A Partial Overview of Complex Networks

Last updated: 2023/05/22, 22:51:18 CEST

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394, 2022–2023| @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
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Outline

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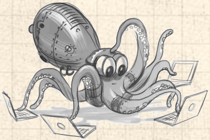
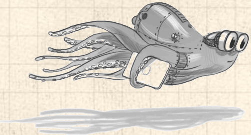




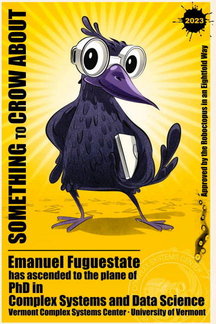
vermontcomplexsystems.org

Leveling up—Scaffolded educational mission:

- Data Science Undergrad.
- Graduate Certificate in Complex Systems and Data Science
- Fall, 2015–: MS in Complex Systems and Data Science
- Fall, 2018–: PhD in ~~The Study of Interesting Things~~ Complex Systems and Data Science

All the words: http://vermontcomplexsystems.org.

Dipoloma-posters:



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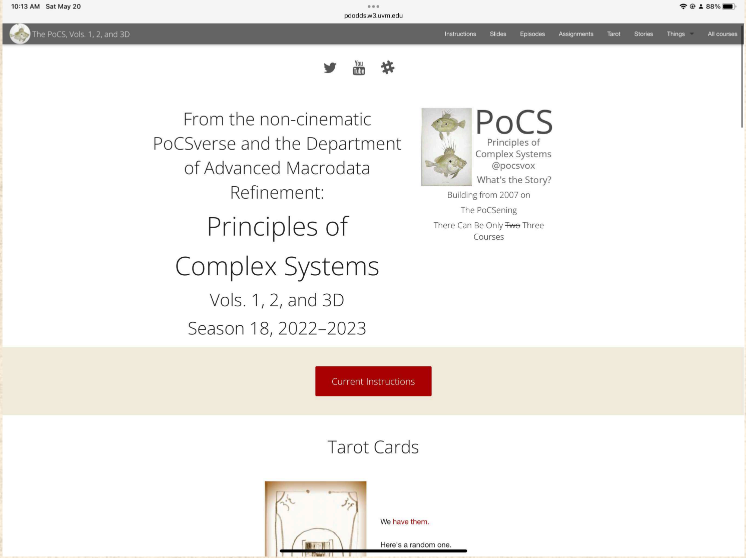

150,000 lines of $\LaTeX$ ...

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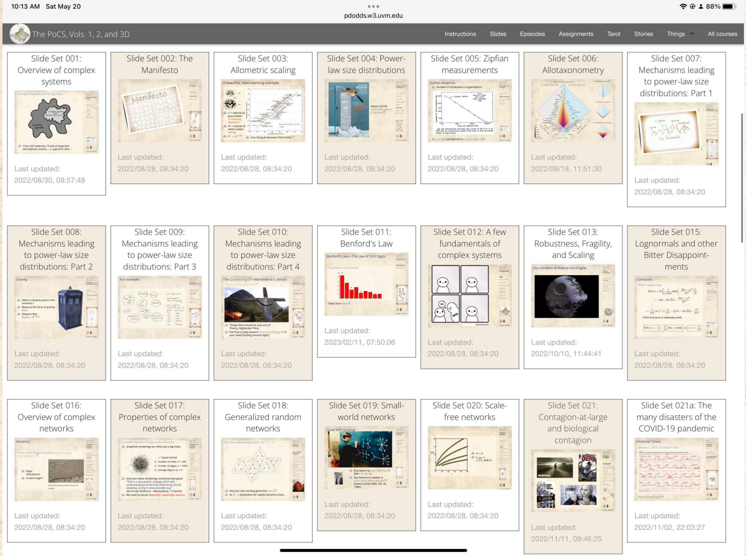

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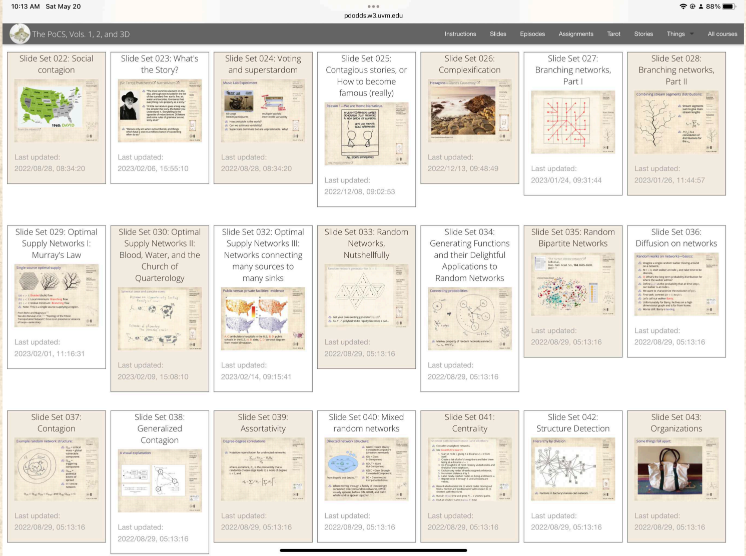

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The PoCS, Vols. 1, 2, and 3D — Instructions | Slides | Episodes | Assignments | Tarot | Stories | Things | All courses

Herbert Simon's rich-get-richer model. Simple, powerful.

Reheated slides on toast:	Freeze-dried snack slides:	Original slides as served in lectures:
64M ; Last updated: 2022/08/28, 03:24:52	9.7M ; Last updated: 2022/08/27, 23:54:10	65M ; Last updated: 2022/08/28, 08:34:20

Covered in these episode(s) and clip(s):

Episode 1: The OG rich-get-richer model (1:52:03)

Clip 1: Intro to Simon vs Mandebrot and the mechanism of rich-get-richerness (6:35)

Clip 2: Observations of Zipfery, 1910 on (12:13)

Clip 3: Herbert Simon #awesomeness (2:18)

Clip 4: Toy model of rich-get-richer (14:51)

Clip 5: Observations about our toy model (7:10)

Clip 6: Krugman's math woes (1:34)

Clip 7: We work through an analysis (14:37)

Clip 8: What we find: Micro-to-macro story and surprising agreement with reality (8:30)

Clip 9: An appraisal of catchphrases (3:53)

Clip 10: Simon's model recap (3:47)

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What's the Story?

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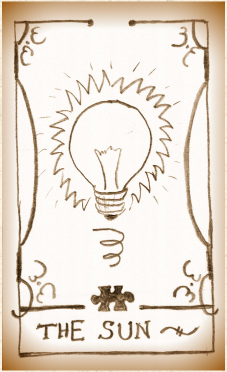
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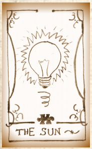
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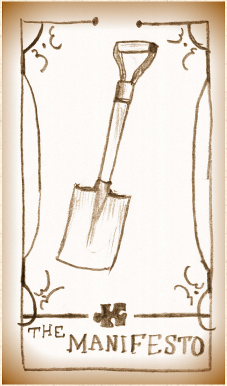
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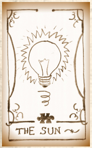
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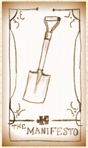
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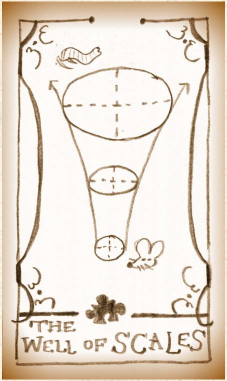
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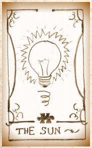
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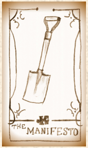
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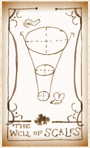
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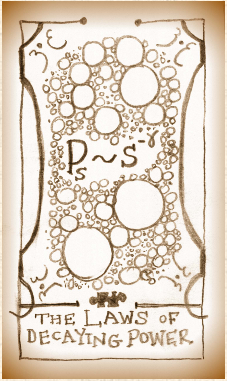
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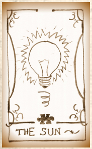
THE LAWS OF DECAYING POWER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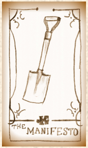
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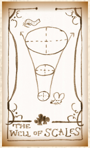
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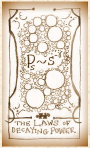
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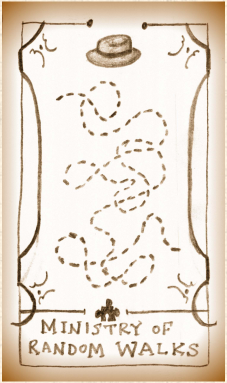
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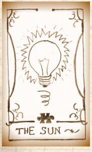
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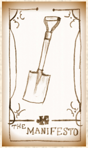
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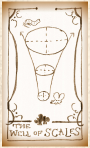
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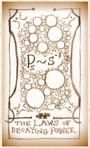
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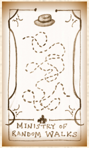
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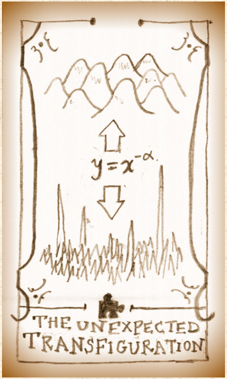
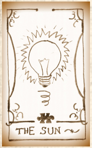

$y=x^{-\alpha}$
THE UNEXPECTED TRANSFIGURATION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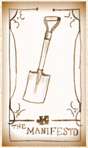
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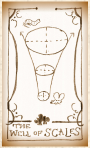
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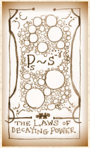
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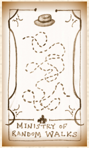
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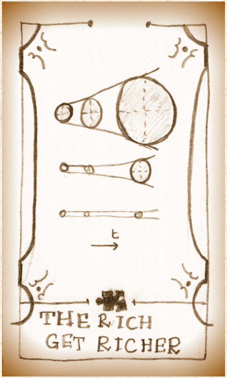
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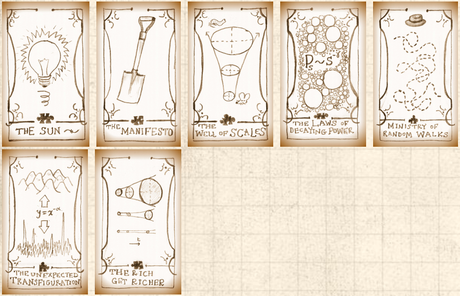
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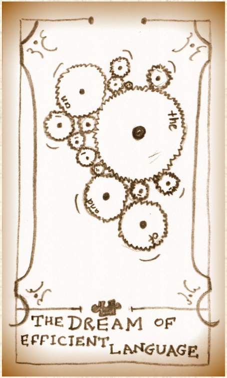
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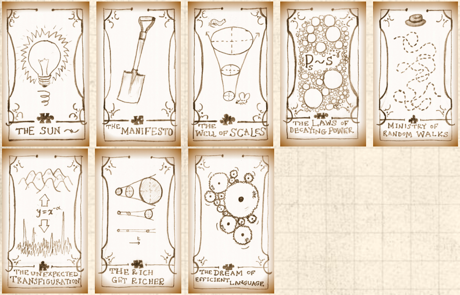
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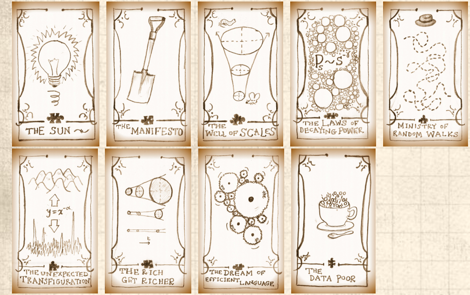
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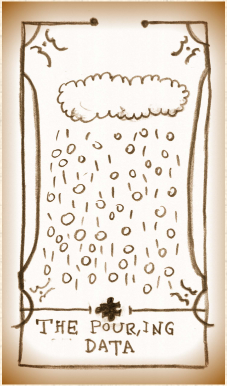
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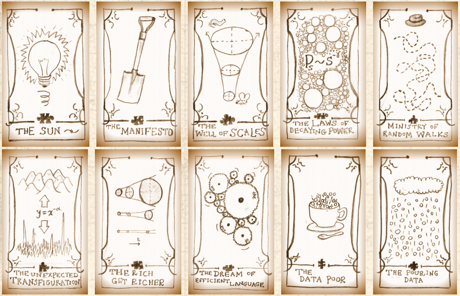
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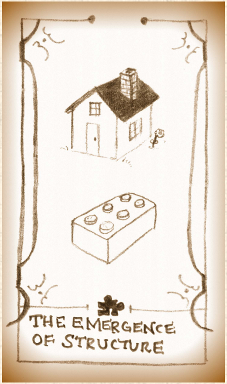
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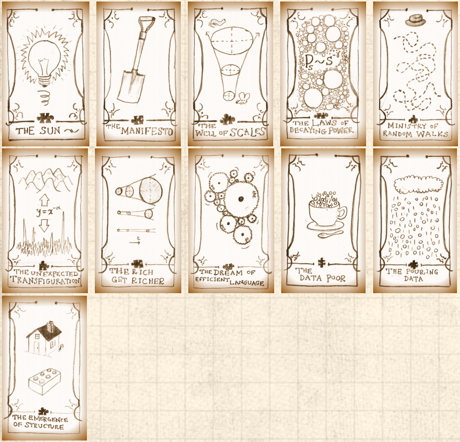
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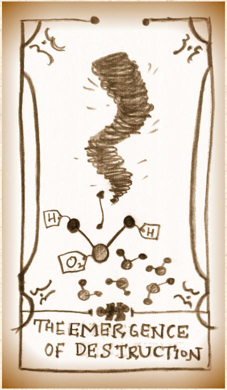
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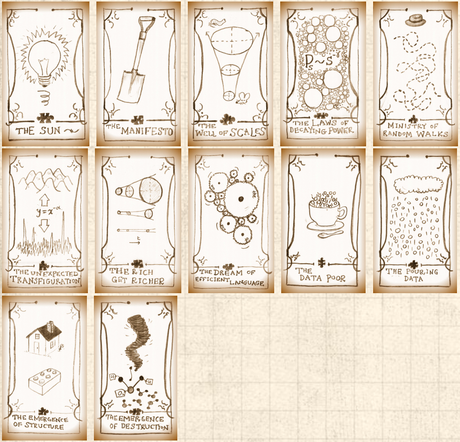
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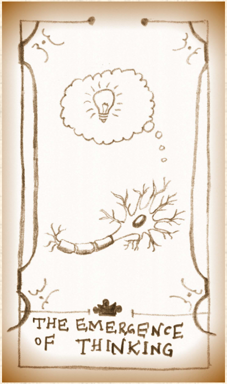
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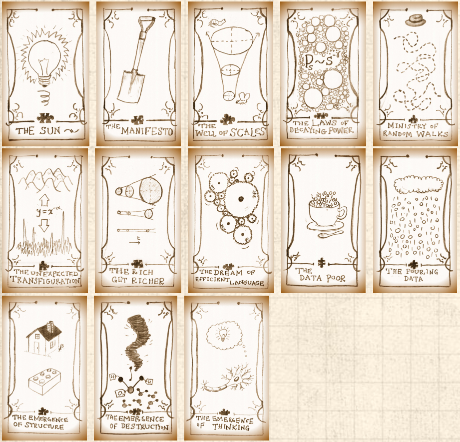

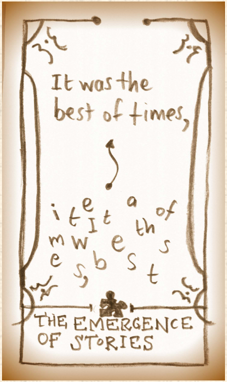

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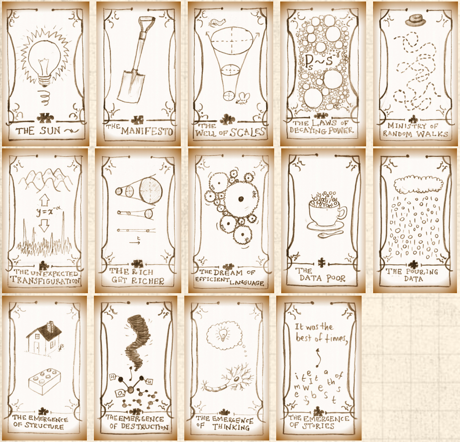
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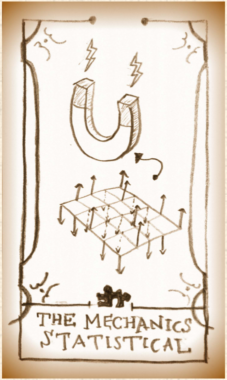
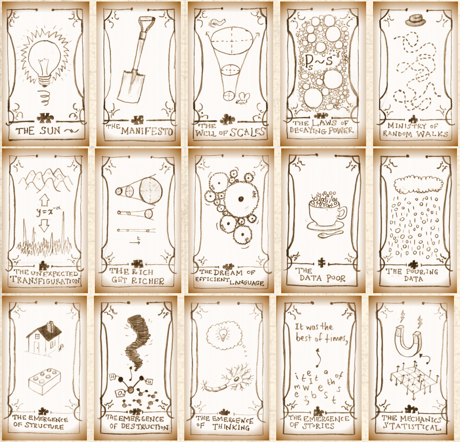
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES


THE MECHANICS STATISTICAL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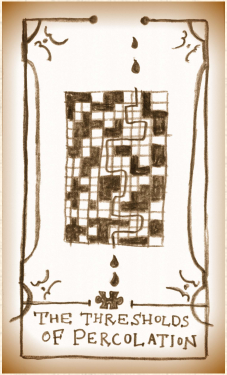
THE THRESHOLDS OF PERCOLATION


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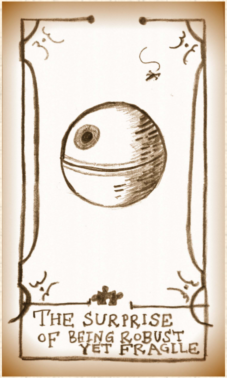
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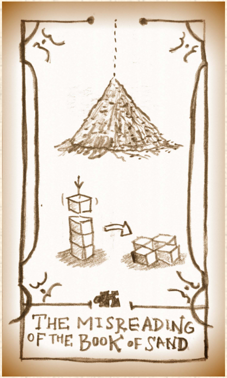
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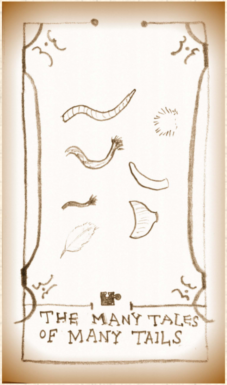
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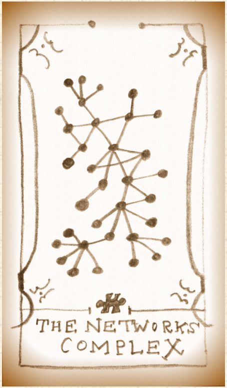
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THE MANIFESTO

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   6.2 We can simulate, model, and create complex systems in extraordinary detail.

net•work |ˈnetˌwərk|
noun
1 an arrangement of intersecting horizontal and vertical lines.
• a complex system of roads, railroads, or other transportation routes *: a network of railroads.*
2 a group or system of interconnected people or things *: a trade network.*
• a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*
• a group of broadcasting stations that connect for the simultaneous broadcast of a program *: the introduction of a second TV network* | [as adj.] *network television.*
• a number of interconnected computers, machines, or operations *: specialized computers that manage multiple outside connections to a network | a local cellular phone network.*
• a system of connected electrical conductors.

verb [trans.]
connect as or operate with a network *: the stock exchanges have proven to be resourceful in networking these deals.*
• link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
• [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career *: the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's etymological investigation:

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

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- 1914–: wireless broadcasting networks

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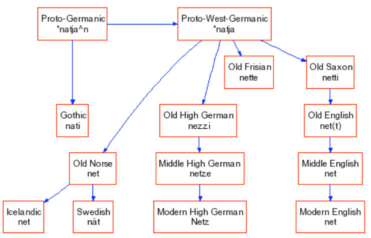
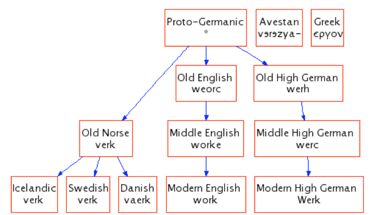
Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appear to have long meant purposeful action.



The network of Germanic `net' words



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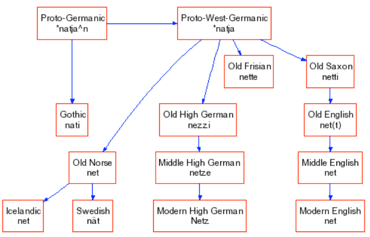

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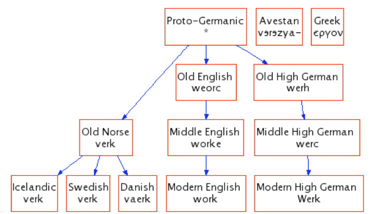

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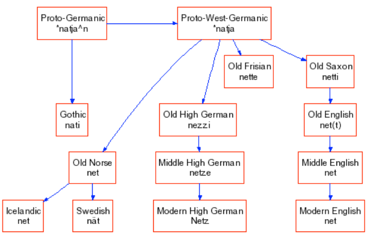

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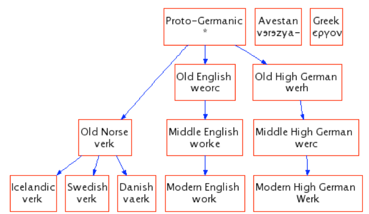

The network of `work' words

- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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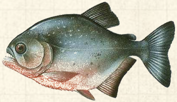

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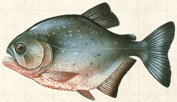

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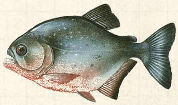

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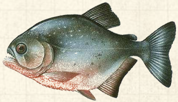

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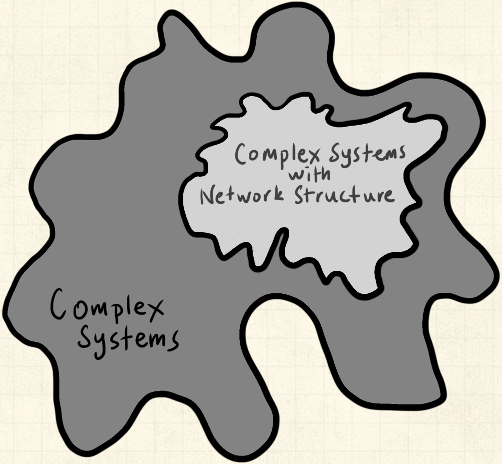
- See also: https://xkcd.com/793/

Complex Systems is the Big Story:



- Only a bit networky: Fluids-at-large (the atmosphere, oceans, ...), organism cells, ...

