.



Building random networks: Stubs

Phase 1:

 **Idea:** start with a soup of unconnected nodes with stubs (half-edges):



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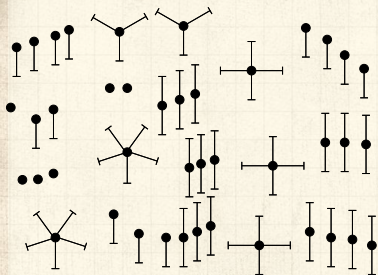
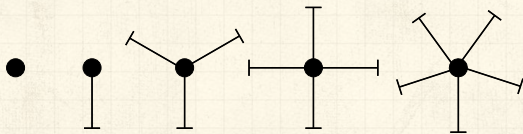
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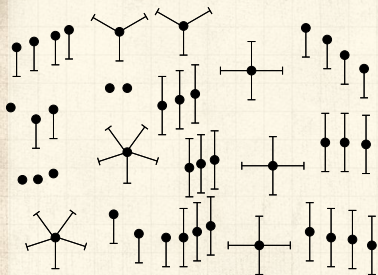
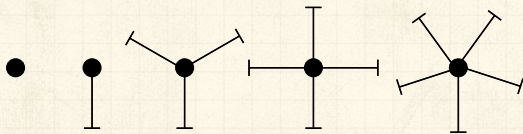
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Building random networks: Stubs

Phase 1:

 **Idea:** start with a soup of unconnected nodes with stubs (half-edges):



Randomly select stubs (not nodes!) and connect them.

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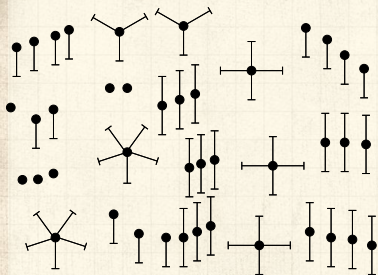
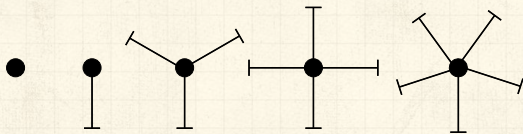
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Building random networks: Stubs

Phase 1:

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Must have an even number of stubs.

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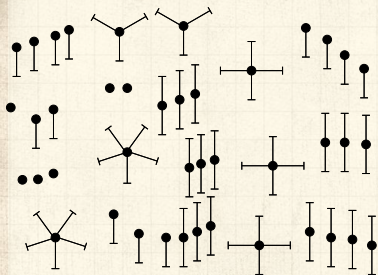
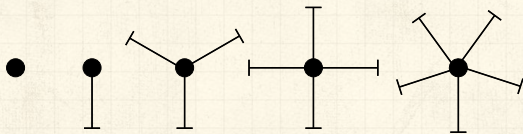
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Building random networks: Stubs


Phase 1:

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 Randomly select stubs (not nodes!) and connect them.

 Must have an even number of stubs.

 Initially allow **self-** and **repeat** connections.

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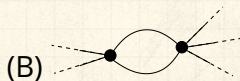
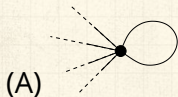
References



Building random networks: First rewiring

Phase 2:

 Now find any (A) self-loops and (B) repeat edges and **randomly rewire** them.



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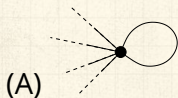
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Phase 2:

 Now find any (A) self-loops and (B) repeat edges and **randomly rewire** them.



 **Being careful:** we can't change the degree of any node, so we can't simply move links around.



Building random networks: First rewiring

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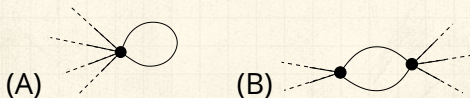
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Phase 2:

 Now find any (A) self-loops and (B) repeat edges and **randomly rewire** them.