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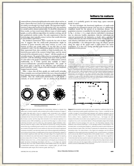

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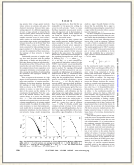

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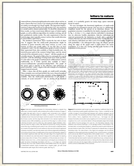

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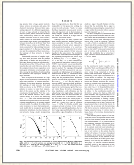

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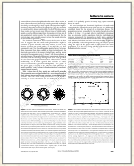
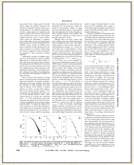

letters to nature

"Collective dynamics of 'small-world' networks"
Watts and Strogatz,
Nature, **393**, 440–442, 1998. [112]

Times cited: $\sim 51,622$ (as of May 19, 2023)



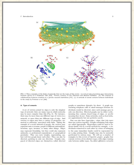
"Emergence of scaling in random networks"
Barabási and Albert,
Science, **286**, 509–511, 1999. [8]

Times cited: $\sim 43,853$ (as of May 19, 2023)

Review articles:

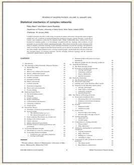
“Complex Networks: Structure and Dynamics”
Boccaletti et al.,
Physics Reports, **424**, 175–308, 2006. [14]

Times cited: ~ 12,318 (as of May 9, 2023)



“The structure and function of complex networks”
M. E. J. Newman,
SIAM Rev., **45**, 167–256, 2003. [77]

Times cited: ~ 23,611 (as of May 9, 2023)



“Statistical mechanics of complex networks”
Albert and Barabási,
Rev. Mod. Phys., **74**, 47–97, 2002. [3]

Times cited: ~ 26,636 (as of May 9, 2023)

Popularity according to textbooks:

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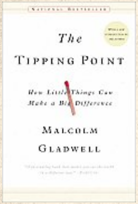
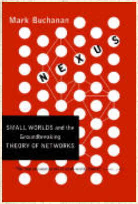
Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction"
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"

Popularity according to popular books:

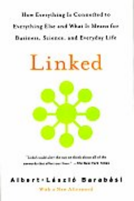


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [43]

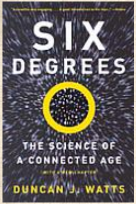


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

Popularity according to popular books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [107]

Numerous others ...

- Complex Social Networks—F. Vega-Redondo [105]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [84]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [19]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [34]

More observations

- But surely networks aren't new ...

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- Graph theory was well established ...

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* *If this is upsetting, maybe string theory is for you ...*

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- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [114]

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But:

- For scientists, description is only part of the battle.
- We still need to understand.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

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Other spiffing words: vertices and edges.

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$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

Super Basic definitions

Adjacency matrix:

- We can represent a network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- For numerical work, we always use sparse matrices.
- For many real networks, A is a function of time.

Examples

So what passes for a complex network?

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- Complex networks are large (in node number)

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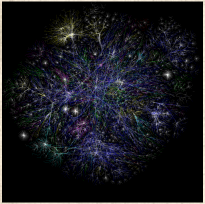
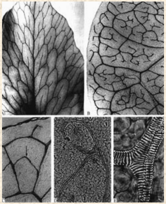
- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The internet (pipes)
- Road networks
- Power grids



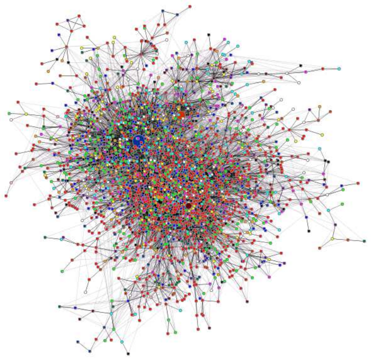


- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks

- The Blogosphere (RIP)
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- Airline networks
- Call networks (AT&T)
- The Media
- The internet (World Wide Web)



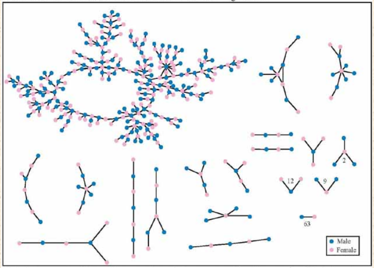
datamining.typepad.com

Examples

Interaction networks: social networks

- Snogging

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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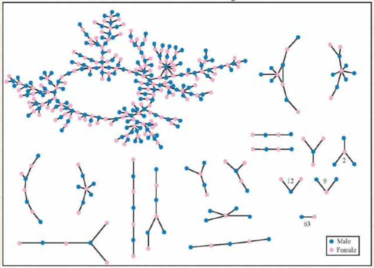

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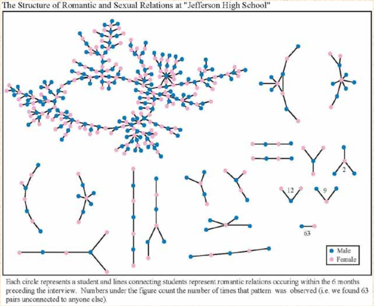

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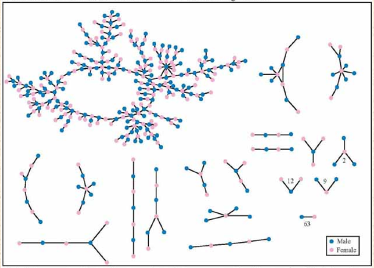

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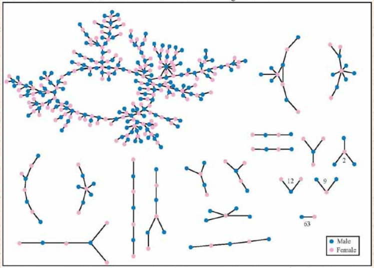

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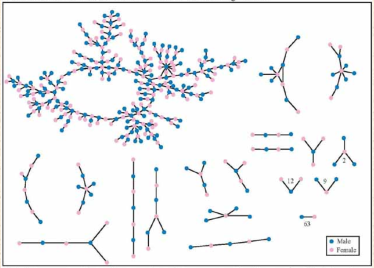

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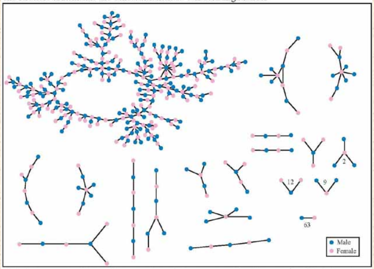

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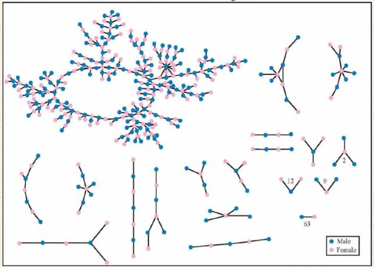

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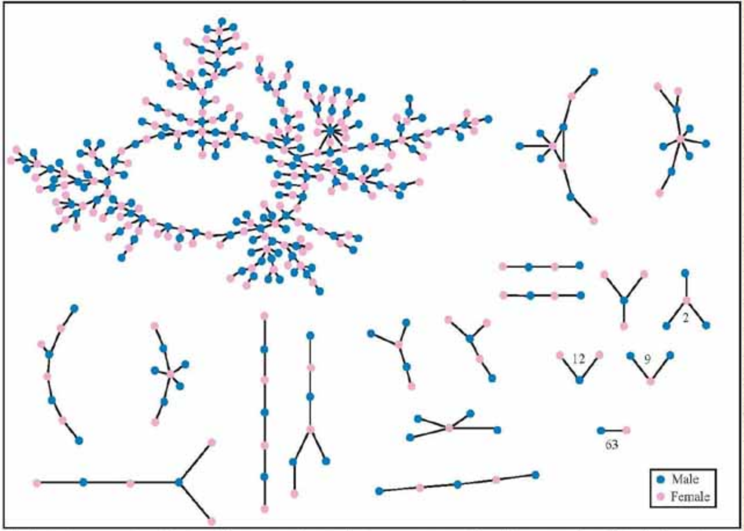

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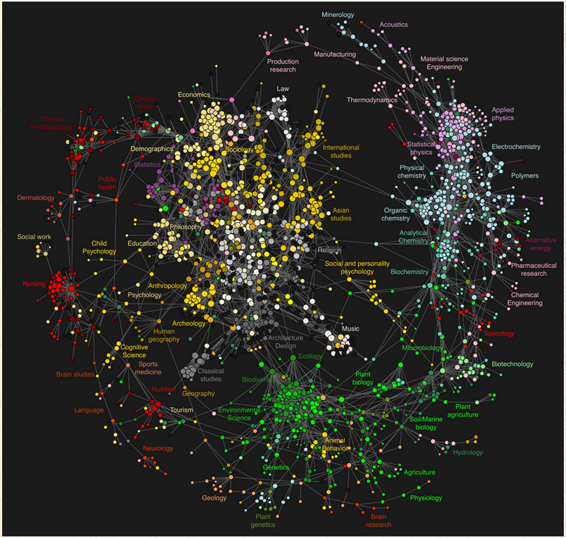
- Large Language Models

Clickworthy Science:



“Clickstream Data Yields High-Resolution Maps of Science”, Bollen et al. [18], 2009.

Neural reboot (NR):

Dog has fun.

https://www.youtube.com/watch?v=7xEX-48RHCY?rel=0


THE SUN
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
MINISTRY OF RANDOM WALKS
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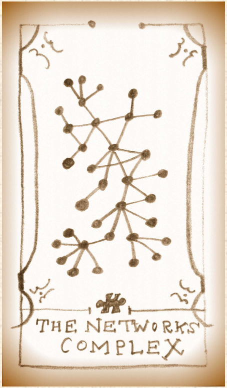
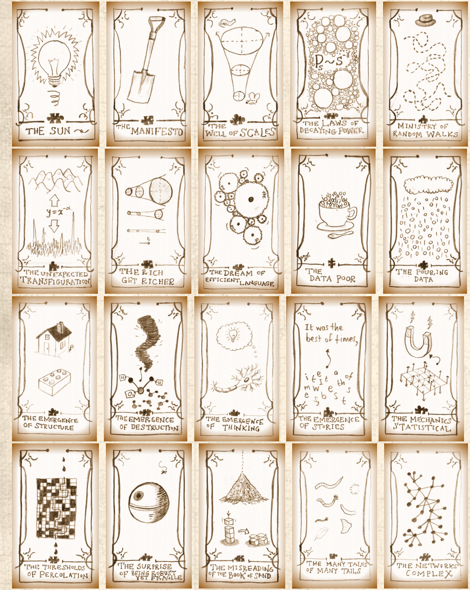

THE NETWORKS COMPLEX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THE ELEMENTS OF WEBS

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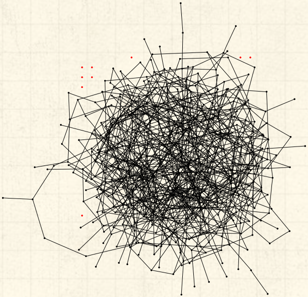

$\Leftarrow$ Typical hairball

- number of nodes N = 500
- number of edges m = 1000
- average degree $\langle k \rangle$ = 4

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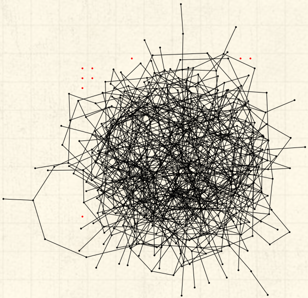

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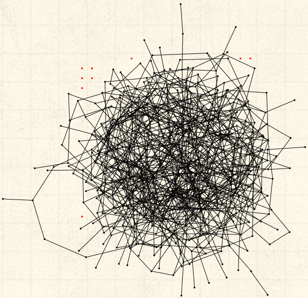

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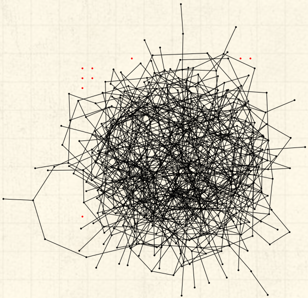

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  said Ponder [Stibbons] —*Making Money*, T. Pratchett.
- We need to extract digestible, meaningful aspects.

Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- interconnectedness
- robustness

- Plus coevolution of network structure and processes on networks.

* Degree distribution is the elephant in the room that we are now all very aware of ...

Properties

1. degree distribution P_k

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- P_k is the probability that a randomly selected node has degree k.

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- P_k is the probability that a randomly selected node has degree k.
- k = node degree = number of connections.

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- "Becoming": Stories = Characters + Time

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  Often techological or biological: internet, WWW, protein interactions, neural networks, food webs.

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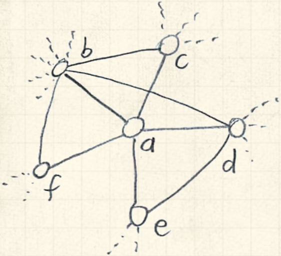
4. Clustering:



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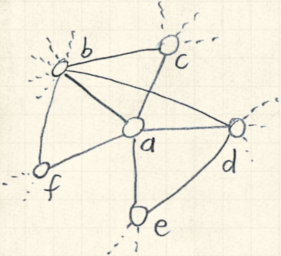

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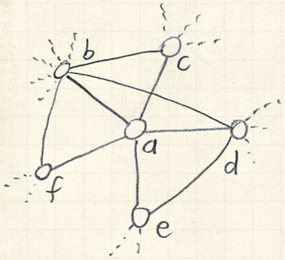


- Your friends tend to know each other.
- Two measures (explained on following slides):
  1. Watts & Strogatz [112]

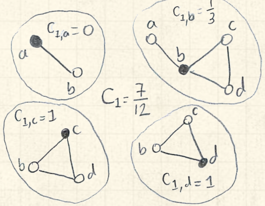
$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

  2. Newman [77]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

Example network:

Calculation of C_1:



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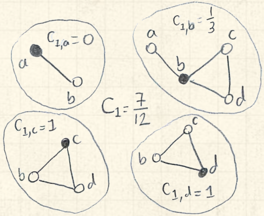

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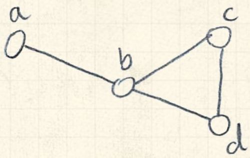

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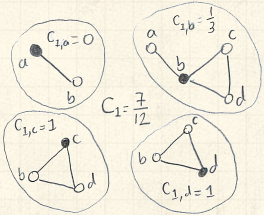

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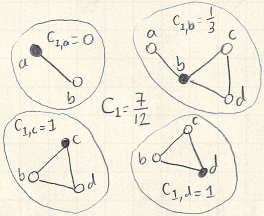

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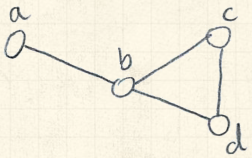

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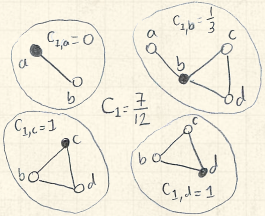

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Triples and triangles

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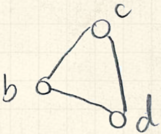

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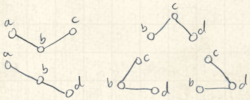

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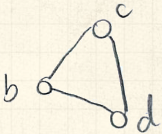

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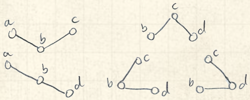

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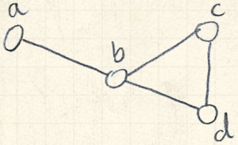

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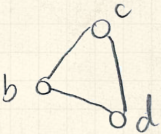

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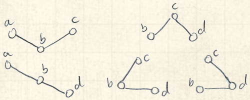

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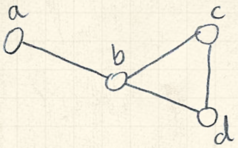

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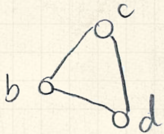

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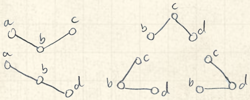

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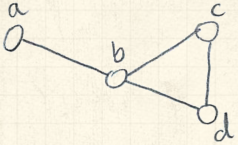

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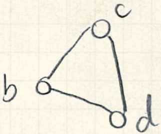

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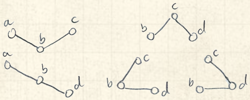

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- Social Network Analysis (SNA): fraction of transitive triples.

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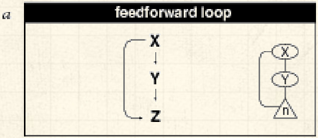
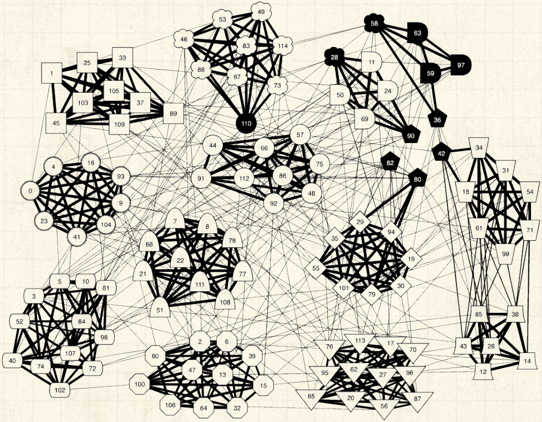


Shen-Orr, Uri Alon, *et al.* [89]

Properties

6. modularity and structure/community detection:



Clauset *et al.*, 2006 [24]: NCAA football

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- Closeness perhaps compresses too much into one number

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- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [56])

Interconnected networks and robustness (two for one deal):

"Catastrophic cascade of failures in interdependent networks" [21]. Buldyrev et al., Nature 2010.

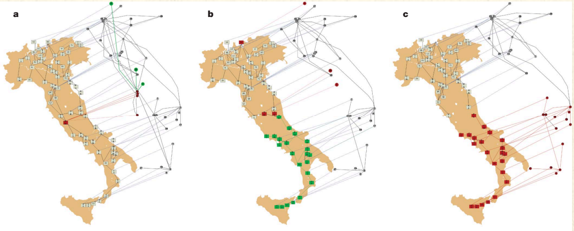

Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003[20]. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a,** One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network) at the next step are marked in green. **b,** Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c,** Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).

Outline


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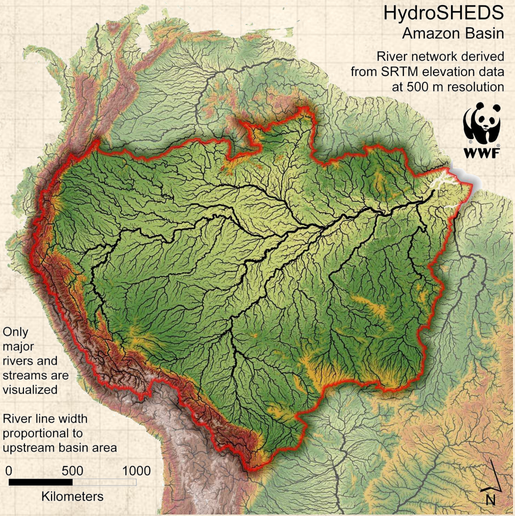
Branching networks are useful things:

- Fundamental to material supply and collection
- Supply: From one source to many sinks in 2- or 3-d.
- Collection: From many sources to one sink in 2- or 3-d.
- Typically observe hierarchical, recursive self-similar structure

Examples:

- River networks
- Cardiovascular networks
- Plants
- Evolutionary trees
- Organizations (only in theory ...)

Branching networks are everywhere ...

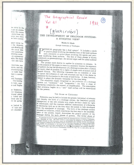


http://hydrosheds.cr.usgs.gov/

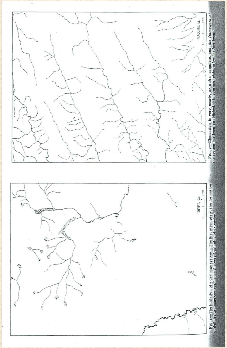
Branching networks are everywhere ...

http://en.wikipedia.org/wiki/Image:Applebox.JPG

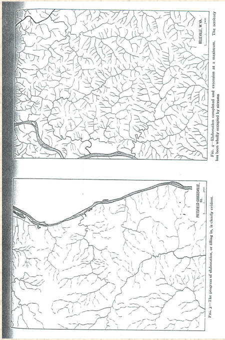
An early thought piece: Extension and Integration



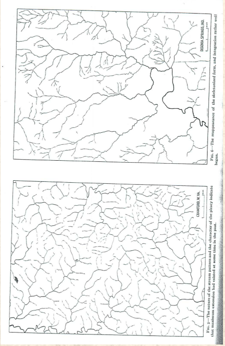
"The Development of Drainage Systems: A Synoptic View"
Waldo S. Glock,
The Geographical Review, **21**, 475–482, 1931. [45]



Initiation, Elongation



Elaboration, Piracy.



Abstraction, Absorption.

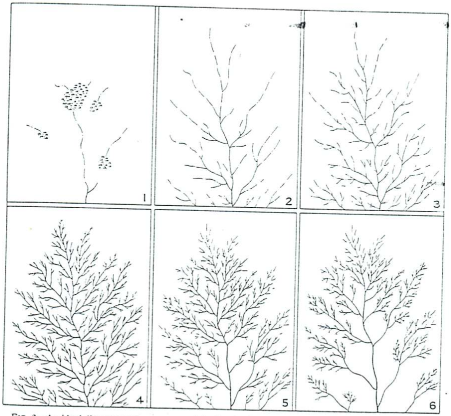
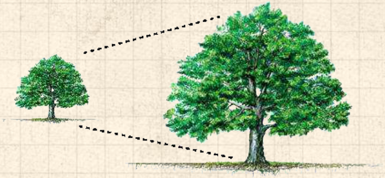


FIG. 8—An ideal diagrammatic summary of the development of a drainage system given for purposes of comparison only. The first four parts show extension, thus: 1, initiation; 2, elongation; 3, elaboration; and 4, maximum extension. Parts 5 and 6 represent steps during integration.

The sequential stages recognized in the evolution of a drainage system are "extension" and "integration"; the first, a stage of increasing complexity; the second, of simplification.

Allometry

- **Isometry:** dimensions scale linearly with each other.



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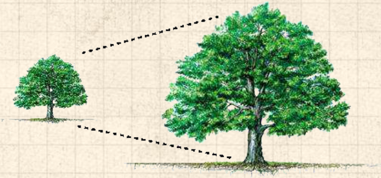
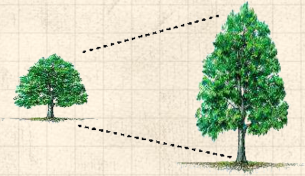
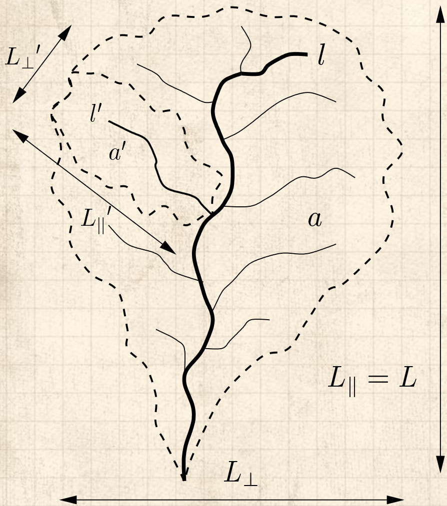


- **Allometry:** dimensions scale nonlinearly.

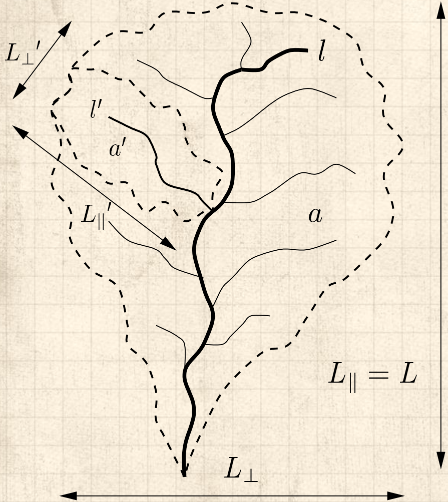


Basin allometry



Allometric relationships:

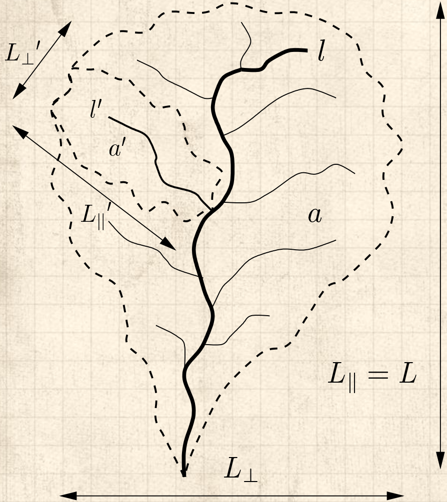
Basin allometry



Allometric relationships:

$$\ell \propto a^h$$

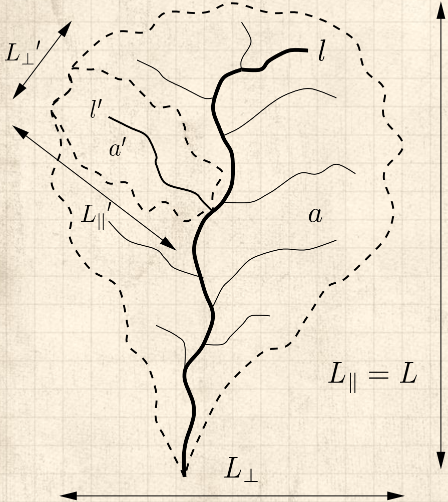
Basin allometry



Allometric relationships:

- $$\ell \propto a^h$$
- $$\ell \propto L^d$$

Basin allometry



Allometric relationships:

- $$\ell \propto a^h$$
- $$\ell \propto L^d$$
- Combine above:

$$a \propto L^{d/h} \equiv L^D$$

'Laws'

- Hack's law (1957) [50]:

$$\boxed{\ell \propto a^h}$$

reportedly $0.5 < h < 0.7$

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reportedly $1.0 < d < 1.1$

- Basin allometry:

$$\boxed{L_{\parallel} \propto a^{h/d} \equiv a^{1/D}}$$

$D < 2 \rightarrow$ basins elongate.

There are a few more 'laws': [31]

Relation:	Name or description:
$T_k = T_1 (R_T)^{k-1}$	Tokunaga's law
$\ell \sim L^d$	self-affinity of single channels
$n_\omega / n_{\omega+1} = R_n$	Horton's law of stream numbers
$\bar{\ell}_{\omega+1} / \bar{\ell}_\omega = R_\ell$	Horton's law of main stream lengths
$\bar{a}_{\omega+1} / \bar{a}_\omega = R_a$	Horton's law of basin areas
$\bar{s}_{\omega+1} / \bar{s}_\omega = R_s$	Horton's law of stream segment lengths
$L_\perp \sim L^H$	scaling of basin widths
$P(a) \sim a^{-\tau}$	probability of basin areas
$P(\ell) \sim \ell^{-\gamma}$	probability of stream lengths
$\ell \sim a^h$	Hack's law
$a \sim L^D$	scaling of basin areas
$\Lambda \sim a^\beta$	Langbein's law
$\lambda \sim L^\varphi$	variation of Langbein's law

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Reported parameter values: [31]

Parameter:	Real networks:
R_n	3.0–5.0
R_a	3.0–6.0
$R_\ell = R_T$	1.5–3.0
T_1	1.0–1.5
d	1.1 ± 0.01
D	1.8 ± 0.1
h	0.50–0.70
τ	1.43 ± 0.05
γ	1.8 ± 0.1
H	0.75–0.80
β	0.50–0.70
φ	1.05 ± 0.05

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Stream Ordering:

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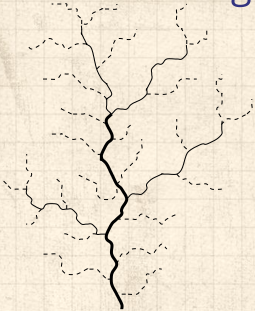

1. Label all source streams as order $\omega = 1$ and remove.

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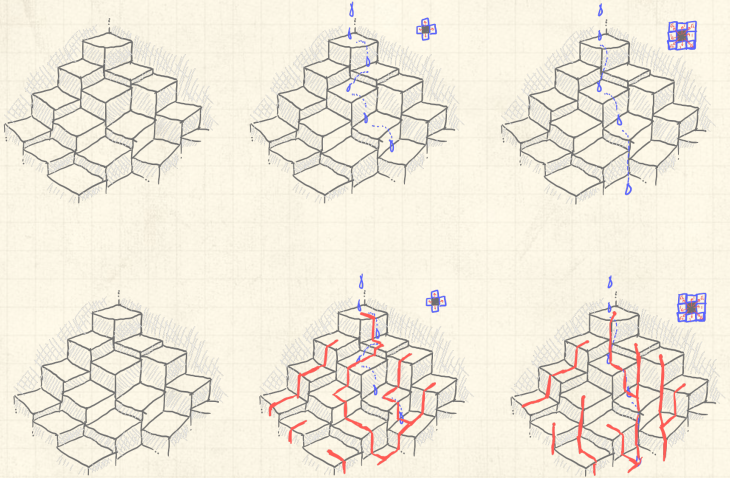
3. Repeat until one stream is left (order = Ω)
4. Basin is said to be of the order of the last stream removed.
5. Example above is a basin of order $\Omega = 3$.

Basic algorithm for extracting networks from Digital Elevation Models (DEMs):



Also:
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Horton's laws

Self-similarity of river networks

Horton's laws

Self-similarity of river networks

- First quantified by Horton (1945) [53], expanded by Schumm (1956) [88]

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Self-similarity of river networks

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Network Architecture

Tokunaga's law [101, 102, 103]

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- We usually write Tokunaga's law as:

$$\boxed{T_k = T_1 (R_T)^{k-1}} \quad \text{where } R_T \simeq 2$$

.

Connecting exponents

Only 3 parameters are independent:
e.g., take d, R_n, and R_s

relation:	scaling relation/parameter: [31]
$\ell \sim L^d$	d
$T_k = T_1 (R_T)^{k-1}$	$T_1 = R_n - R_s - 2 + 2R_s/R_n$
	$R_T = R_s$
$n_\omega / n_{\omega+1} = R_n$	R_n
$\bar{a}_{\omega+1} / \bar{a}_\omega = R_a$	$R_a = R_n$
$\bar{\ell}_{\omega+1} / \bar{\ell}_\omega = R_\ell$	$R_\ell = R_s$
$\ell \sim a^h$	$h = \ln R_s / \ln R_n$
$a \sim L^D$	$D = d/h$
$L_\perp \sim L^H$	$H = d/h - 1$
$P(a) \sim a^{-\tau}$	$\tau = 2 - h$
$P(\ell) \sim \ell^{-\gamma}$	$\gamma = 1/h$
$\Lambda \sim a^\beta$	$\beta = 1 + h$
$\lambda \sim L^\varphi$	$\varphi = d$

PoCS
Principles of Complex Systems
@pocsvox
What's the Story?

Outline


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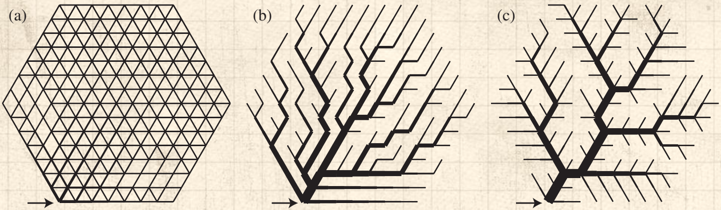
Single source optimal supply



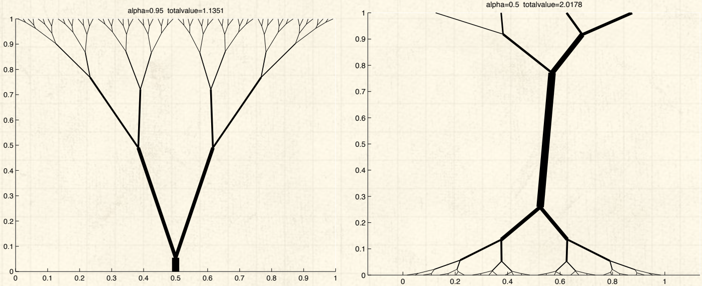
(a) $\gamma > 1$: Braided (bulk) flow
(b) $\gamma < 1$: Local minimum: Branching flow
(c) $\gamma < 1$: Global minimum: Branching flow

- Note: This is a single source supplying a region.

From Bohn and Magnasco [16]
See also Banavar *et al.* [6]: “Topology of the Fittest Transportation Network”; focus is on presence or absence of loops—same story

Single source optimal supply

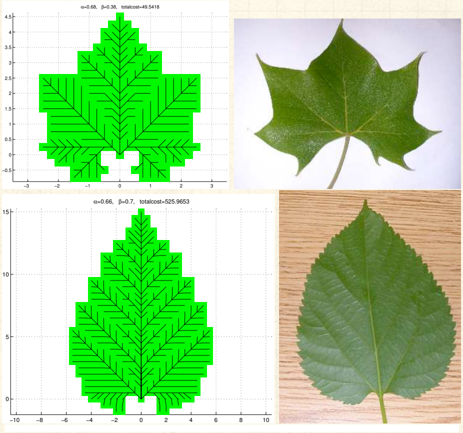
Optimal paths related to transport (Monge) problems:



"Optimal paths related to transport problems"
Qinglan Xia,
Communications in Contemporary Mathematics, **5**, 251–279, 2003. [116]

Growing networks: [117]

FIGURE 3. A maple leaf



- Top: $\alpha = 0.66$, $\beta = 0.38$; Bottom: $\alpha = 0.66$, $\beta = 0.70$

Single source optimal supply

An immensely controversial issue ...

- The form of natural branching networks: Random, optimal, or some combination? [55, 113, 7, 33, 27]

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- Real networks differ in details of scaling but reasonably agree in scaling relations.


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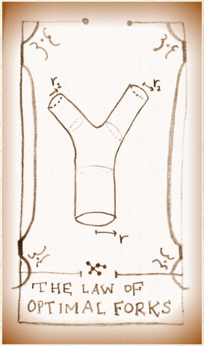
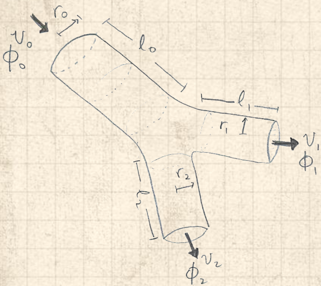

r_1
r_2
r
THE LAW OF OPTIMAL FORKS

Optimization—Murray's law



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$$\boxed{r_{\text{parent}}^3 = r_{\text{offspring1}}^3 + r_{\text{offspring2}}^3}$$

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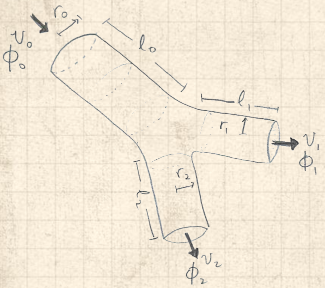

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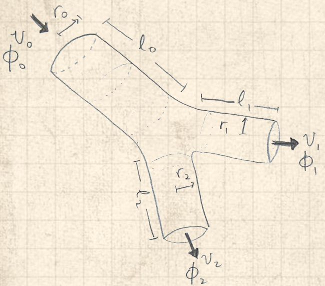

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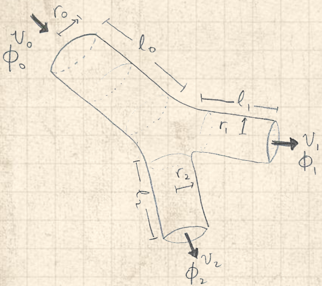

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- See D'Arcy Thompson's "On Growth and Form" for background and general inspiration [99, 100].


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THE CHURCH OF QUARTEROLOGY

Animal power

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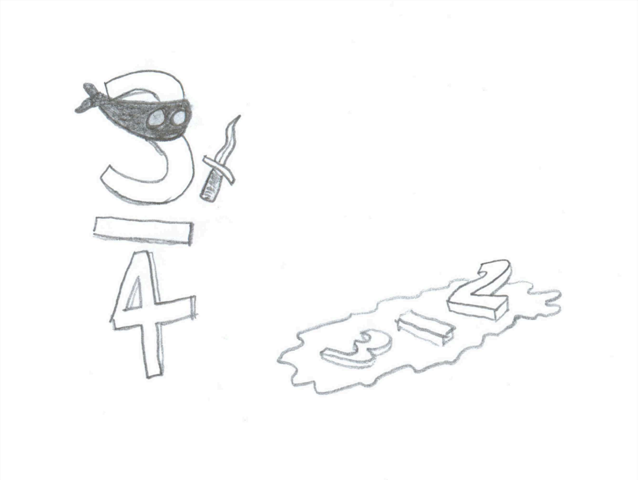
Stories—The Fraction Assassin:



Quarterology spreads throughout the land:

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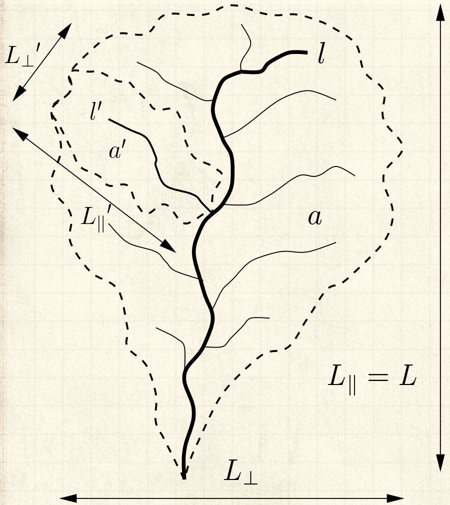
- "Energy Metabolism; Proceedings of the 3rd symposium held at Troon, Scotland, May 1964," Ed. Sir Kenneth Blaxter [13]

Somehow, optimal river networks are connected:



- a = drainage basin area
- ℓ = length of longest (main) stream
- $L = L_\|$ = longitudinal length of basin

Mysterious allometric scaling in river networks

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  "Studies of Longitudinal Stream Profiles in Virginia and Maryland"

$$\ell \sim a^{h}$$

$$h \sim 0.6$$

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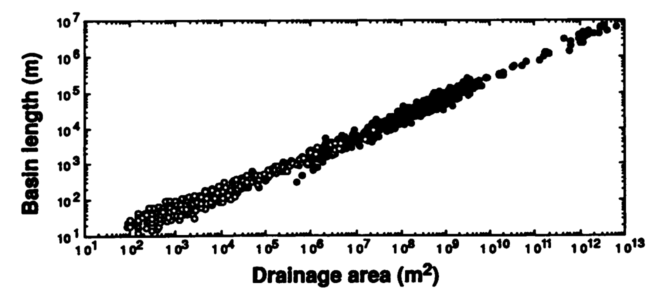
- A catch: studies done on small scales.

Large-scale networks:

(1992) Montgomery and Dietrich [69]:

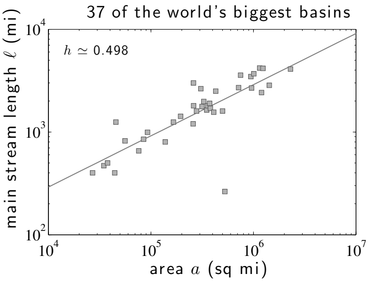


- Composite data set: includes everything from unchanneled valleys up to world's largest rivers.
- Estimated fit:

$$L \simeq 1.78a^{0.49}$$

- Mixture of basin and main stream lengths.

World's largest rivers only:



- Data from Leopold (1994) [60, 32]
- Estimate of Hack exponent: $h = 0.50 \pm 0.06$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THE CHURCH OF QUARTEROLOGY

Nutrient delivering networks:

- 1960's: Rashevsky considers blood networks and finds a $2/3$ scaling.

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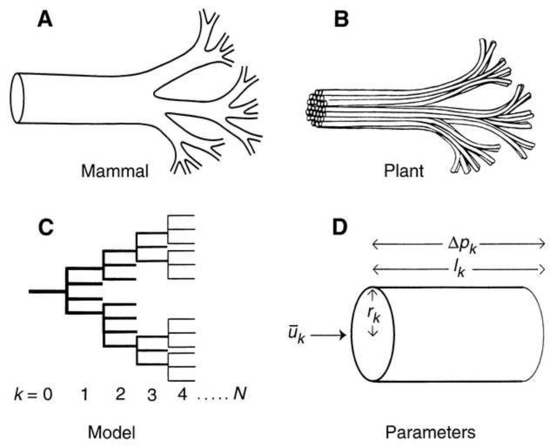
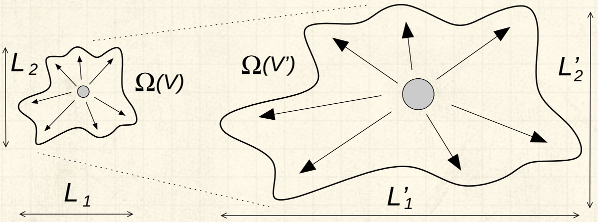
- 1997: West *et al.* [113] use a network story to find $3/4$ scaling.



Geometric argument

- Allometrically growing regions:



- Have d length scales which scale as

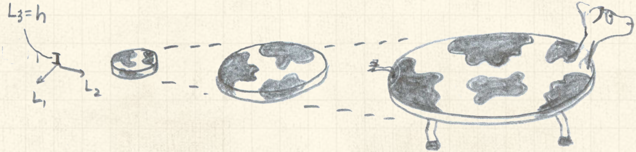
$$L_i \propto V^{\gamma_i} \text{ where } \gamma_1 + \gamma_2 + \ldots + \gamma_d = 1.$$

- For isometric growth, $\gamma_i = 1/d$.
- For allometric growth, we must have at least two of the $\{\gamma_i\}$ being different

Spherical cows and pancake cows:

Assume an isometrically Scaling family of cows:

Extremes of allometry:
The pancake cows—

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THE FAMILY OF SPHERICAL COWS

Minimal network volume:

Real supply networks are close to optimal:

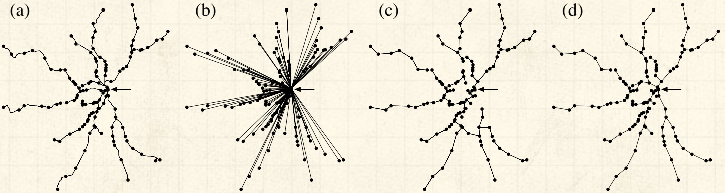
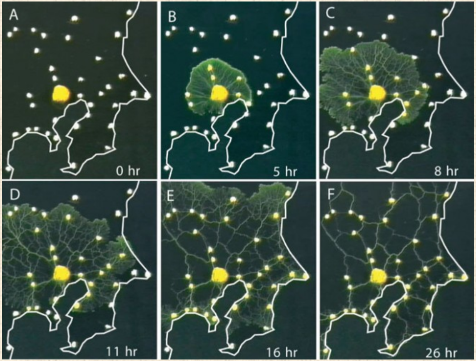


Figure 1. (a) Commuter rail network in the Boston area. The arrow marks the assumed root of the network. (b) Star graph. (c) Minimum spanning tree. (d) The model of equation (3) applied to the same set of stations.

Gastner and Newman (2006): "Shape and efficiency in spatial distribution networks" [41]

"Rules for Biologically Inspired Adaptive Network Design"

Tero et al.,
Science, **327**, 439-442, 2010. [98]



Urban deslime in action:
https://www.youtube.com/watch?v=GwKuFREOgmo

Blood networks

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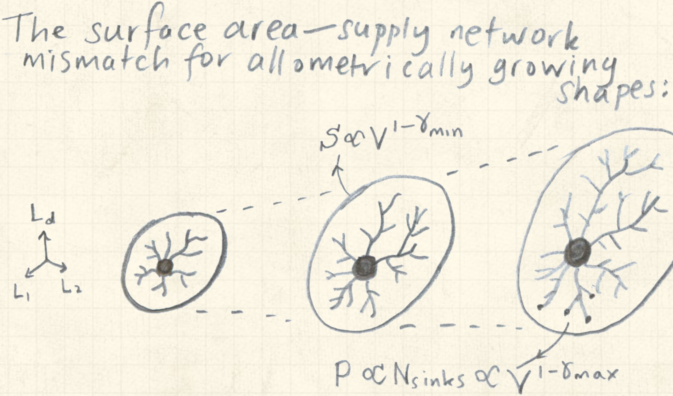
- Including other constraints may raise scaling exponent to a higher, less efficient value.

- Exciting bonus: Scaling obtained by the supply network story and the surface-area law only match for isometrically growing shapes.



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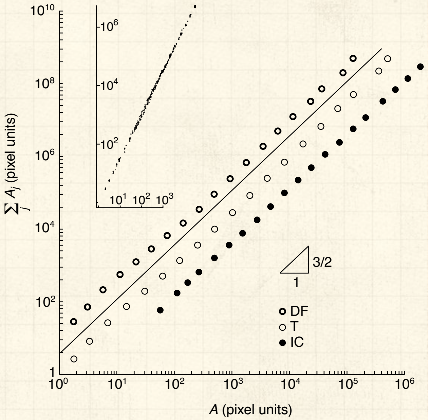

Figure 2 Allometric scaling in river networks. Double logarithmic plot of $C \propto \Sigma_{x\in\gamma} A_x$ versus A for three river networks characterized by different climates, geology and geographic locations (Dry Fork, West Virginia, 586 km^2, digital terrain map (DTM) size 30 × 30 m^2; Island Creek, Idaho, 260 km^2, DTM size 30 × 30 m^2; Tirso, Italy, 2,024 km^2, DTM size 237 × 237 m^2). The experimental points are obtained by binning total contributing areas, and computing the ensemble average of the sum of the inner areas for each sub-basin within the binned interval. The figure uses pixel units in which the smallest area element is assigned a unit value. Also plotted is the predicted scaling relationship with slope 3/2. The inset shows the raw data from the Tirso basin before any binning has been done.