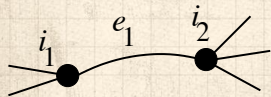


 **Being careful:** we can't change the degree of any node, so we can't simply move links around.

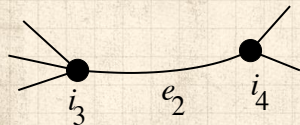
 **Simplest solution:** randomly rewire **two edges** at a time.



General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
a random edge)



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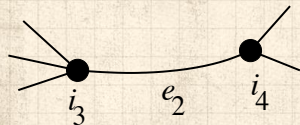
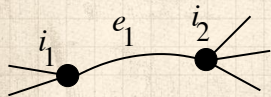
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General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
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Check to make sure edges are
disjoint.

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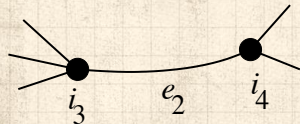
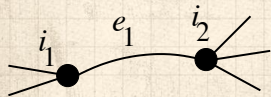
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General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
a random edge)



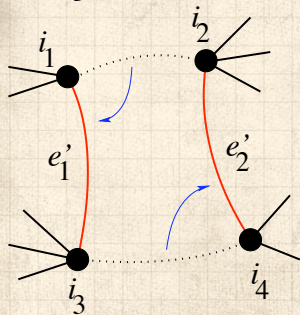
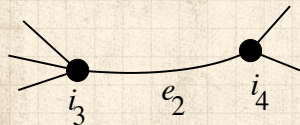
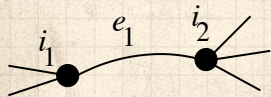
Check to make sure edges are
disjoint.



Rewire one end of each edge.



General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
a random edge)



Check to make sure edges are
disjoint.



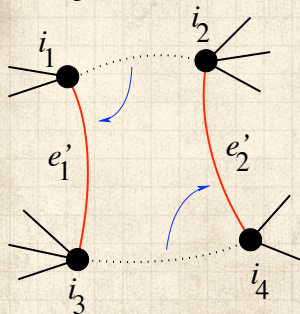
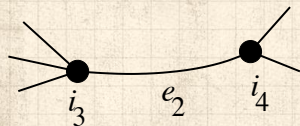
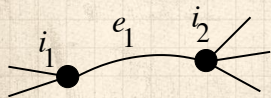
Rewire one end of each edge.



Node degrees **do not change**.



General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
a random edge)



Check to make sure edges are
disjoint.



Rewire one end of each edge.



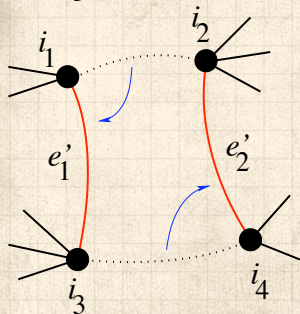
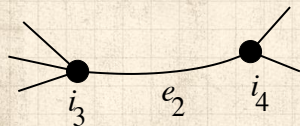
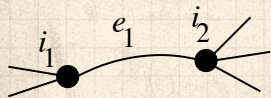
Node degrees **do not change**.



Works if e_1 is a self-loop or
repeated edge.



General random rewiring algorithm



Randomly choose **two edges**.
(Or choose problem edge and
a random edge)



Check to make sure edges are
disjoint.



Rewire one end of each edge.



Node degrees **do not change**.



Works if e_1 is a self-loop or
repeated edge.



Same as finding on/off/on/off
4-cycles. and rotating them.



Sampling random networks

Phase 2:



Use rewiring algorithm to remove all self and repeat loops.

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Phase 2:

 Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

 **Randomize network** wiring by applying rewiring algorithm liberally.

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Sampling random networks

Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

- Randomize network** wiring by applying rewiring algorithm liberally.
- Rule of thumb:** # Rewirings $\simeq 10 \times$ # edges [5].