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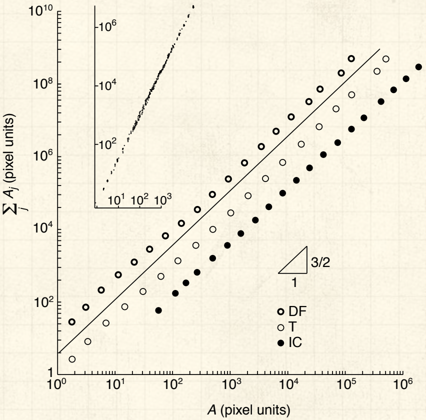

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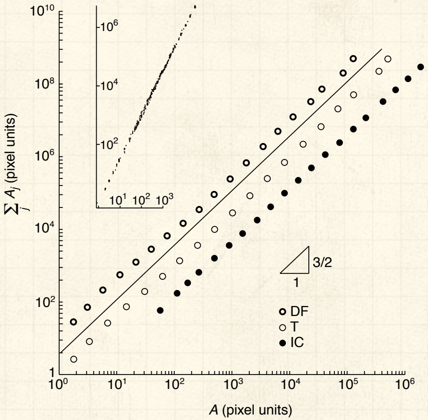

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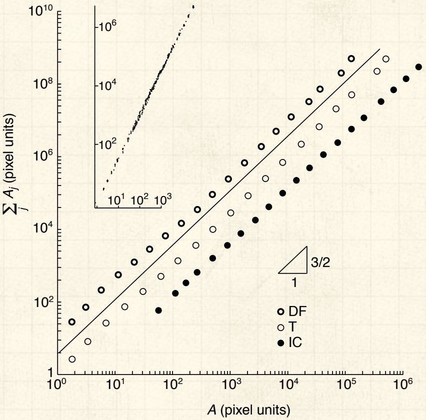

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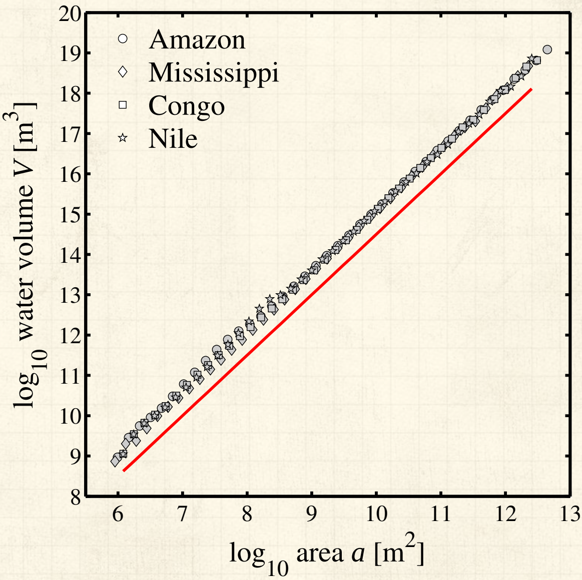
Even better—prefactors match up:

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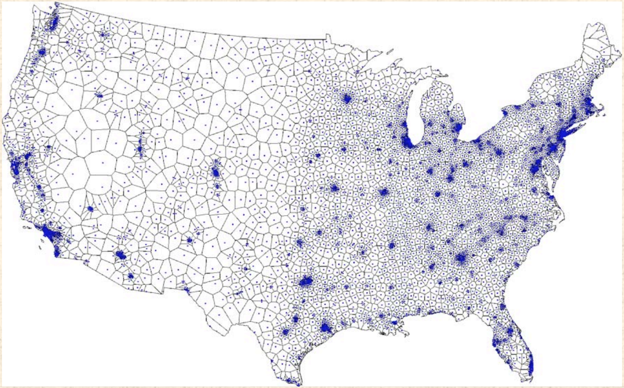
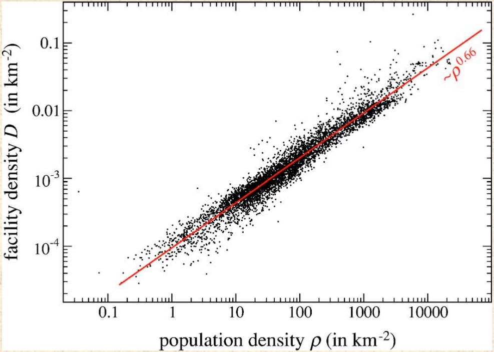
"Optimal design of spatial distribution networks"
Gastner and Newman,
Phys. Rev. E, **74**, 016117, 2006. [40]



- Approximately optimal location of 5000 facilities.
- Based on 2000 Census data.
- Simulated annealing + Voronoi tessellation.

Optimal source allocation



- Optimal facility density ρ_{fac} vs. population density ρ_{pop}.

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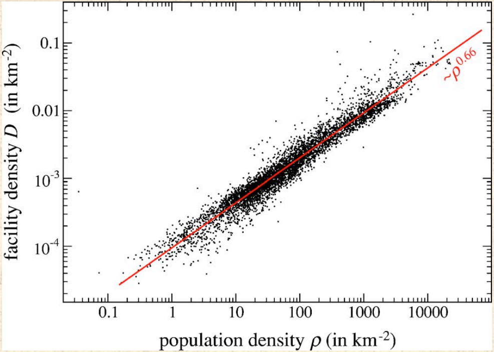

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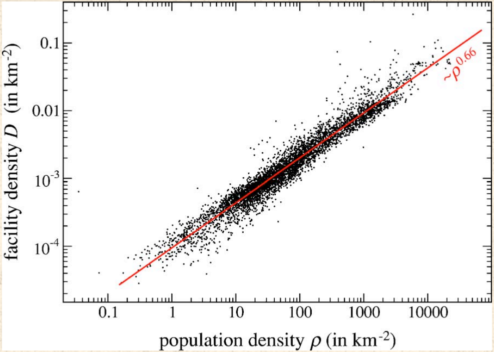

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- Also known as the p-median problem, and connected to cluster analysis.

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- Assume given a fixed population density ρ_{pop} defined on a spatial region Ω.
- Formally, we want to find the locations of n sources $\{\vec{x}_1, \dots, \vec{x}_n\}$ that minimizes the cost function

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- Approximate solution originally due to Gusein-Zade [49].

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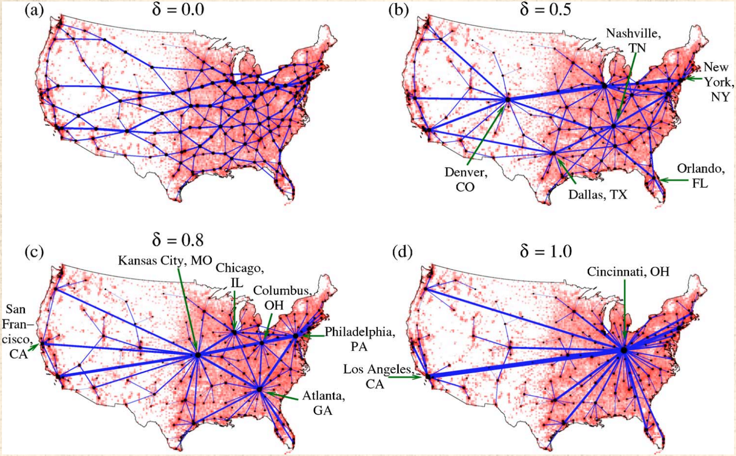

From Gastner and Newman (2006) [40]


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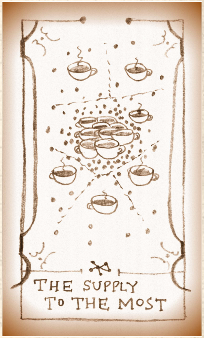
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Public versus private facilities

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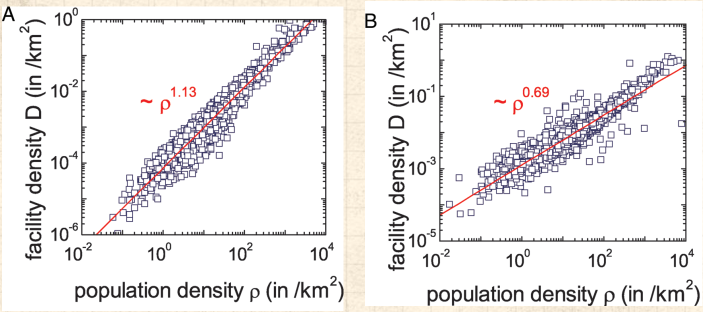
  1. For-profit, commercial facilities: $\alpha = 1$;
  2. Pro-social, public facilities: $\alpha = 2/3$.
- Um *et al.* investigate facility locations in the United States and South Korea.

Public versus private facilities: evidence



- Left plot: ambulatory hospitals in the U.S.
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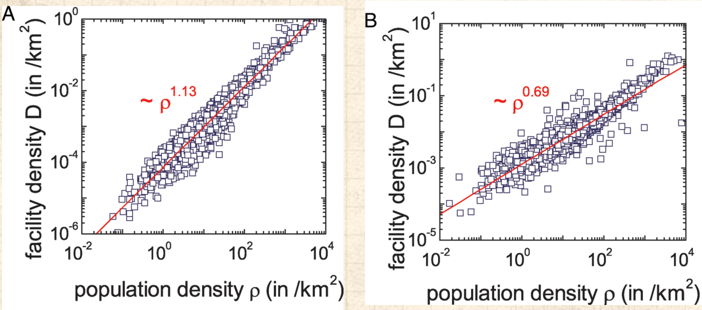

- Left plot: ambulatory hospitals in the U.S.
- Right plot: public schools in the U.S.
- Note: break in scaling for public schools. Transition from $\alpha \simeq 2/3$ to $\alpha = 1$ around $\rho_{\text{pop}} \simeq 100$.

Public versus private facilities: evidence

US facility	α (SE)	R^2
Ambulatory hospital	1.13(1)	0.93
Beauty care	1.08(1)	0.86
Laundry	1.05(1)	0.90
Automotive repair	0.99(1)	0.92
Private school	0.95(1)	0.82
Restaurant	0.93(1)	0.89
Accommodation	0.89(1)	0.70
Bank	0.88(1)	0.89
Gas station	0.86(1)	0.94
Death care	0.79(1)	0.80
* Fire station	0.78(3)	0.93
* Police station	0.71(6)	0.75
Public school	0.69(1)	0.87

SK facility	α (SE)	R^2
Bank	1.18(2)	0.96
Parking place	1.13(2)	0.91
* Primary clinic	1.09(2)	1.00
* Hospital	0.96(5)	0.97
* University/college	0.93(9)	0.89
Market place	0.87(2)	0.90
* Secondary school	0.77(3)	0.98
* Primary school	0.77(3)	0.97
Social welfare org.	0.75(2)	0.84
* Police station	0.71(5)	0.94
Government office	0.70(1)	0.93
* Fire station	0.60(4)	0.93
* Public health center	0.09(5)	0.19

Rough transition between public and private at $\alpha \simeq 0.8$.

Note: * indicates analysis is at state/province level; otherwise county level.


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"I am the Monarch of the Sea."
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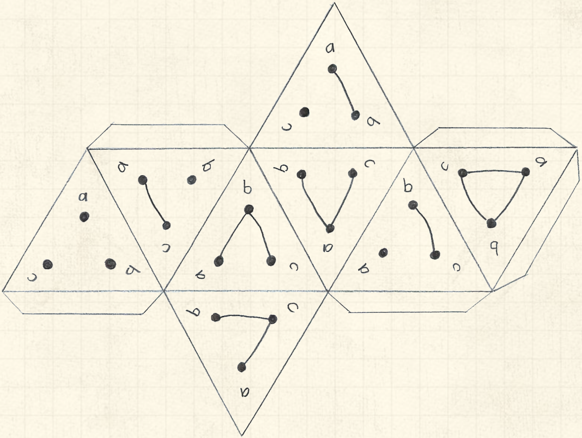
Random network generator for $N = 3$:

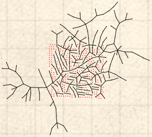


- Get your own exciting generator here.
- As $N \nearrow$, polyhedral die rapidly becomes a ball ...

Random networks: examples for N=500

m = 100
$\langle k \rangle$ = 0.4

m = 200
$\langle k \rangle$ = 0.8



m = 230
$\langle k \rangle$ = 0.92

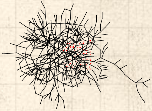
m = 240
$\langle k \rangle$ = 0.96

m = 250
$\langle k \rangle$ = 1

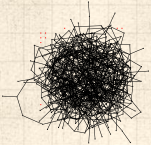
m = 260
$\langle k \rangle$ = 1.04

m = 280
$\langle k \rangle$ = 1.12

m = 300
$\langle k \rangle$ = 1.2

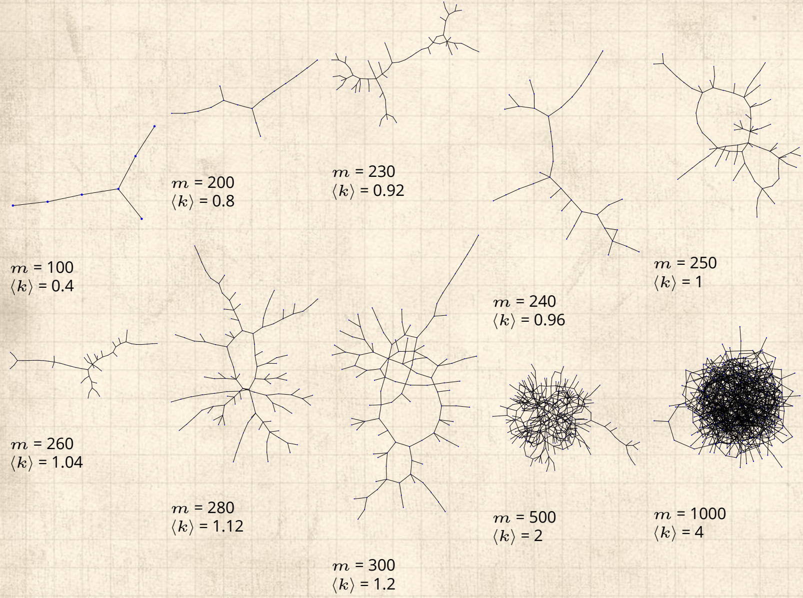


m = 500
$\langle k \rangle$ = 2

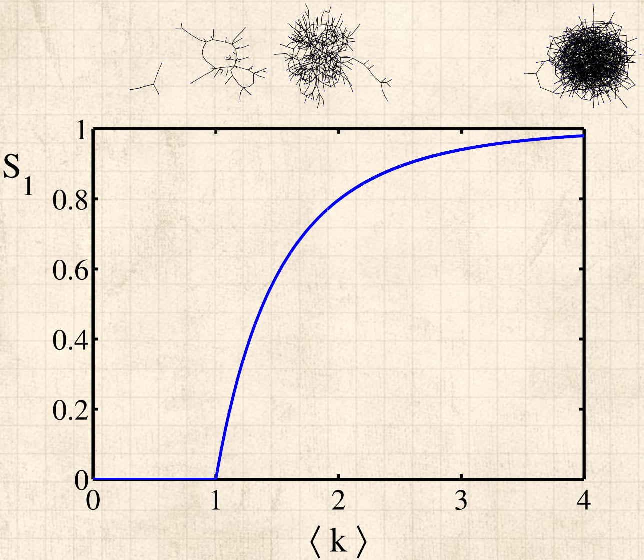


m = 1000
$\langle k \rangle$ = 4

Random networks: largest components



Giant component



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- Therefore have a binomial distribution:

$$P(k;p,N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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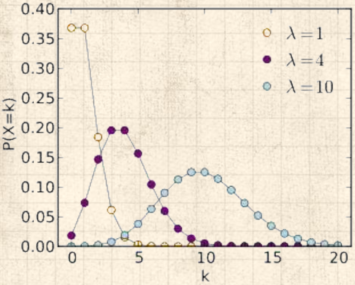
- This is a Poisson distribution with mean $\langle k \rangle$.

Poisson basics:

$$P(k;\lambda) = \frac{\lambda^k}{k!}e^{-\lambda}$$



- $\lambda > 0$
- $k = 0, 1, 2, 3, \ldots$
- Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
- e.g.: phone calls/minute, horse-kick deaths.
- 'Law of small numbers'

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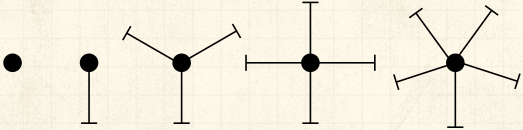
Building random networks: Stubs

Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):



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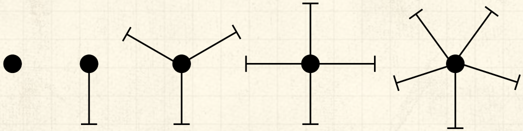

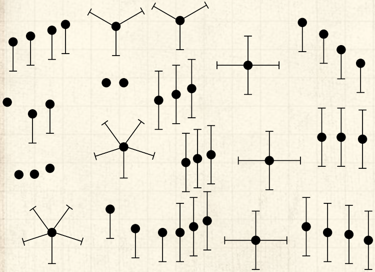

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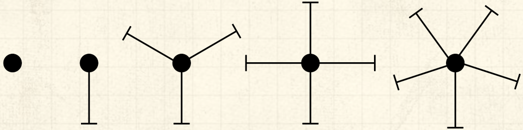

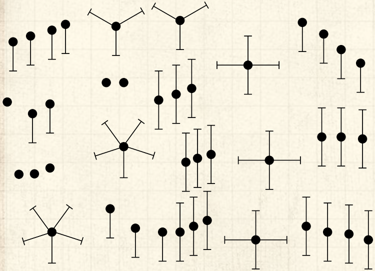

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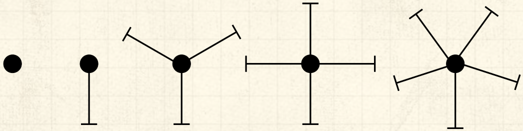

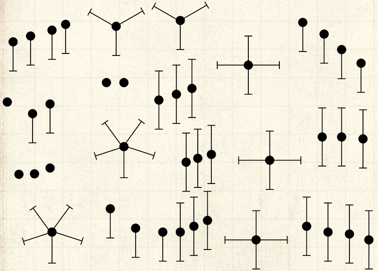

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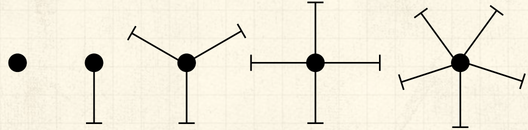

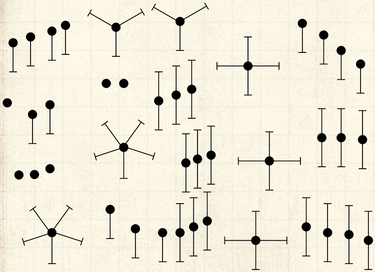

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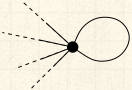
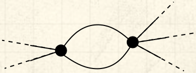
- Must have an even number of stubs.
- Initially allow self- and repeat connections.

Building random networks: First rewiring

Phase 2:

- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.

(A)  (B) 

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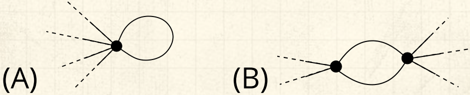

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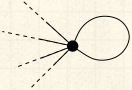
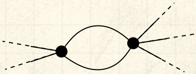
(A)  (B) 

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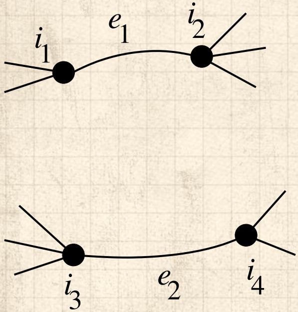
- Simplest solution: randomly rewire two edges at a time.

General random rewiring algorithm



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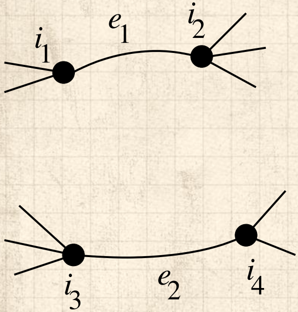

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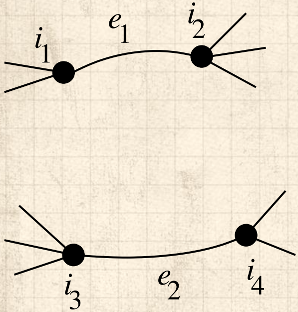

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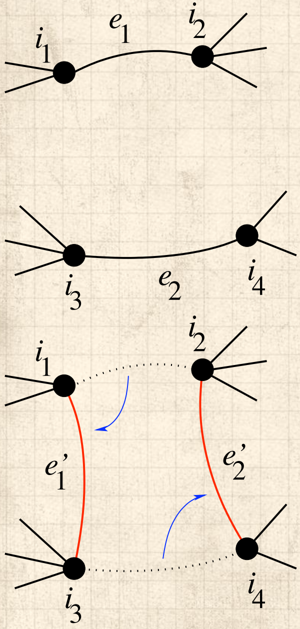

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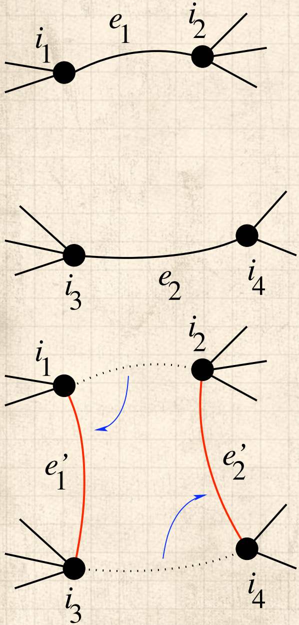

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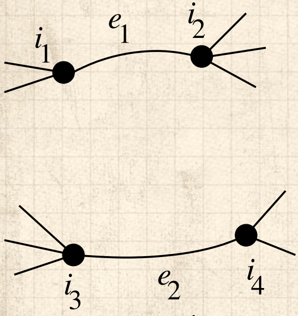

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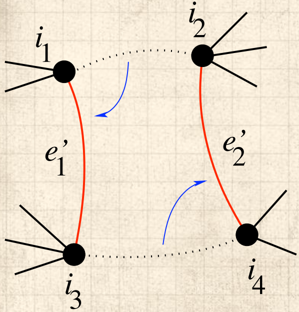

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- Same as finding on/off/on/off 4-cycles. and rotating them.

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- Rule of thumb: # Rewirings $\simeq 10 \times$ # edges [68].

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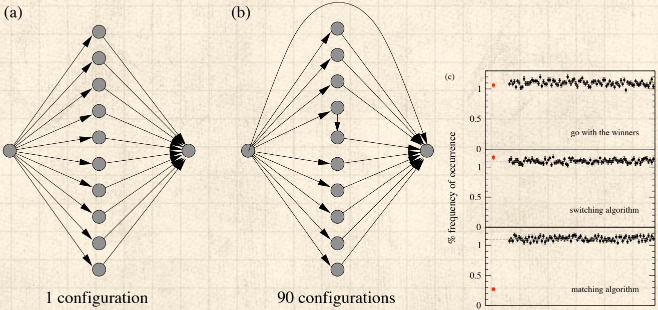
- Example from Milo et al. (2003) [68]:



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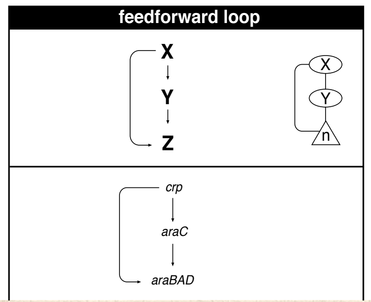

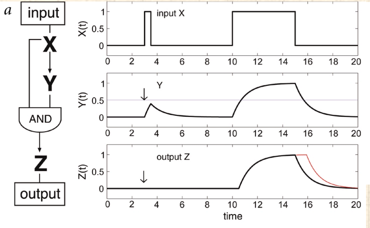

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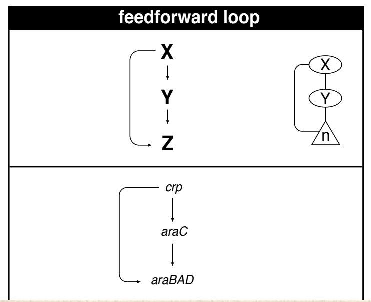

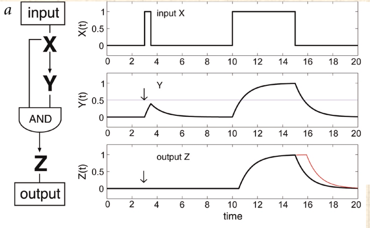

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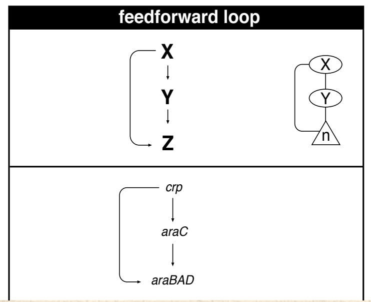

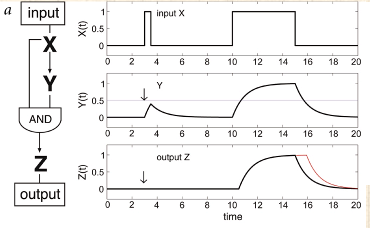

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- Analogy to elevator doors.