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Random sampling



Problem with only joining up stubs is **failure** to randomly sample from all possible networks.

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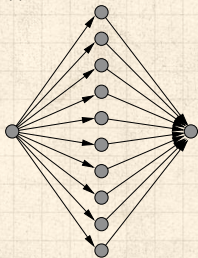
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 **Problem** with only joining up stubs is **failure** to randomly sample from all possible networks.

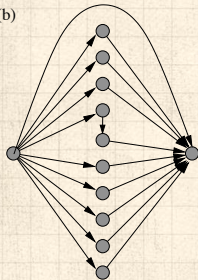
 Example from Milo et al. (2003) [5]:

(a)

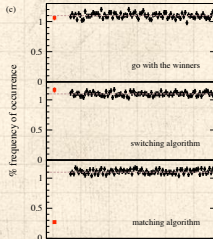


1 configuration

(b)



90 configurations



Sampling random networks



What if we have P_k instead of N_k ?

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Sampling random networks



What if we have P_k instead of N_k ?



Must now create nodes before start of the construction algorithm.

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Sampling random networks

- What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .

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Sampling random networks

- What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .
- Easy to do exactly numerically since k is discrete.

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Sampling random networks

- What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .
- Easy to do exactly numerically since k is discrete.
- Note:** not all P_k will always give nodes that can be wired together.

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Network motifs



Idea of **motifs** ^[8] introduced by Shen-Orr, Alon et al. in 2002.

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Network motifs

 Idea of **motifs** ^[8] introduced by Shen-Orr, Alon et al. in 2002.

 Looked at gene expression within full context of transcriptional regulation networks.

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Network motifs

- 🧱 Idea of **motifs** [8] introduced by Shen-Orr, Alon et al. in 2002.
- 🧱 Looked at gene expression within full context of transcriptional regulation networks.
- 🧱 Specific example of Escherichia coli.

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Network motifs

- Idea of **motifs** [8] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of **transcriptional regulation networks**.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).

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Network motifs

- Idea of **motifs** [8] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of **transcriptional regulation networks**.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .

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Network motifs

- Idea of **motifs** ^[8] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of **transcriptional regulation networks**.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for **certain subnetworks (motifs)** that appeared more or less often than expected

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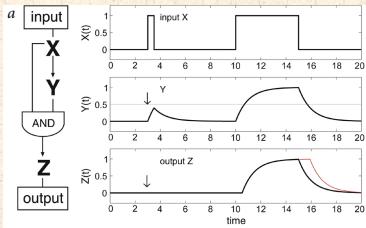
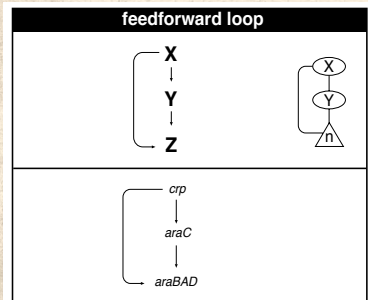
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 Z only turns on in response to sustained activity in X .



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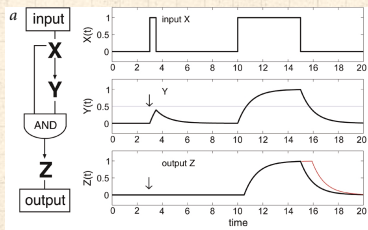
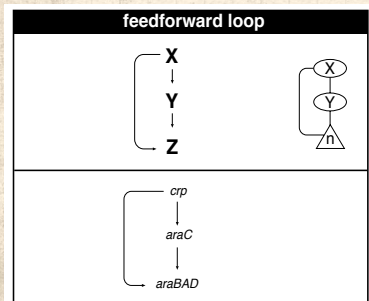
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 Z only turns on in response to sustained activity in X .

 Turning off X rapidly turns off Z .

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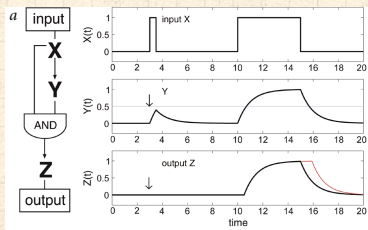
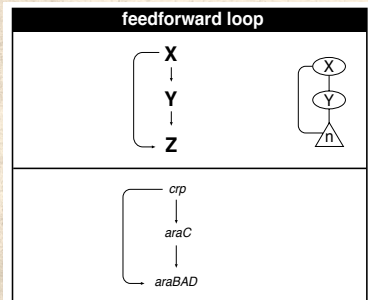
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 Z only turns on in response to sustained activity in X .

 Turning off X rapidly turns off Z .

 Analogy to elevator doors.

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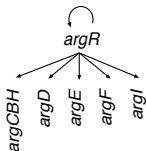
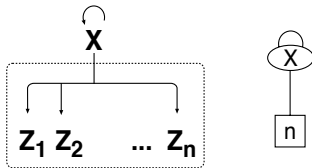
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single input module (SIM)



Master switch.

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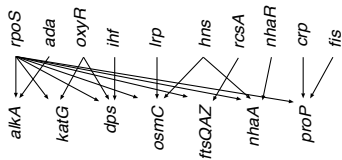
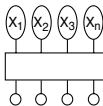
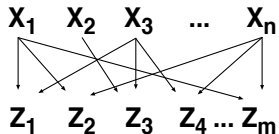
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dense overlapping regulons (DOR)



Network motifs



Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

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Network motifs



Note: selection of motifs to test is reasonable but nevertheless ad-hoc.



For more, see work carried out by Wiggins *et al.* at Columbia.

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The edge-degree distribution:



The degree distribution P_k is fundamental for our description of many complex networks

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The edge-degree distribution:

-  The degree distribution P_k is fundamental for our description of many complex networks
-  Again: P_k is the degree of **randomly chosen node**.

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The edge-degree distribution:

- 🧱 The degree distribution P_k is fundamental for our description of many complex networks
- 🧱 Again: P_k is the degree of **randomly chosen node**.
- 🧱 A second very important distribution arises from **choosing randomly on edges** rather than on nodes.

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The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks
- Again: P_k is the degree of **randomly chosen node**.
- A second very important distribution arises from **choosing randomly on edges** rather than on nodes.
- Define Q_k to be the probability the node at a **random end** of a **randomly chosen edge** has degree k .

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The edge-degree distribution:

- 🧱 The degree distribution P_k is fundamental for our description of many complex networks
- 🧱 Again: P_k is the degree of **randomly chosen node**.
- 🧱 A second very important distribution arises from **choosing randomly on edges** rather than on nodes.
- 🧱 Define Q_k to be the probability the node at a **random end** of a **randomly chosen edge** has degree k .
- 🧱 Now choosing nodes based on their degree (i.e., size):

$$Q_k \propto kP_k$$

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The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks
- Again: P_k is the degree of **randomly chosen node**.
- A second very important distribution arises from **choosing randomly on edges** rather than on nodes.
- Define Q_k to be the probability the node at a **random end** of a **randomly chosen edge** has degree k .
- Now choosing nodes based on their degree (i.e., size):

$$Q_k \propto kP_k$$

- Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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- Big deal:** Rich-get-richer mechanism is built into this selection process.