THE SUN
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THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
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THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
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THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
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THE JOHN DORY
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THE STORYTELLERS
THE FATED ASTRAL PATH
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- Big deal: Rich-get-richer mechanism is built into this selection process.

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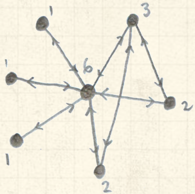
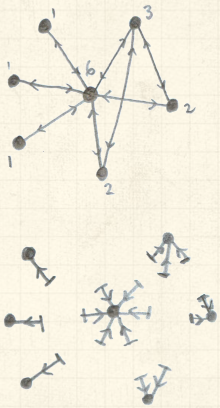
- Natural question: what's the expected number of other friends that one friend has?



Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7$, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.



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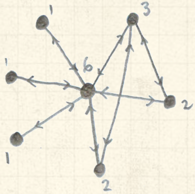

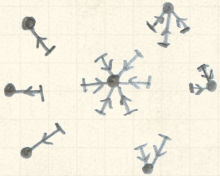

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  $R_0 = 3/16$ $R_1 = 4/16$,

  $R_2 = 3/16$, $R_5 = 6/16$.

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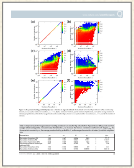
  4. See also: class size paradoxes (nod to: Gelman)



"Generalized friendship paradox in complex networks: The case of scientific collaboration"

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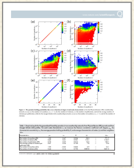

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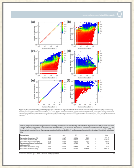

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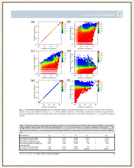

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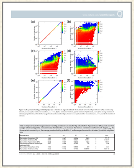

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- Research possibility: The Frenemy Paradox.

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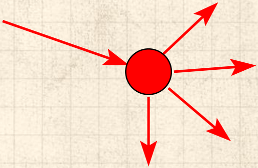

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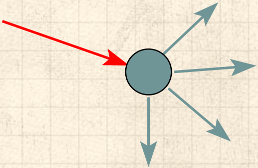

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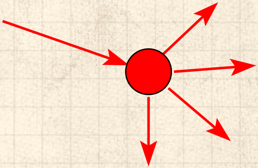

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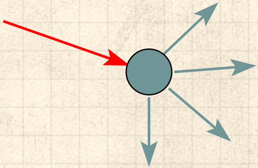

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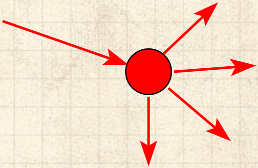

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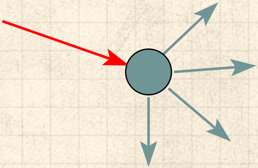

- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?

Global spreading condition

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- Our global spreading condition is then:

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- Good: This is just our giant component condition again.

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- A fraction (1-β) of edges do not transmit infection.
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- Aka bond percolation.
- Resulting degree distribution $\tilde{P}_k$:

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

Random directed networks:

- So far, we've largely studied networks with undirected, unweighted edges.
- Now consider directed, unweighted edges.
- Nodes have k_i and k_o incoming and outgoing edges, otherwise random.

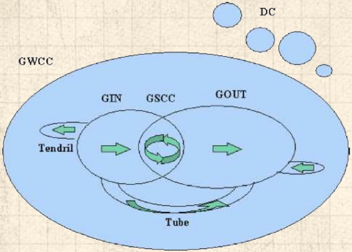
- Network defined by joint in- and out-degree distribution: $P_{k_\mathrm{i},k_\mathrm{o}}$
- Normalization: $\sum_{k_\mathrm{i}=0}^{\infty} \sum_{k_\mathrm{o}=0}^{\infty} P_{k_\mathrm{i},k_\mathrm{o}} = 1$
- Marginal in-degree and out-degree distributions:

$$P_{k_\mathrm{i}} = \sum_{k_\mathrm{o}=0}^{\infty} P_{k_\mathrm{i},k_\mathrm{o}} \text{ and } P_{k_\mathrm{o}} = \sum_{k_\mathrm{i}=0}^{\infty} P_{k_\mathrm{i},k_\mathrm{o}}$$

- Required balance:

$$\langle k_\mathrm{i} \rangle = \sum_{k_\mathrm{i}=0}^{\infty} \sum_{k_\mathrm{o}=0}^{\infty} k_\mathrm{i} P_{k_\mathrm{i},k_\mathrm{o}} = \sum_{k_\mathrm{i}=0}^{\infty} \sum_{k_\mathrm{o}=0}^{\infty} k_\mathrm{o} P_{k_\mathrm{i},k_\mathrm{o}} = \langle k_\mathrm{o} \rangle$$

Directed network structure:



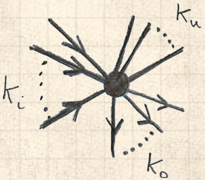
From Boguñá and Serano. [15]

- GWCC = Giant Weakly Connected Component (directions removed);
- GIN = Giant In-Component;
- GOUT = Giant Out-Component;
- GSCC = Giant Strongly Connected Component;
- DC = Disconnected Components (finite).

- When moving through a family of increasingly connected directed random networks, GWCC usually appears before GIN, GOUT, and GSCC which tend to appear together. [80, 15]

Observation:

- Directed and undirected random networks are separate families ...
- ...and analyses are also disjoint.
- Need to examine a larger family of random networks with mixed directed and undirected edges.



- Consider nodes with three types of edges:
  1. k_{u} undirected edges,
  2. k_{i} incoming directed edges,
  3. k_{o} outgoing directed edges.
- Define a node by generalized degree:

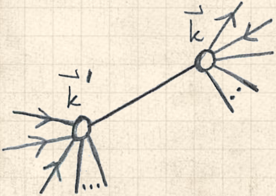
$$\vec{k} = [\, k_{\mathrm{u}} \;\; k_{\mathrm{i}} \;\; k_{\mathrm{o}} \,]^{\mathsf{T}}.$$

Correlations:

- Now add correlations (two point or Markovian):
  1. $P^{(\mathrm{u})}(\vec{k}\,|\,\vec{k}')$ = probability that an undirected edge leaving a degree $\vec{k}'$ nodes arrives at a degree $\vec{k}$ node.
  2. $P^{(\mathrm{i})}(\vec{k}\,|\,\vec{k}')$ = probability that an edge leaving a degree $\vec{k}'$ nodes arrives at a degree $\vec{k}$ node is an in-directed edge relative to the destination node.
  3. $P^{(\mathrm{o})}(\vec{k}\,|\,\vec{k}')$ = probability that an edge leaving a degree $\vec{k}'$ nodes arrives at a degree $\vec{k}$ node is an out-directed edge relative to the destination node.
- Now require more refined (detailed) balance.
- Conditional probabilities cannot be arbitrary.
  1. $P^{(\mathrm{u})}(\vec{k}\,|\,\vec{k}')$ must be related to $P^{(\mathrm{u})}(\vec{k}'\,|\,\vec{k})$.
  2. $P^{(\mathrm{o})}(\vec{k}\,|\,\vec{k}')$ and $P^{(\mathrm{i})}(\vec{k}\,|\,\vec{k}')$ must be connected.

Correlations—Undirected edge balance:

- Randomly choose an edge, and randomly choose one end.
- Say we find a degree $\vec{k}$ node at this end, and a degree $\vec{k}'$ node at the other end.
- Define probability this happens as $P^{(\mathrm{u})}(\vec{k}, \vec{k}')$.
- Observe we must have $P^{(\mathrm{u})}(\vec{k}, \vec{k}') = P^{(\mathrm{u})}(\vec{k}', \vec{k})$.



- Conditional probability connection:

$$P^{(\mathrm{u})}(\vec{k}, \vec{k}') = P^{(\mathrm{u})}(\vec{k} \,|\, \vec{k}') \frac{k'_{\mathrm{u}} P(\vec{k}')}{\langle k'_{\mathrm{u}} \rangle}$$

$$P^{(\mathrm{u})}(\vec{k}', \vec{k}) = P^{(\mathrm{u})}(\vec{k}' \,|\, \vec{k}) \frac{k_{\mathrm{u}} P(\vec{k})}{\langle k_{\mathrm{u}} \rangle}.$$

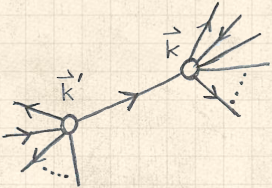
Correlations—Directed edge balance:

- The quantities

$$\frac{k_{\mathrm{o}} P(\vec{k})}{\langle k_{\mathrm{o}} \rangle} \text{ and } \frac{k_{\mathrm{i}} P(\vec{k})}{\langle k_{\mathrm{i}} \rangle}$$

give the probabilities that in starting at a random end of a randomly selected edge, we begin at a degree $\vec{k}$ node and then find ourselves travelling:

1. along an outgoing edge, or
2. against the direction of an incoming edge.



- We therefore have

$$P^{(\mathrm{dir})}(\vec{k},\vec{k}') = P^{(\mathrm{i})}(\vec{k}\,|\,\vec{k}')\frac{k_{\mathrm{o}}' P(\vec{k}')}{\langle k_{\mathrm{o}}' \rangle} = P^{(\mathrm{o})}(\vec{k}'\,|\,\vec{k})\frac{k_{\mathrm{i}} P(\vec{k})}{\langle k_{\mathrm{i}} \rangle}.$$

- Note that $P^{(\mathrm{dir})}(\vec{k},\vec{k}')$ and $P^{(\mathrm{dir})}(\vec{k}',\vec{k})$ are in general not related if $\vec{k} \neq \vec{k}'$.

Summary of contagion conditions for uncorrelated networks:

- I. Undirected, Uncorrelated—$f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathrm{u}}} P^{(\mathrm{u})}(k_{\mathrm{u}} \,|\, *) \bullet (k_{\mathrm{u}} - 1) \bullet B_{k_{\mathrm{u}},*}$$

- II. Directed, Uncorrelated—$f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathrm{i}},k_{\mathrm{o}}} P^{(\mathrm{i})}(k_{\mathrm{i}}, k_{\mathrm{o}} \,|\, *) \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}},*}$$

- III. Mixed Directed and Undirected, Uncorrelated—

$$\begin{bmatrix} f^{(\mathrm{u})}(d+1) \\ f^{(\mathrm{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathrm{u})}(d) \\ f^{(\mathrm{o})}(d) \end{bmatrix}$$

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Summary of contagion conditions for correlated networks:

- IV. Undirected, Correlated— $f_{k_\mathsf{u}}(d+1) = \sum_{k'_\mathsf{u}} R_{k_\mathsf{u} k'_\mathsf{u}} f_{k'_\mathsf{u}}(d)$

$$R_{k_\mathsf{u} k'_\mathsf{u}} = P^{(\mathsf{u})}(k_\mathsf{u} \,|\, k'_\mathsf{u}) \bullet (k_\mathsf{u} - 1) \bullet B_{k_\mathsf{u} k'_\mathsf{u}}$$

- V. Directed, Correlated— $f_{k_\mathsf{i} k_\mathsf{o}}(d+1) = \sum_{k'_\mathsf{i}, k'_\mathsf{o}} R_{k_\mathsf{i} k_\mathsf{o} k'_\mathsf{i} k'_\mathsf{o}} f_{k'_\mathsf{i} k'_\mathsf{o}}(d)$

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- VI. Mixed Directed and Undirected, Correlated—

$$\left[\begin{array}{c} f^{(\mathsf{u})}_{\vec{k}}(d+1) \\ f^{(\mathsf{o})}_{\vec{k}}(d+1) \end{array} \right] = \sum_{k'} \mathbf{R}_{\vec{k}\vec{k}'} \left[\begin{array}{c} f^{(\mathsf{u})}_{\vec{k}'}(d) \\ f^{(\mathsf{o})}_{\vec{k}'}(d) \end{array} \right]$$

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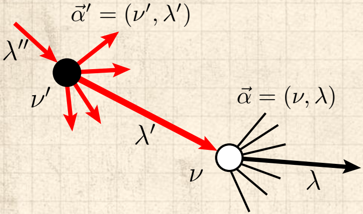
Full generalization:



$$f_{\vec{\alpha}}(d+1) = \sum_{\vec{\alpha}'} R_{\vec{\alpha}\vec{\alpha}'} f_{\vec{\alpha}'}(d)$$

$R_{\vec{\alpha}\vec{\alpha}'}$ is the gain ratio matrix and has the form:

$$R_{\vec{\alpha}\vec{\alpha}'} = P_{\vec{\alpha}\vec{\alpha}'} \bullet k_{\vec{\alpha}\vec{\alpha}'} \bullet B_{\vec{\alpha}\vec{\alpha}'}.$$

- $P_{\vec{\alpha}\vec{\alpha}'}$ = conditional probability that a type λ' edge emanating from a type ν' node leads to a type ν node.
- $k_{\vec{\alpha}\vec{\alpha}'}$ = potential number of newly infected edges of type λ emanating from nodes of type ν.
- $B_{\vec{\alpha}\vec{\alpha}'}$ = probability that a type ν node is eventually infected by a single infected type λ' link arriving from a neighboring node of type ν'.
- Generalized contagion condition:

$$\max|\mu| : \mu \in \sigma(\mathbf{R}) > 1$$

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- Stories $\sim$ Characters + Time.
- Characters are shortcuts to stories.

Outline

For novel diseases:

1. Can we predict the size of an epidemic?
2. How important is the reproduction number R_0?

R_0 approximately same for all of the following:

- 1918-19 “Spanish Flu” $\sim$ 75,000,000 world-wide, 500,000 deaths in US.
- 1957-58 “Asian Flu” $\sim$ 2,000,000 world-wide, 70,000 deaths in US.
- 1968-69 “Hong Kong Flu” $\sim$ 1,000,000 world-wide, 34,000 deaths in US.
- 2003 “SARS Epidemic” $\sim$ 800 deaths world-wide.

Improving simple models

Idea for social networks: incorporate identity

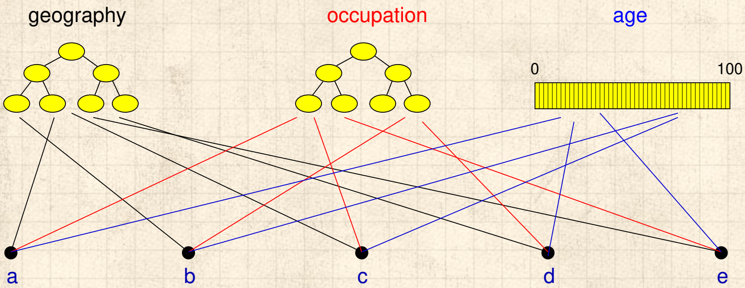
Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Age
- Recreational activities

Groups are crucial ...

- formed by people with at least one similar attribute
- Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks. [110]

Generalized context space



(Blau & Schwartz [12], Simmel [91], Breiger [20])

A toy agent-based model:


Multiscale, resurgent epidemics in a hierarchical metapopulation model

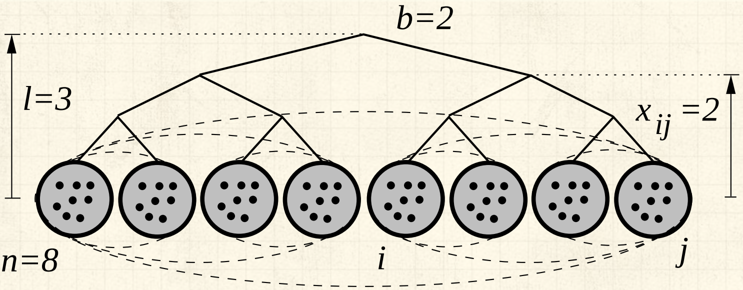
"Multiscale, resurgent epidemics in a hierarchcial metapopulation model"
Watts et al.,
Proc. Natl. Acad. Sci., **102**, 11157–11162, 2005. [111]

Geography: allow people to move between contexts

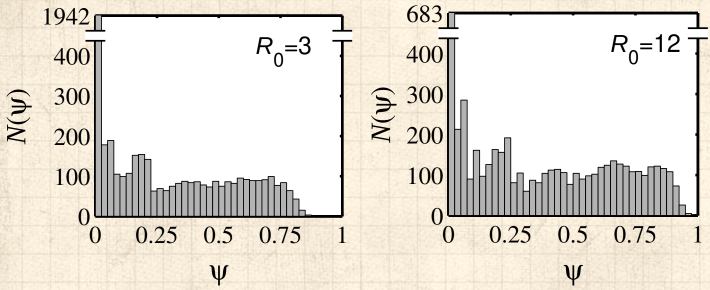
- Locally: standard SIR model with random mixing
- discrete time simulation
- β = infection probability
- γ = recovery probability
- P = probability of travel
- Movement distance: $\Pr(d) \propto \exp(-d/\xi)$
- ξ = typical travel distance

A toy agent-based model

Schematic:



Example model output: size distributions



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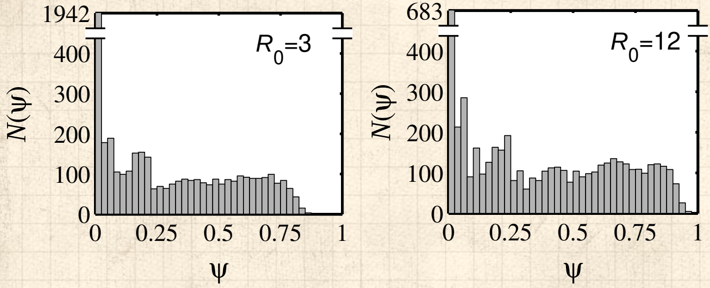

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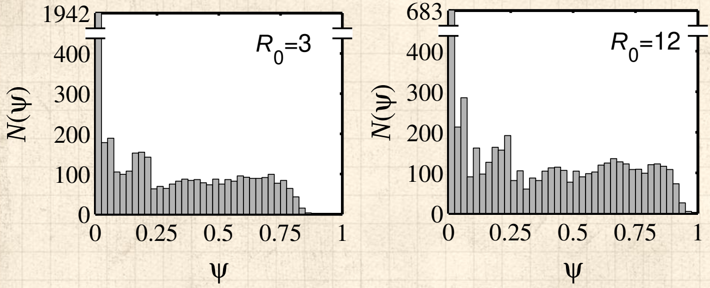

- Flat distributions are possible for certain ξ and P.

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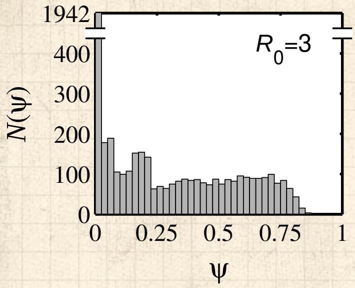

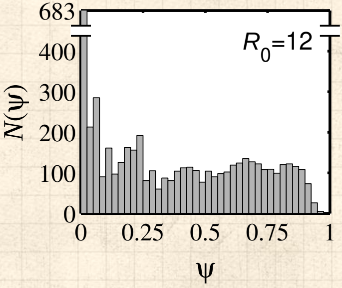

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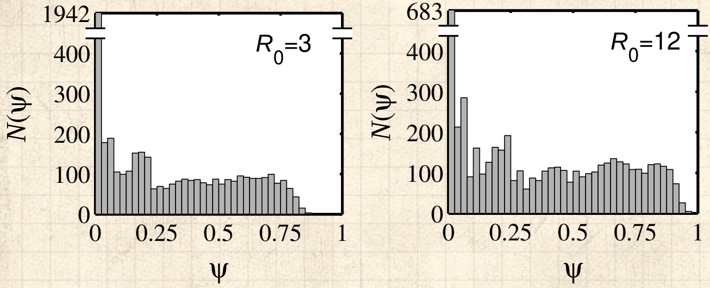
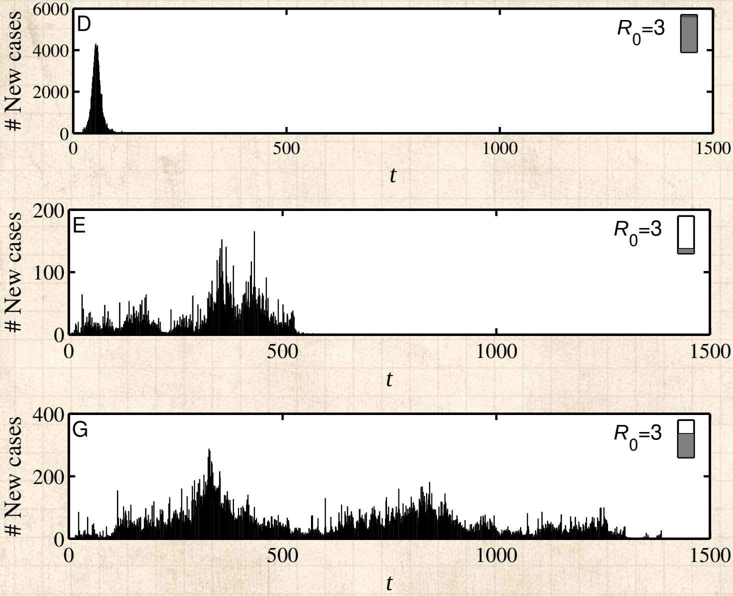


- Flat distributions are possible for certain ξ and P.
- Different R_0's may produce similar distributions
- Same epidemic sizes may arise from different R_0's

Model output—resurgence



Journal entry, 2020/02/21:

Twitter DMs to Sam Scarpino:

- Okay: The scientists studying pandemics need to be able to present some kind set of numbers that show how bad things are. The whole R_0 disaster has been waiting to happen because people have been … lazily having fun with math models? Unconcerned about how to communicate vital scientific information? Stupid? I don't know. Maybe a radar plot visualization. I don't know.

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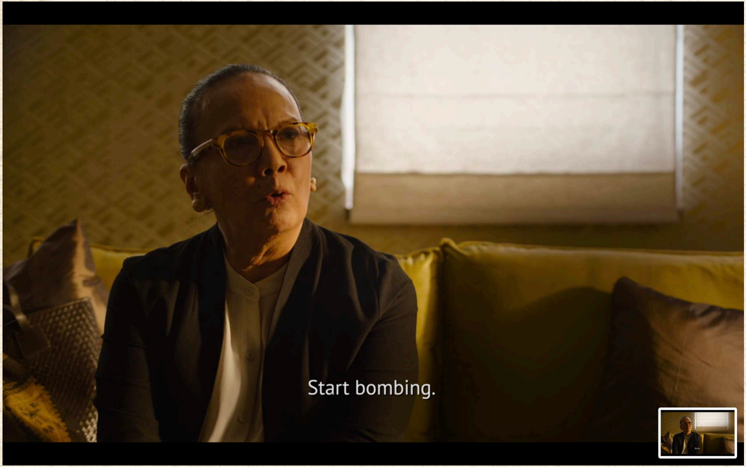
- "When these three boundaries are crossed, we are in trouble"
- Measles has an R_0 of 20. We should all have it. Of course, there's no f**king time scale for R_0 so we don't know when that happens.