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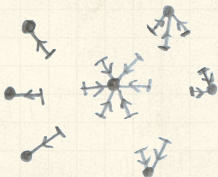
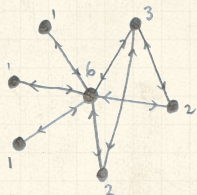
References





Probability of randomly selecting a node of degree k by choosing from nodes:

$$P_1 = 3/7, P_2 = 2/7, P_3 = 1/7, P_6 = 1/7.$$



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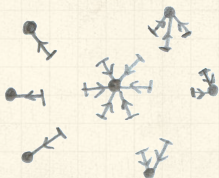
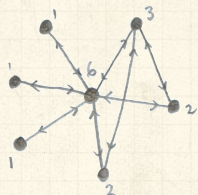
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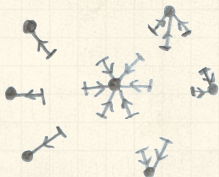
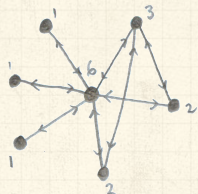
$$P_1 = 3/7, P_2 = 2/7, P_3 = 1/7, P_6 = 1/7.$$



Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:

$$Q_1 = 3/16, Q_2 = 4/16, Q_3 = 3/16, Q_6 = 6/16.$$





Probability of randomly selecting a node of degree k by choosing from nodes:

$$P_1 = 3/7, P_2 = 2/7, P_3 = 1/7, P_6 = 1/7.$$



Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:


$$Q_1 = 3/16, Q_2 = 4/16, Q_3 = 3/16, Q_6 = 6/16.$$



Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$R_0 = 3/16, R_1 = 4/16, R_2 = 3/16, R_5 = 6/16.$$

The edge-degree distribution:

 For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.

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The edge-degree distribution:

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 Useful variant on Q_k :

R_k = probability that a friend of a random node has k other friends.

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0} (k'+1)P_{k'+1}}$$

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 Equivalent to friend having degree $k+1$.



The edge-degree distribution:


 For networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.


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 Equivalent to friend having degree $k+1$.

 **Natural question:** what's the expected number of other friends that one friend has?



The edge-degree distribution:

 Given R_k is the probability that a friend has k other friends, then the average number of **friends' other friends** is

$$\langle k \rangle_R = \sum_{k=0}^{\infty} k R_k$$

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$$\begin{aligned}\langle k \rangle_R &= \sum_{k=0}^{\infty} k R_k = \sum_{k=0}^{\infty} k \frac{(k+1)P_{k+1}}{\langle k \rangle} \\ &= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1)P_{k+1}\end{aligned}$$

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(where we have sneakily matched up indices)

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(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad (\text{using } j = k+1)$$

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The edge-degree distribution:

 Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for **all** random networks, **independent of degree distribution**.

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 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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 Therefore:

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 Again, neatness of results is a special property of the Poisson distribution.

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 Again, neatness of results is a special property of the Poisson distribution.

 So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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In fact, R_k is rather special for pure random networks ...

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 Substituting

$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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 #samesies.

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Two reasons why this matters

Reason #1:

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Two reasons why this matters

Reason #1:

 Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

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 Key: Average depends on the **1st and 2nd moments** of P_k and **not just the 1st moment.**

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 Key: Average depends on the **1st and 2nd moments** of P_k and **not just the 1st moment**.

 Three peculiarities:

1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k - 1) \rangle$.

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Two reasons why this matters

Reason #1:

 Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle) = \langle k^2 \rangle - \langle k \rangle.$$

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4. See also: class size paradoxes (nod to: Gelman)

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Two reasons why this matters

More on peculiarity #3:

 A node's average # of friends: $\langle k \rangle$

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Two reasons why this matters

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 A node's average # of friends: $\langle k \rangle$

 Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$

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 Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$

 Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

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$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2}$$

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 So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as its friends.

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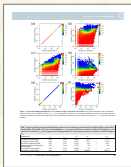
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 So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as its friends.

 Intuition: for networks, the more connected a node, the more likely it is to be chosen as a friend.