The Last of Us: Groups.

The Last of Us: Groups.



The Last of Us: Groups.

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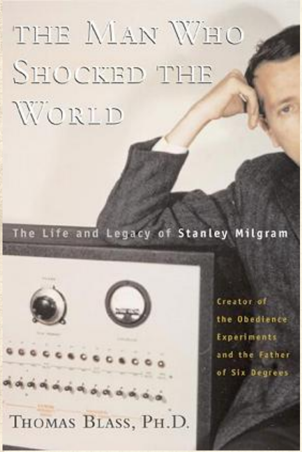
Understanding distributed social search

Milgram's social search experiment



http://www.stanleymilgram.com

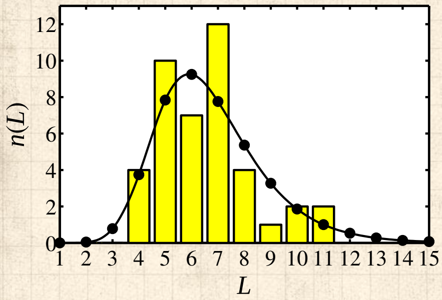
- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- chain length $\simeq 6.5$.

Popular terms:

- The Small World Phenomenon;
- "Six Degrees of Separation."

The model—results

Milgram's Nebraska-Boston data:



Model parameters:

- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;

- $\langle L_{\text{model}} \rangle \simeq 6.7$
- $L_{\text{data}} \simeq 6.5$

Social search—the Columbia experiment

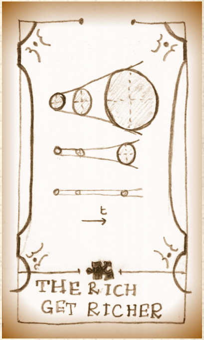
- 60,000+ participants in 166 countries
- 18 targets in 13 countries including
  - a professor at an Ivy League university,
  - an archival inspector in Estonia,
  - a technology consultant in India,
  - a policeman in Australia,
    and
  - a veterinarian in the Norwegian army.
- 24,000+ chains

We were lucky and contagious:

"Using E-Mail to Count Connections", Sarah Milstein, New York Times, Circuits Section (December, 2001)

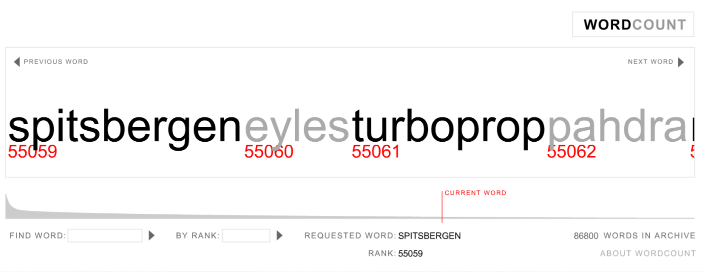
THE CHURCH OF QUARTEROLOGY


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE DREAM OF EFFICIENT LANGUAGE
1 2 5 4 3 6 7 8 9
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
BIO HAZARD
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
D-1
D
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END


t
THE RICH GET RICHER

Jonathan Harris's Wordcount:

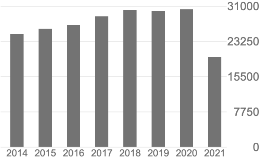
A word frequency distribution explorer:



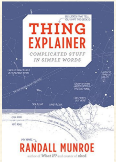
The long tail of knowledge:

Cited by VIEW ALL

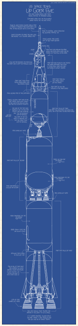
	All	Since 2016
Citations	432350	165872
h-index	155	104
i10-index	369	277

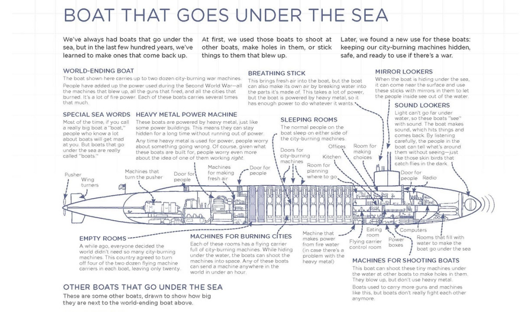


Take a scrolling voyage
to the citational abyss,
starting at the surface with
the lonely, giant citaceans,
moving down
to the legion of strange,
sometimes misplaced,
unloved creatures,
that dwell in
Kahneman's Google Scholar page

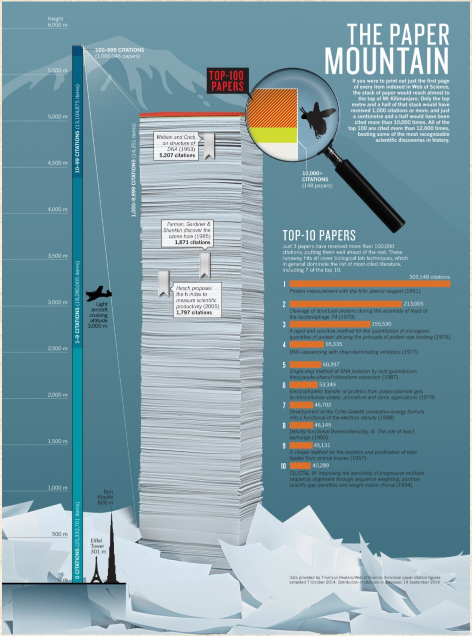


"Thing Explainer: Complicated Stuff in Simple Words" by Randall Munroe (2015). [70]





Up goer five

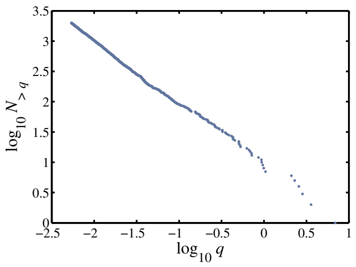


Nature (2014):
Most cited papers of all time

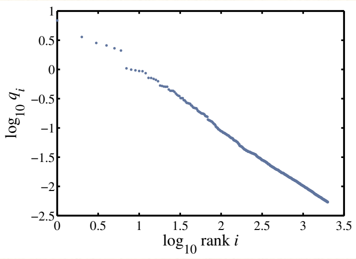
Size distributions:

Brown Corpus (1,015,945 words):

CCDF:



Zipf:



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- ‘Size’ = word frequency

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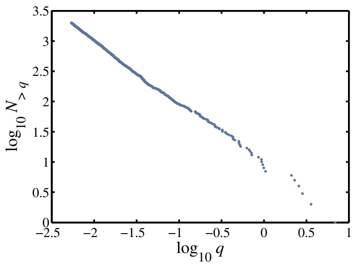

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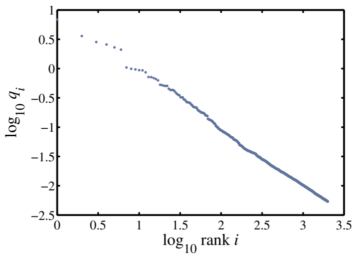

- The, of, and, to, a, ...= ‘objects’
- ‘Size’ = word frequency
- Beep: (Important) CCDF and Zipf plots are related ...

Pre-Zipf's law observations of Zipf's law

- 1910s: Word frequency examined re Stenography (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup [36].

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- 1926: Lotka [61]:
  # Scientific papers per author (Lotka's law)

Theoretical Work of Yore:

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  Network of Scientific Citations.
- 1999: Barabasi and Albert [8]:
  The World Wide Web, networks-at-large.

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Random Competitive Replication (RCR):

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Note: This is a terrible way to write a novel.

For example:

- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
$6/21$	ook
$4/21$	the
$3/21$	and
$2/21$	penguin
$\vdots$	
$1/21$	library

Micro-to-Macro story with ρ and γ measurable.

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

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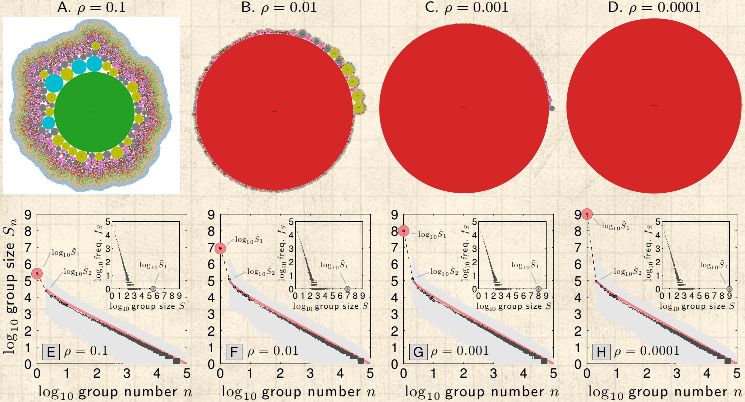
- All elephants have different flavors.
- Upshot: Tunable mechanism producing a family of universality classes.

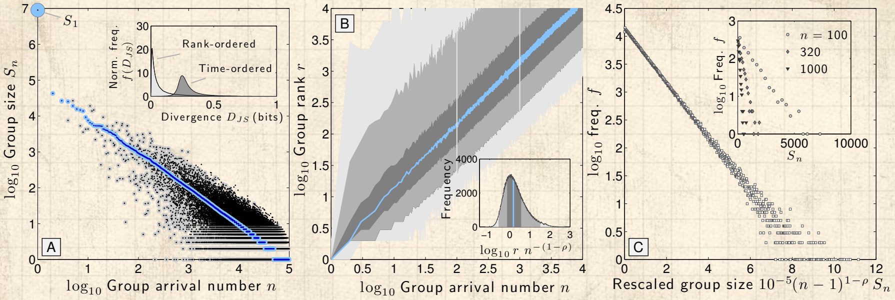
"Simon's fundamental rich-get-richer model entails a dominant first-mover advantage"

Dodds et al.,
Physical Review E, **95**, 052301, 2017. [29]



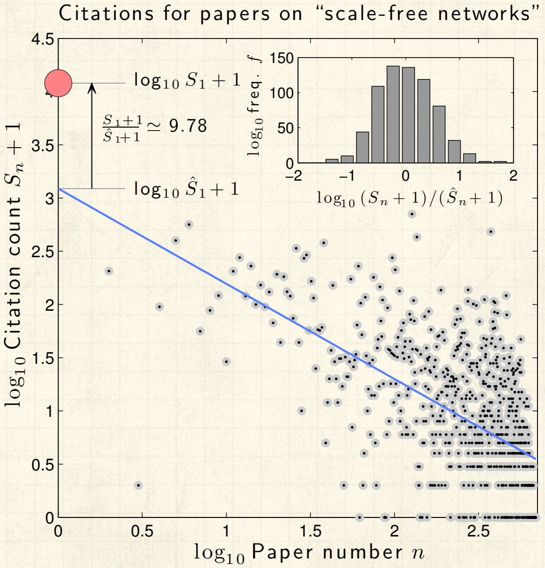
See visualization at paper's online app-endices

Arrival variability:



- Any one simulation shows a high amount of disorder.
- Two orders of magnitude variation in possible rank.
- Rank ordering creates a smooth Zipf distribution.
- Size distribution for the nth arriving group show exponential decay.

Self-referential citation data:



The Quickening—Mandelbrot v. Simon:

There Can Be Only One:

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There Can Be Only One:

- Things there should be only one of: Theory, Highlander Films.

The Quickening—Mandelbrot v. Simon:

There Can Be Only One:

- Things there should be only one of: Theory, Highlander Films.
- Feel free to play Queen's It's a Kind of Magic in your head (funding remains tight).

vs.

Mandelbrot vs. Simon:

We were born to be Princes of the Universe

vs.

Mandelbrot vs. Simon:

- Mandelbrot (1953): "An Informational Theory of the Statistical Structure of Languages" [62]

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- Simon (1955): "On a class of skew distribution functions" [92]
- Mandelbrot (1959): "A note on a class of skew distribution functions: analysis and critique of a paper by H.A. Simon" [63]
- Simon (1960): "Some further notes on a class of skew distribution functions" [93]

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THE NETWORKS FREE OF SCALE

Scale-free networks

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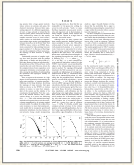

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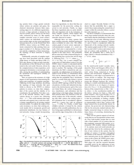

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- Somewhat misleading nomenclature ...

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- KR also looked at changing the details of the attachment kernel.

Outline


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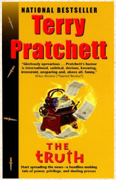
'The rumor spread through the city like wildfire



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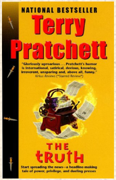

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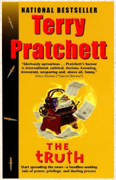

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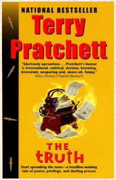

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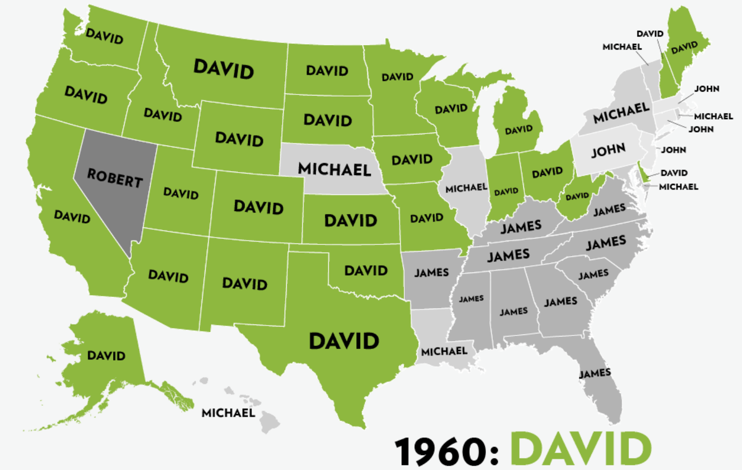

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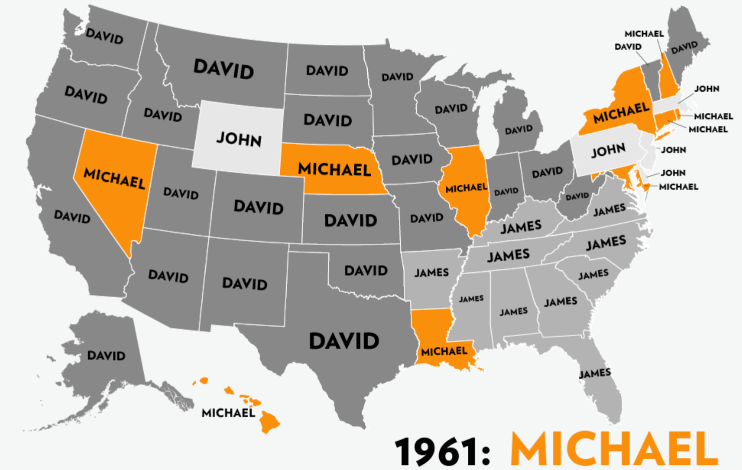

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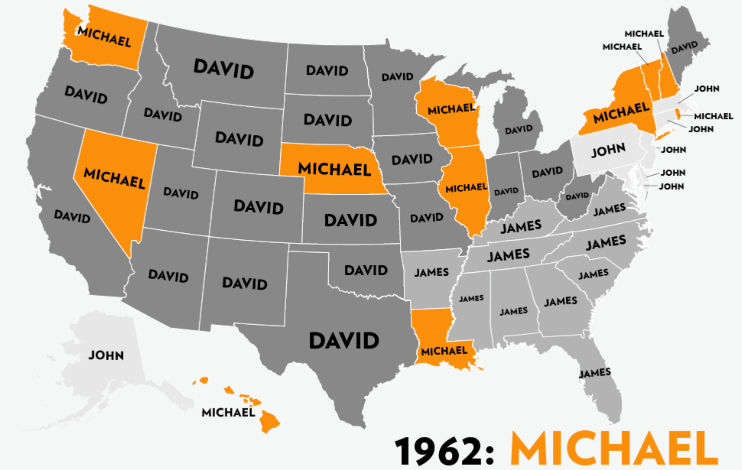

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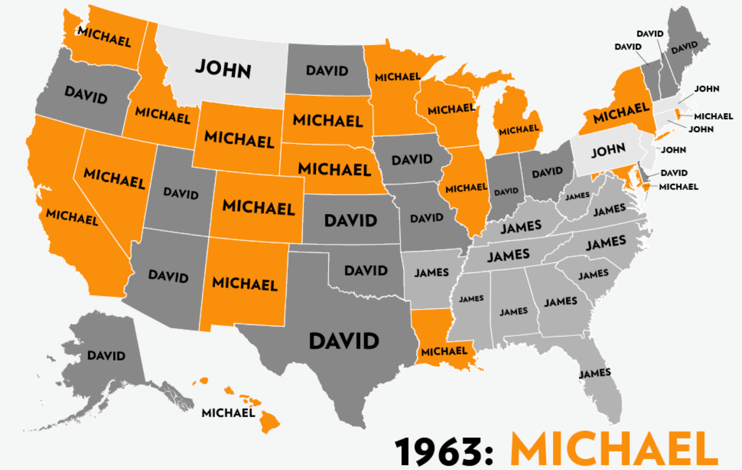
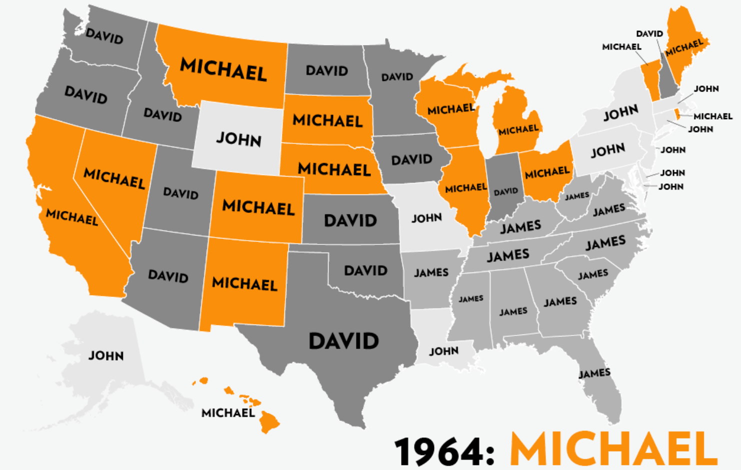


From the Atlantic

The PoCSverse
Basic definitions
Examples
Basic Properties
Branching Networks
Supply Networks
Random networks
Major Models
Generalized Affiliation Networks
Thresholds
Generating Functions
Structure Detection
Big Nutshell
References



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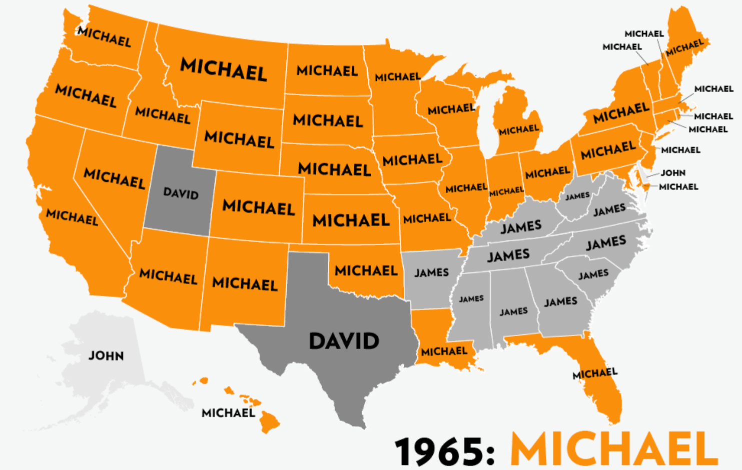

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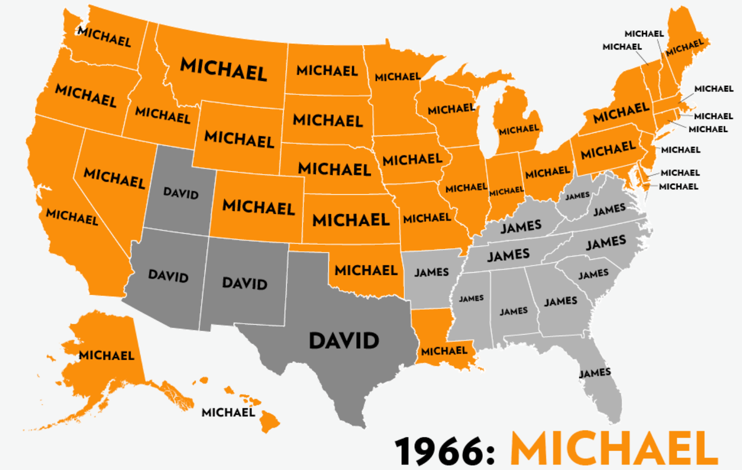

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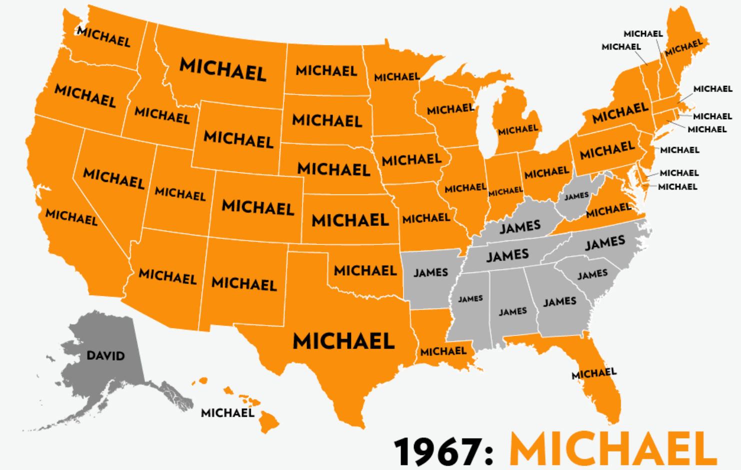

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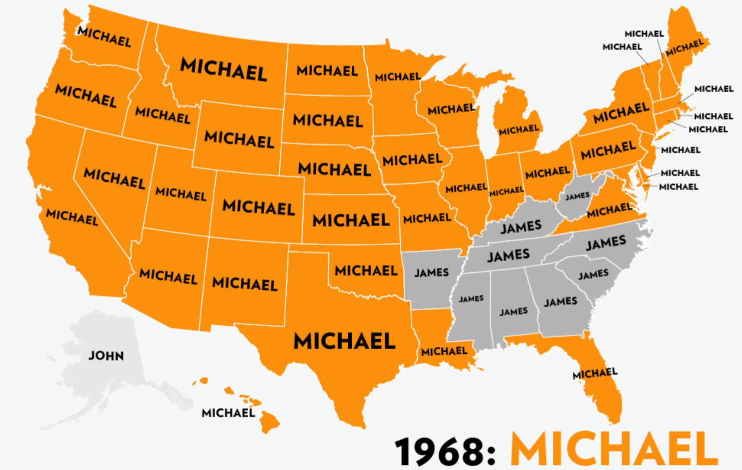

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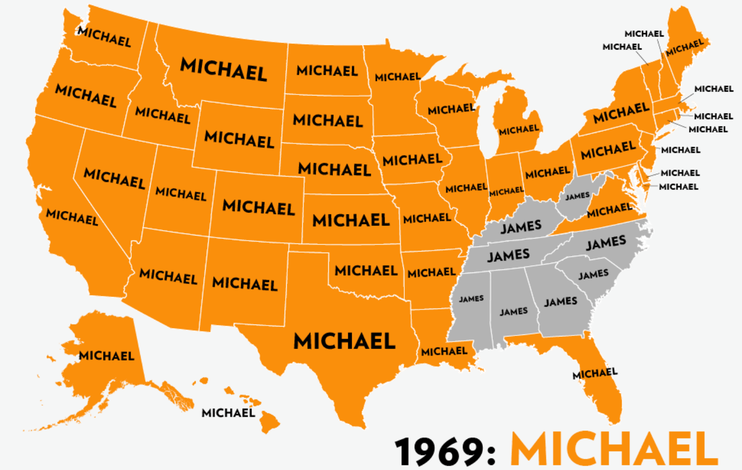

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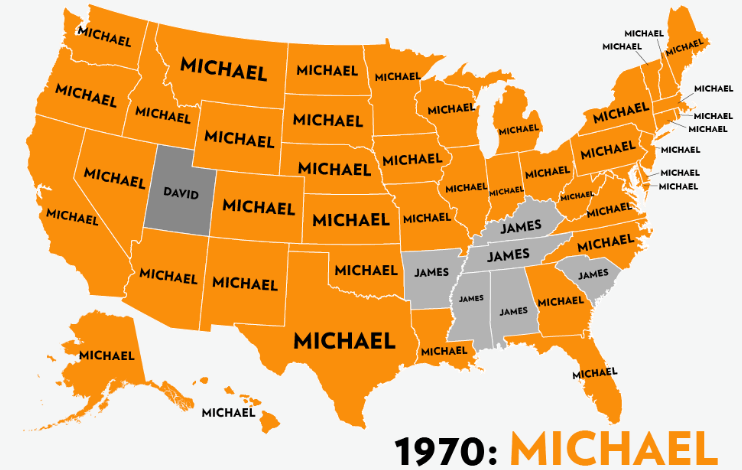

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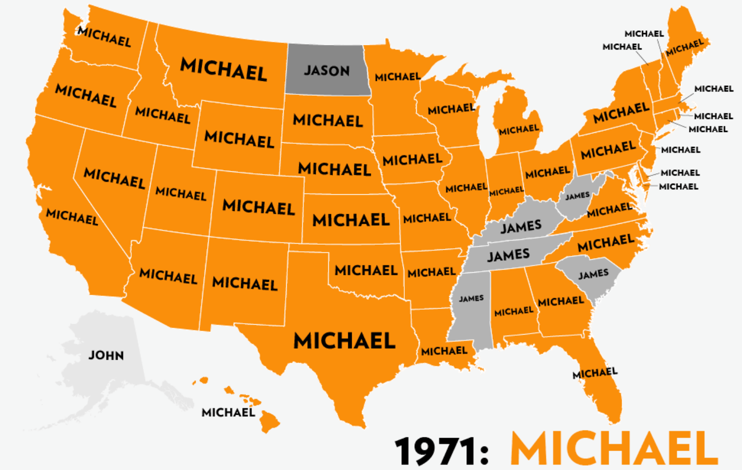

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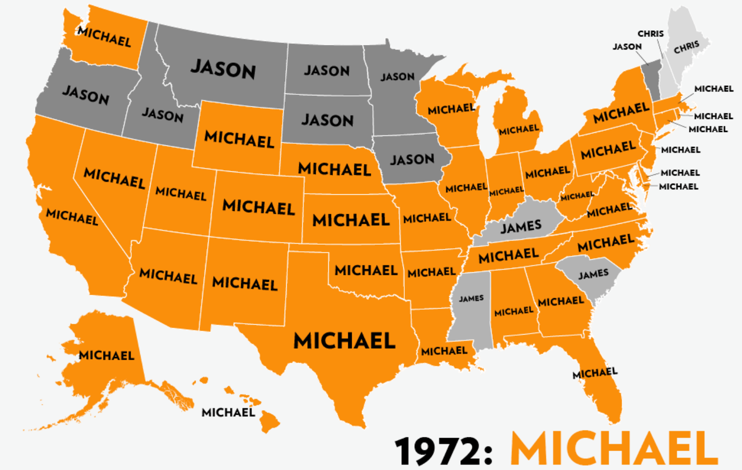

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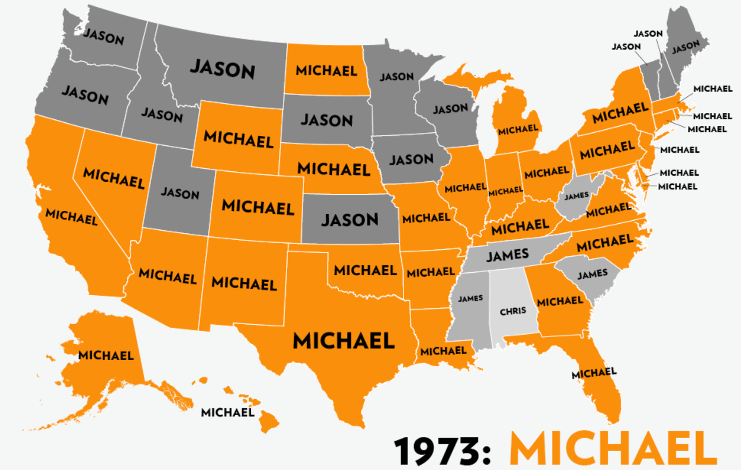

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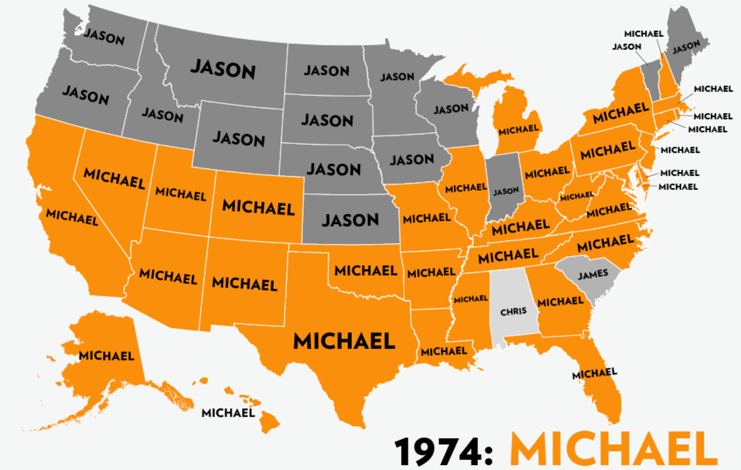

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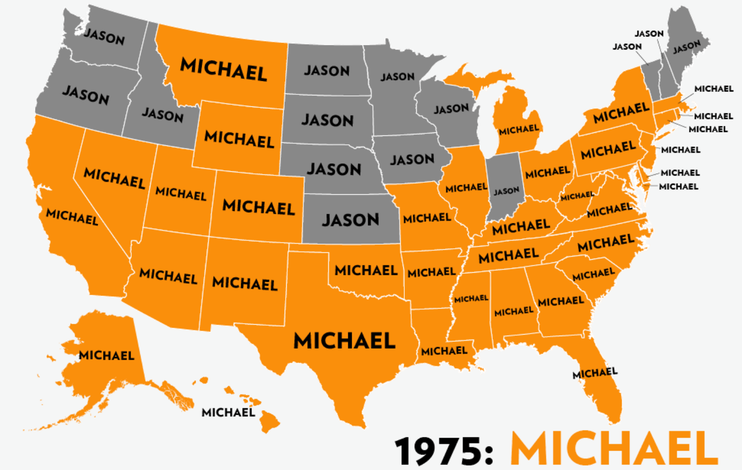

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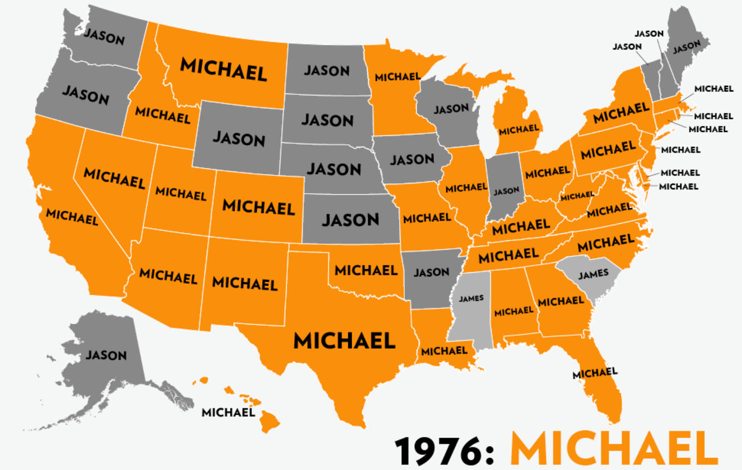

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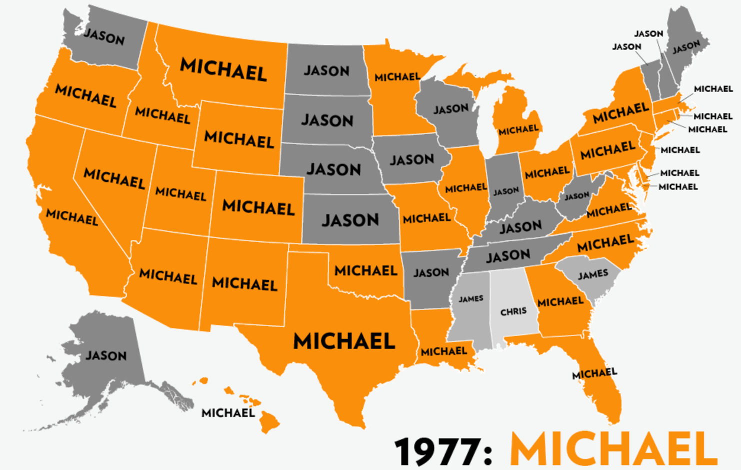

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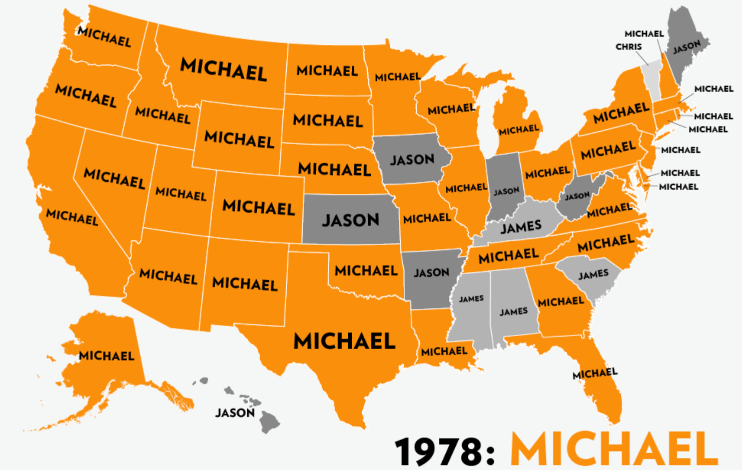

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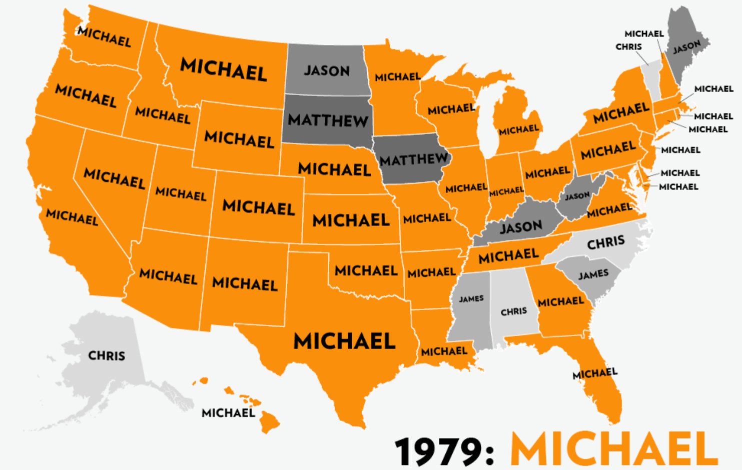

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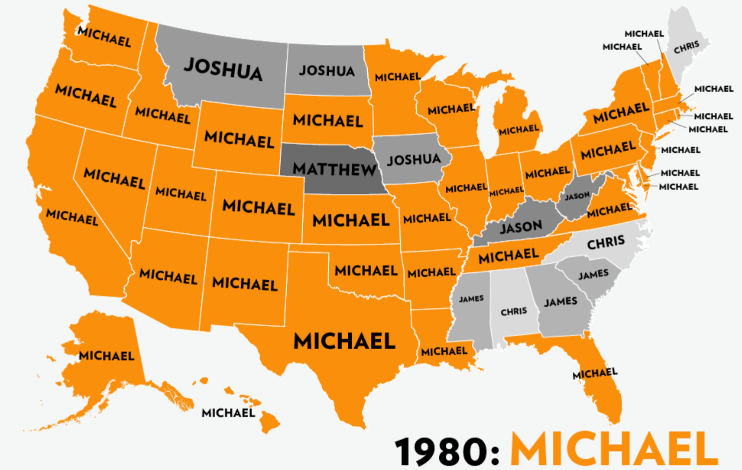

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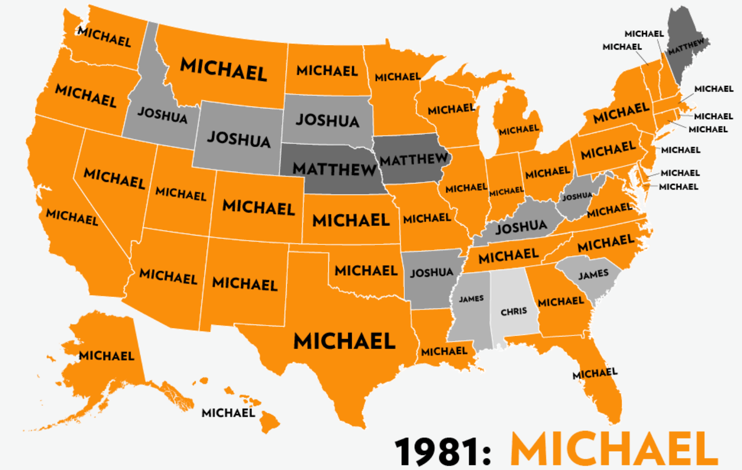

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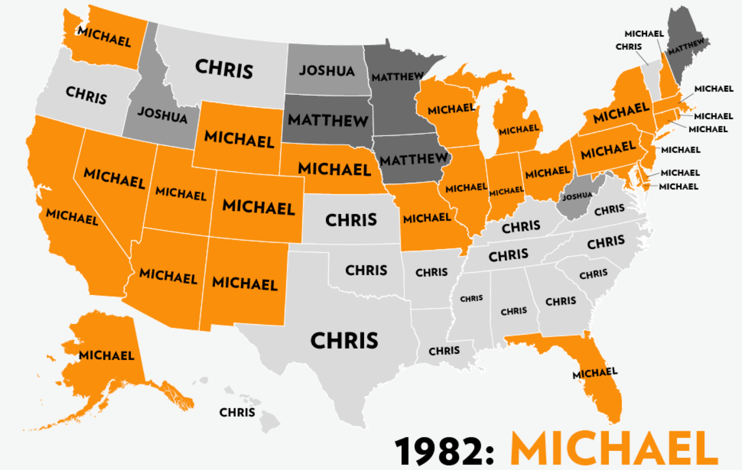

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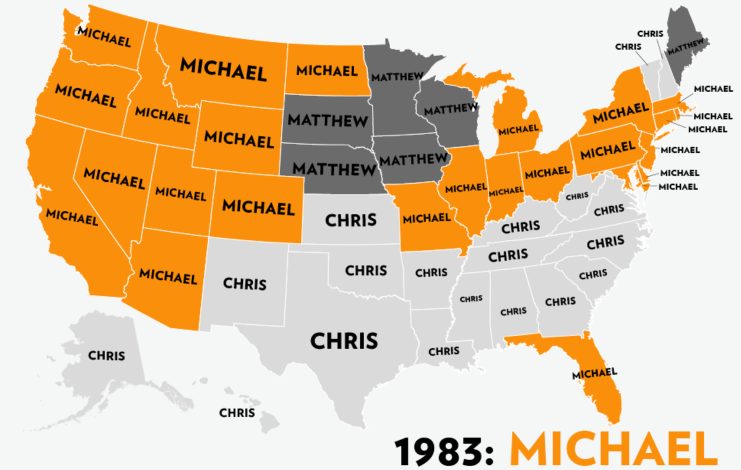

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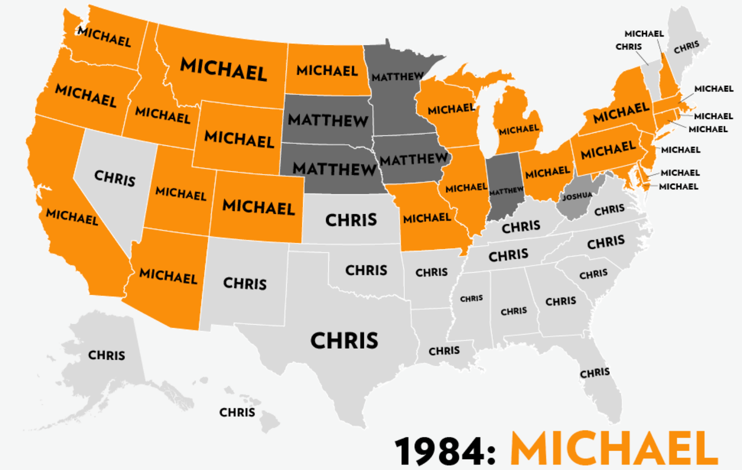

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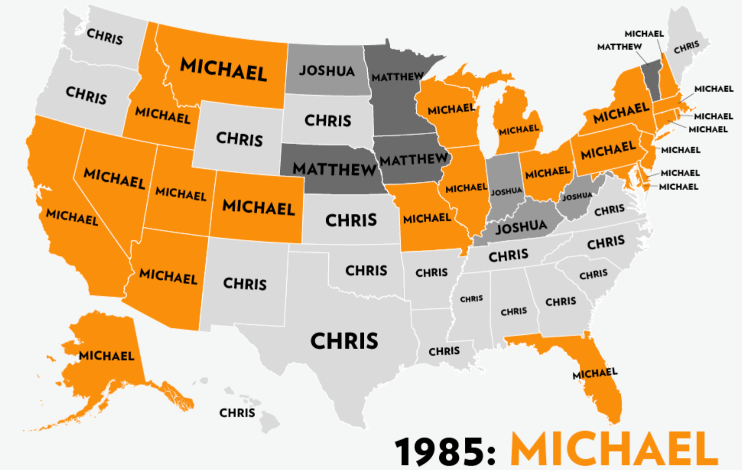

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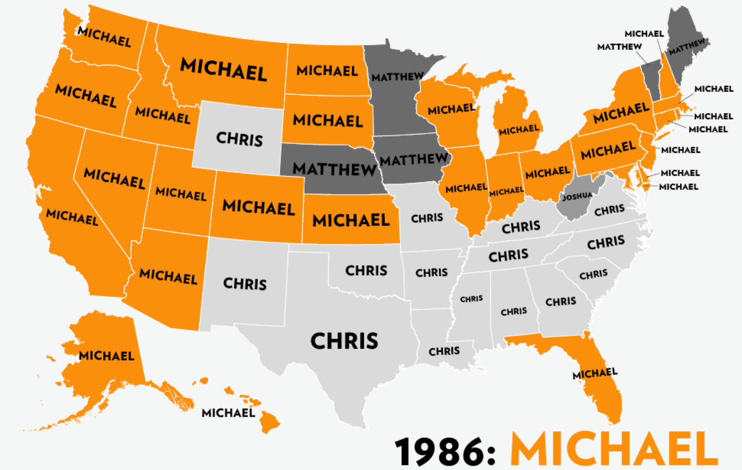

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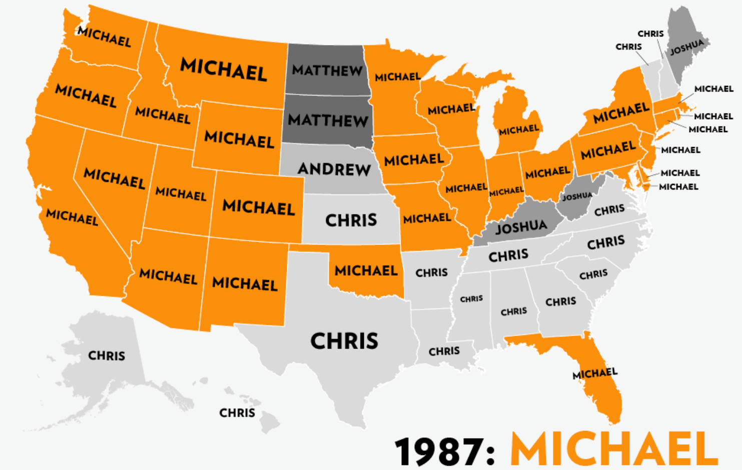

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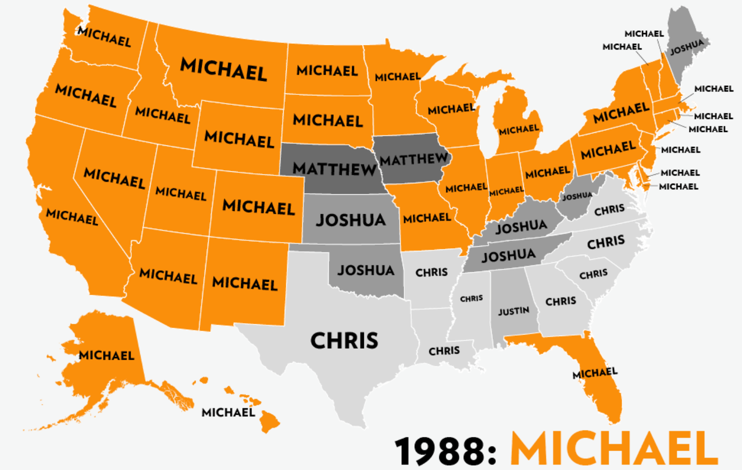

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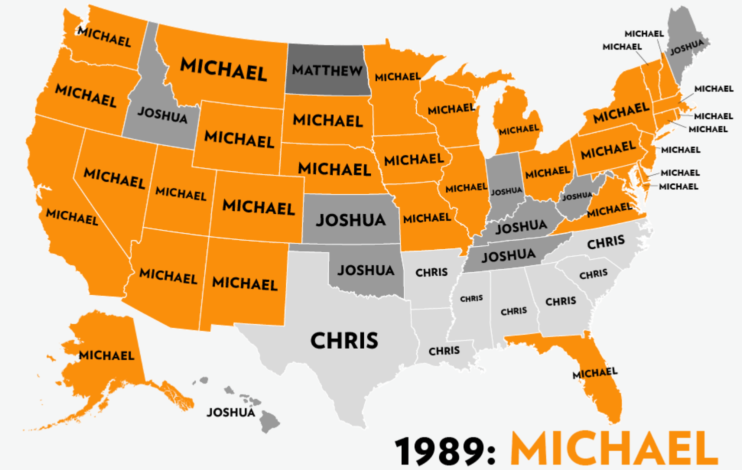