“Generalized friendship paradox in complex networks: The case of scientific collaboration” [↗](#)

Eom and Jo,
Nature Scientific Reports, **4**, 4603, 2014. ^[3]

Your friends really are ~~monsters~~ #winners:¹

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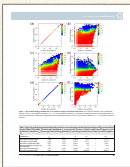
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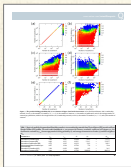
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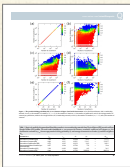
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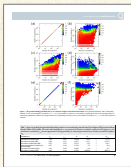
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-  Research possibility: The Frenemy Paradox.

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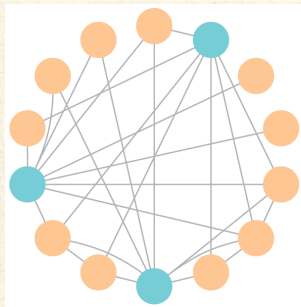
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Related disappointment:

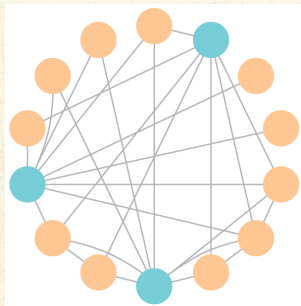


Nodes see their friends'
color choices.



¹<https://www.washingtonpost.com/graphics/business/wonkblog/majority-illusion/>

Related disappointment:



Nodes see their friends'
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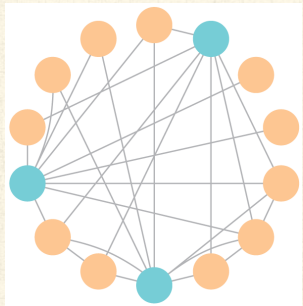


Which color is more
popular?¹



¹<https://www.washingtonpost.com/graphics/business/wonkblog/majority-illusion/>

Related disappointment:



Nodes see their friends' color choices.



Which color is more popular?¹




Again: thinking in edge space changes everything.



¹<https://www.washingtonpost.com/graphics/business/wonkblog/majority-illusion/>

Two reasons why this matters

(Big) Reason #2:

 $\langle k \rangle_R$ is **key** to understanding how well random networks are connected together.

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Two reasons why this matters

(Big) Reason #2:

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-  Note: Component = Cluster

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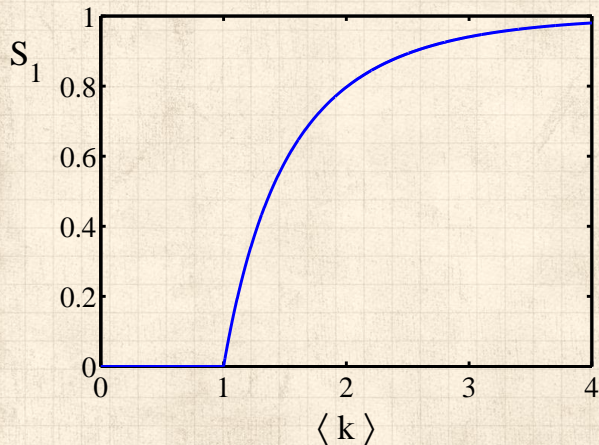
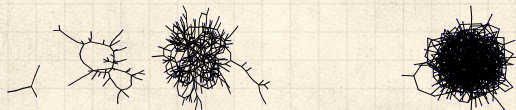
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Giant component



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Structure of random networks

Giant component:

 A giant component exists if when we follow a random edge, we are likely to hit a node with **at least 1** other outgoing edge.

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Structure of random networks

Giant component:

-  A giant component exists if when we follow a random edge, we are likely to hit a node with **at least 1** other outgoing edge.
-  Equivalently, expect exponential growth in node number as we move out from a random node.

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 **Giant component condition** (or percolation condition):


$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$





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 Again, see that the second moment is an essential part of the story.





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 **Giant component condition** (or percolation condition):

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 Again, see that the second moment is an essential part of the story.

 Equivalent statement: $\langle k^2 \rangle > 2\langle k \rangle$



Spreading on Random Networks



For random networks, we know local structure is pure branching.

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Spreading on Random Networks

- For random networks, we know local structure is pure branching.
- Successful spreading is \therefore contingent on **single edges** infecting nodes.

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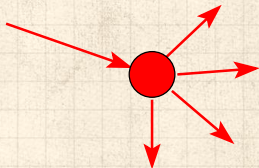


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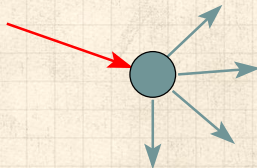
For random networks, we know local structure is pure branching.

Successful spreading is \therefore contingent on **single edges** infecting nodes.

Success



Failure:



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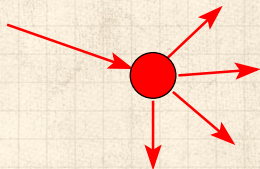


Spreading on Random Networks

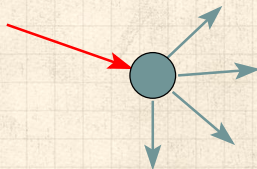
For random networks, we know local structure is pure branching.

Successful spreading is \therefore contingent on **single edges** infecting nodes.

Success



Failure:



Focus on **binary** case with edges and nodes either infected or not.

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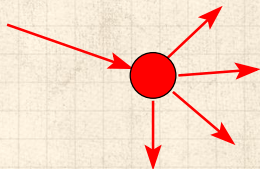


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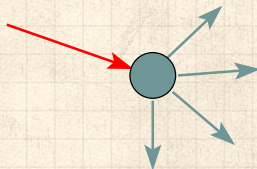
For random networks, we know local structure is pure branching.

Successful spreading is \therefore contingent on **single edges** infecting nodes.

Success



Failure:



Focus on **binary** case with edges and nodes either infected or not.

First big question: for a given network and contagion process, can global spreading from a single seed occur?

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Global spreading condition



We need to find: [2]

R = the average # of infected edges that one random infected edge brings about.



Call **R** the **gain ratio**.

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Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.

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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle}$$

prob. of
connecting to
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$$\underbrace{(k-1)}$$

outgoing
infected
edges

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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \cdot \underbrace{(k-1)}_{\text{\# outgoing infected edges}} \cdot \underbrace{B_{k1}}_{\text{Prob. of infection}}$$

prob. of connecting to a degree k node



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Global spreading condition



Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \cdot (k - 1) \cdot B_{k1} > 1.$$

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Global spreading condition

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 Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k - 1) \bullet B_{k1} > 1.$$

 Case 1–Rampant spreading:

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 **Case 1-Rampant spreading:** If $B_{k1} = 1$

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 **Case 1-Rampant spreading:** If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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 **Good:** This is just our giant component condition again.

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Case 2—Simple disease-like:

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Case 2—Simple disease-like: If $B_{k1} = \beta < 1$

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 A fraction $(1-\beta)$ of edges do not transmit infection.

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 Analogous phase transition to giant component case but **critical value** of $\langle k \rangle$ is **increased**.

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 Aka bond percolation .

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 Aka bond percolation .

 Resulting degree distribution \tilde{P}_k :

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$



Giant component for standard random networks:

 Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

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Giant component for standard random networks:

 Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

 Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

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🧱 Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

When $\langle k \rangle < 1$, all components are finite.



Giant component for standard random networks:

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🧱 Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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🧱 Fine example of a continuous phase transition ↗.



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Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.


When $\langle k \rangle < 1$, all components are finite.

Fine example of a continuous phase transition ↗.

We say $\langle k \rangle = 1$ marks the critical point of the system.



Random networks with skewed P_k :

 e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \geq 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

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 So giant component **always exists** for these kinds of networks.




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 Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

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🧱 So giant component **always exists** for these kinds of networks.


🧱 Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

🧱 How about $P_k = \delta_{kk_0}$?



Giant component

And how big is the largest component?

 Define S_1 as the **size of the largest component**.

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Giant component

And how big is the largest component?

- Define S_1 as the **size of the largest component**.
- Consider an infinite ER random network with average degree $\langle k \rangle$.

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- Let's find S_1 with a back-of-the-envelope argument.

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- Simple connection: $\delta = 1 - S_1$.
- Dirty trick:** If a randomly chosen node is not part of the largest component, then none of its neighbors are.

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Giant component

And how big is the largest component?

- Define S_1 as the **size of the largest component**.
- Consider an infinite ER random network with average degree $\langle k \rangle$.
- Let's find S_1 with a back-of-the-envelope argument.
- Define δ as the probability that a randomly chosen node **does not** belong to the largest component.
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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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- Substitute in Poisson distribution...

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Giant component



Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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Giant component



Carrying on:

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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 Carrying on:

$$\begin{aligned}\delta &= \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ &= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}\end{aligned}$$

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 Carrying on:

$$\begin{aligned}\delta &= \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ &= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \\ &= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle(1-\delta)}.\end{aligned}$$

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Carrying on:

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Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$



Giant component



We can figure out some limits and details for

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$

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Giant component

 We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.

 First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

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Giant component

🧱 We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.

🧱 First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

🧱 As $\langle k \rangle \rightarrow 0$, $S_1 \rightarrow 0$.

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$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

🧱 As $\langle k \rangle \rightarrow 0$, $S_1 \rightarrow 0$.

🧱 As $\langle k \rangle \rightarrow \infty$, $S_1 \rightarrow 1$.

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$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

🧱 As $\langle k \rangle \rightarrow 0$, $S_1 \rightarrow 0$.

🧱 As $\langle k \rangle \rightarrow \infty$, $S_1 \rightarrow 1$.

🧱 Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.



Giant component

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🧱 Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

🧱 Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.

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🧱 As $\langle k \rangle \rightarrow \infty$, $S_1 \rightarrow 1$.

🧱 Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

🧱 Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.

🧱 Really a transcritical bifurcation. ^[9]

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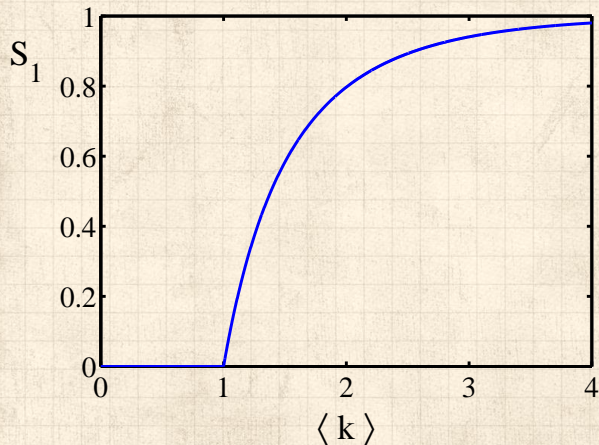
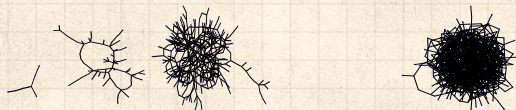
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Turns out we were lucky...

 Our dirty trick **only works for** ER random networks.

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Turns out we were lucky...

 Our dirty trick **only works for** ER random networks.

 **The problem:** We assumed that neighbors have the same probability δ of belonging to the largest component.

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Turns out we were lucky...

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The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.

But we know our friends are different from us...

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Works for ER random networks because
 $\langle k \rangle = \langle k \rangle_R$.

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We need a separate probability δ' for the chance that an edge **leads to** the giant (infinite) component.

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We can sort many things out with **sensible probabilistic arguments...**

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More detailed investigations will profit from a spot of **Generatingfunctionology**.^[10]

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CocoNuTs: We figure out the final size and complete dynamics.

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Neural reboot (NR):

Falling maple leaf

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