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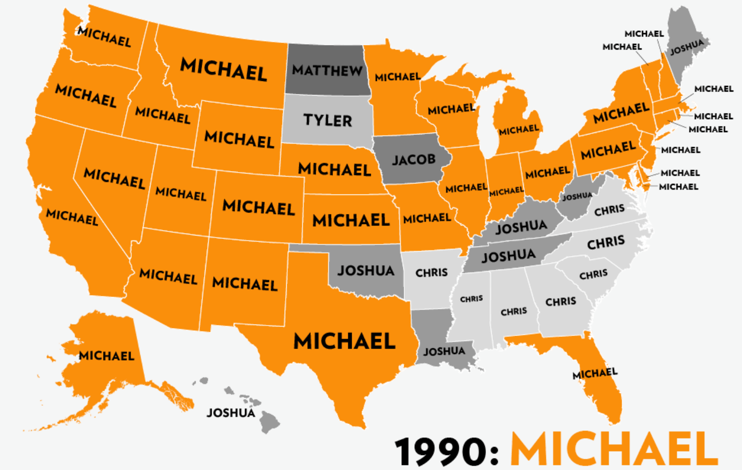

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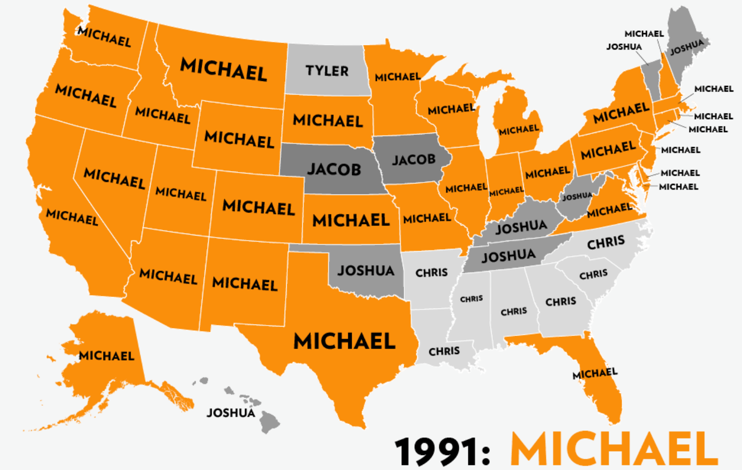

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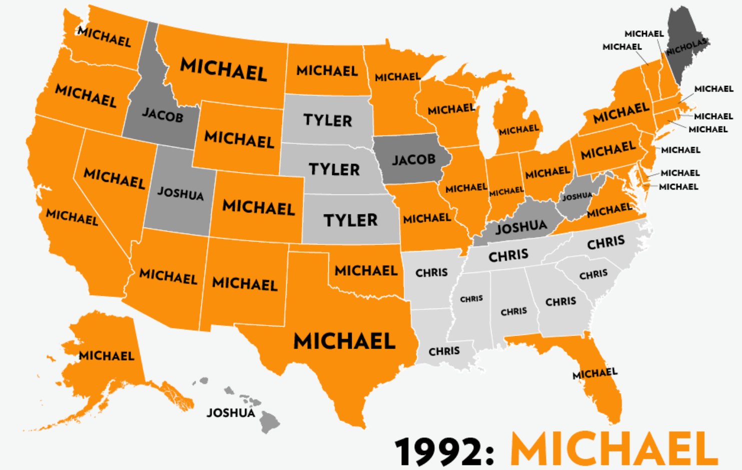

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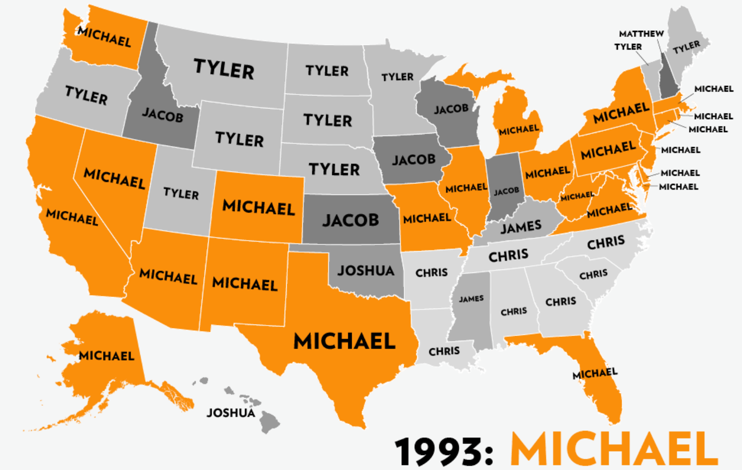

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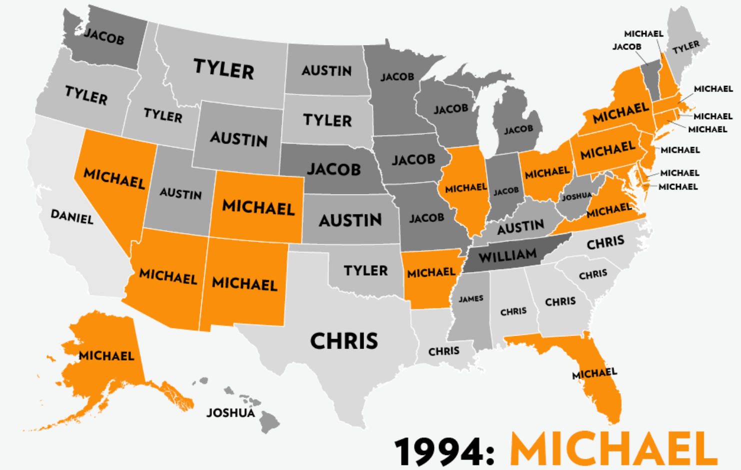

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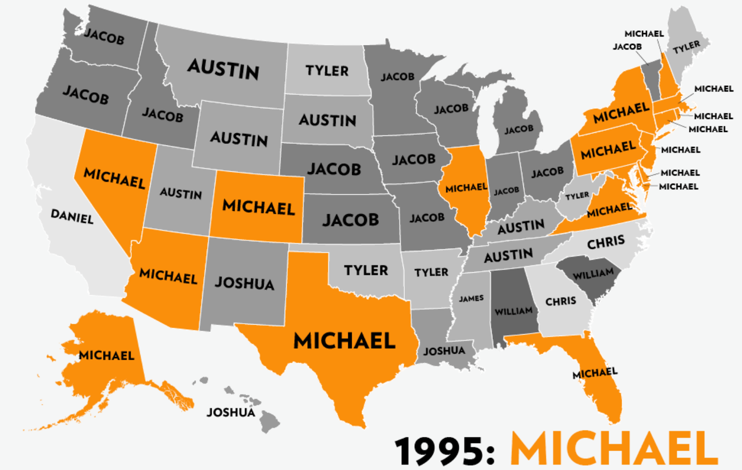

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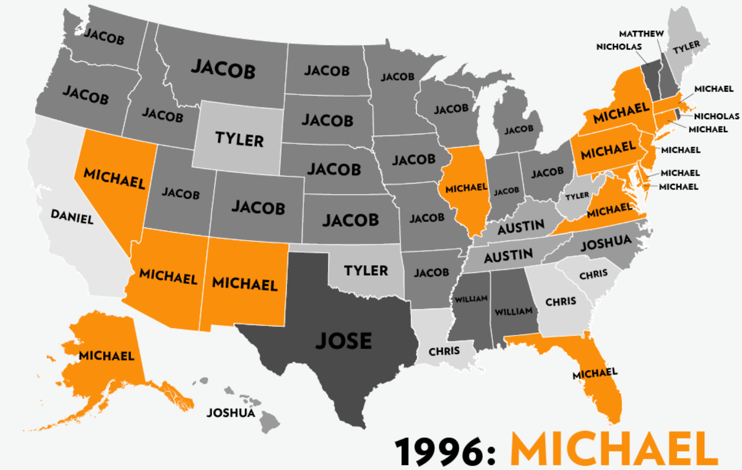

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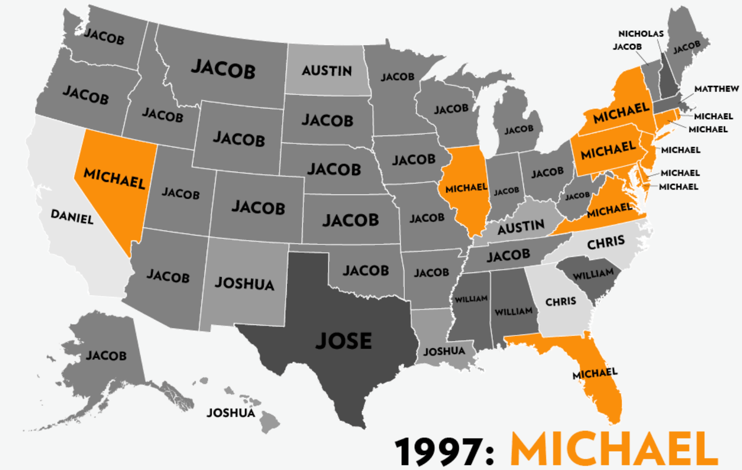

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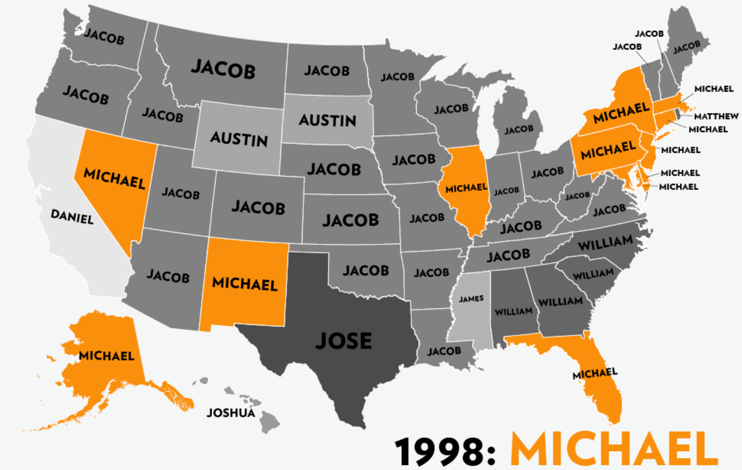

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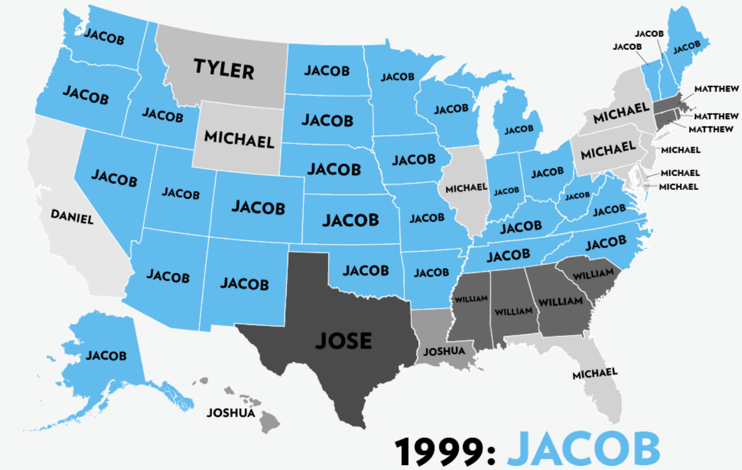

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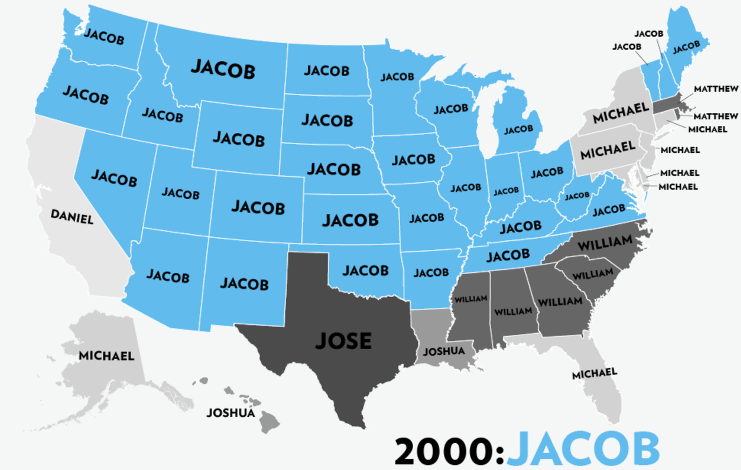

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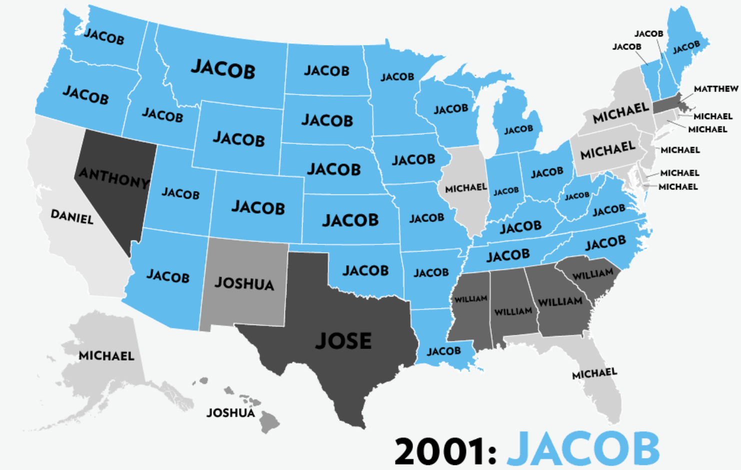

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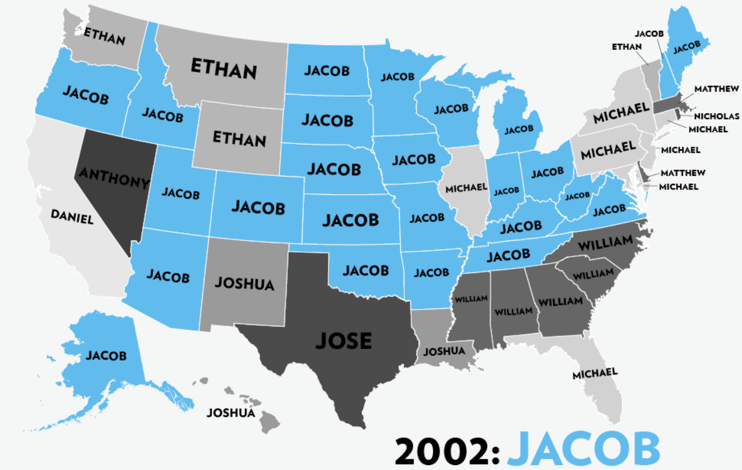

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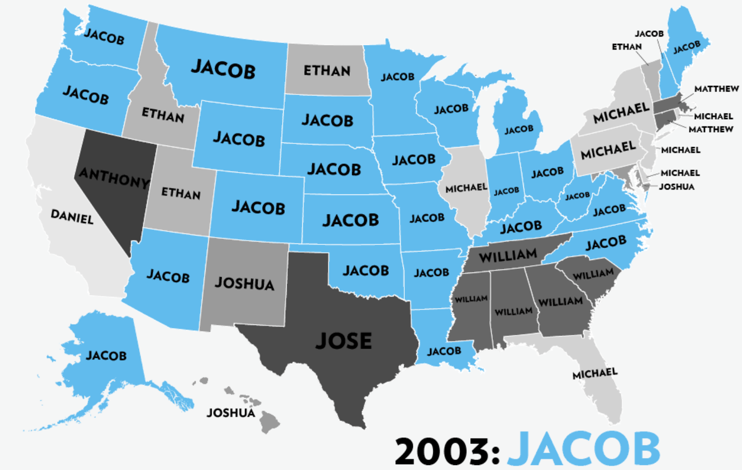

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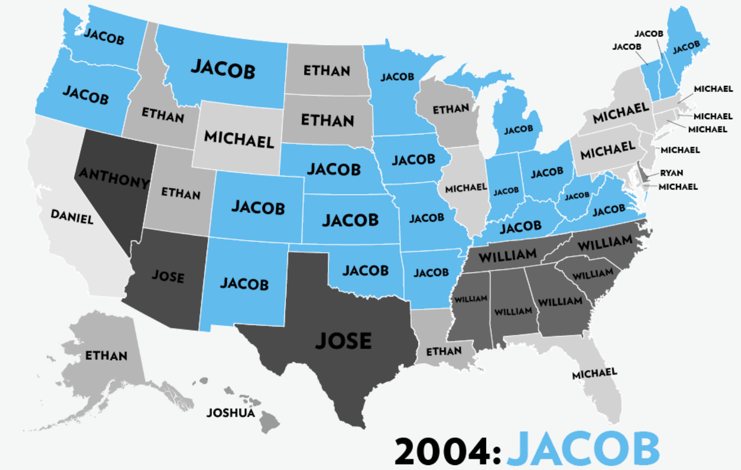

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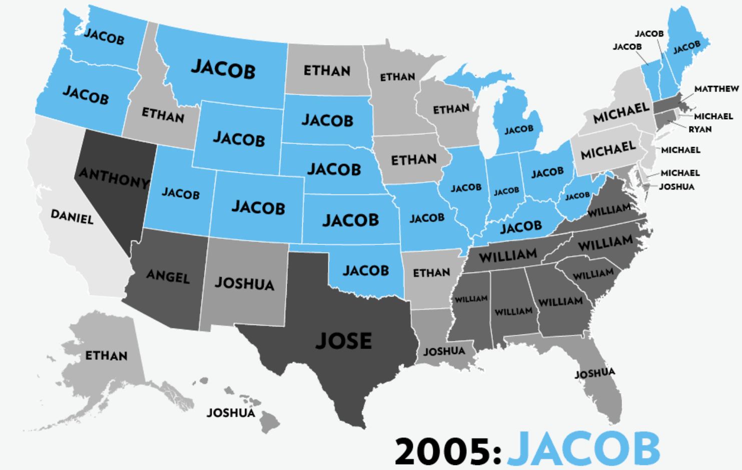

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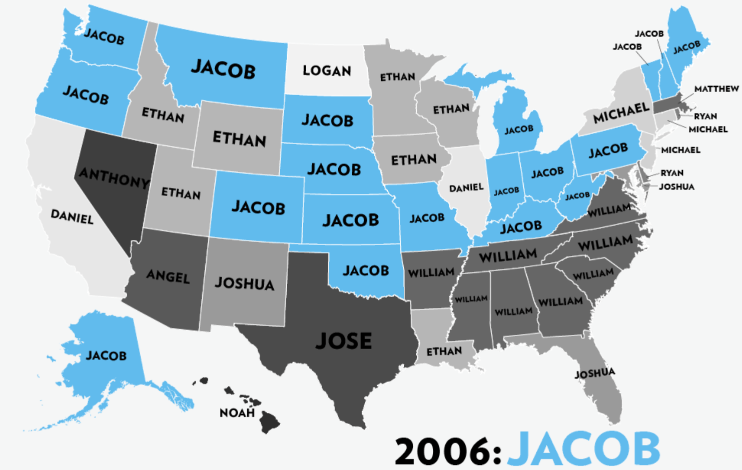

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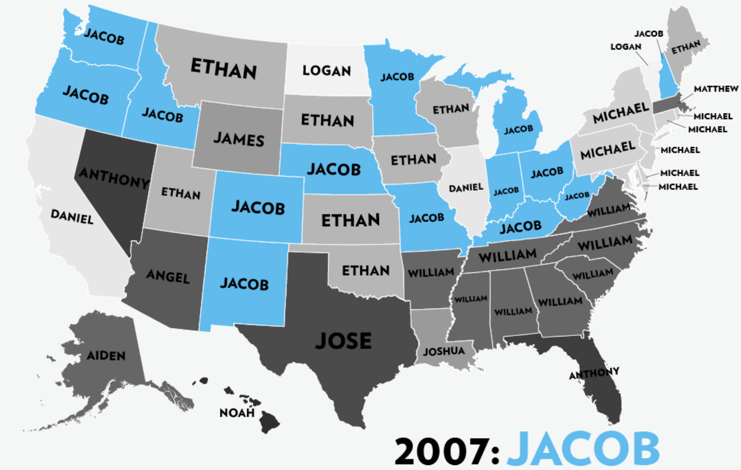

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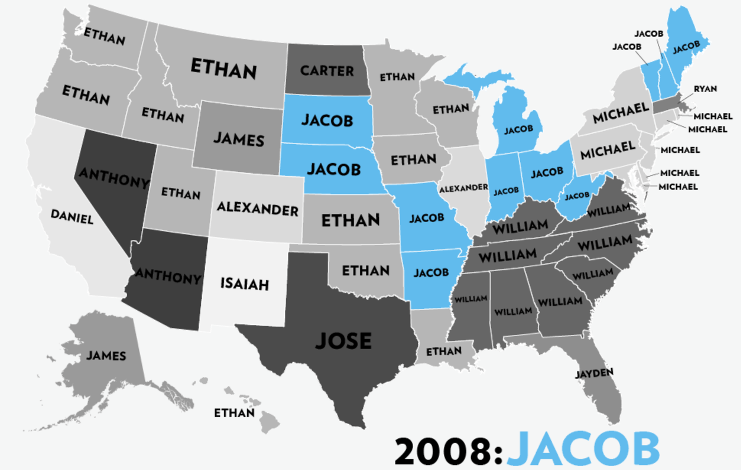

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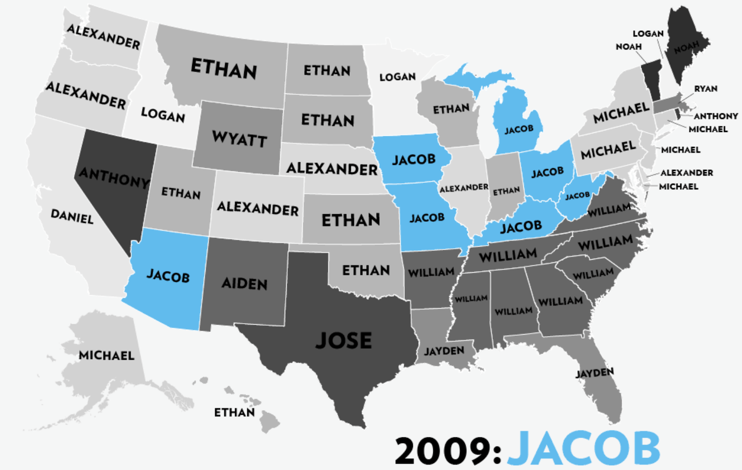

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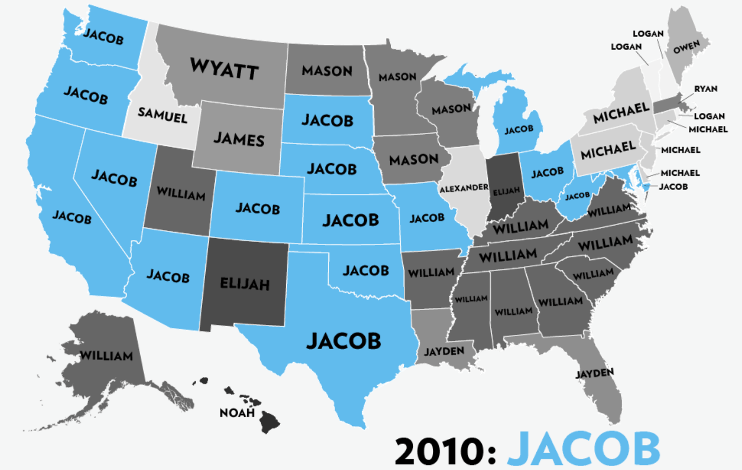

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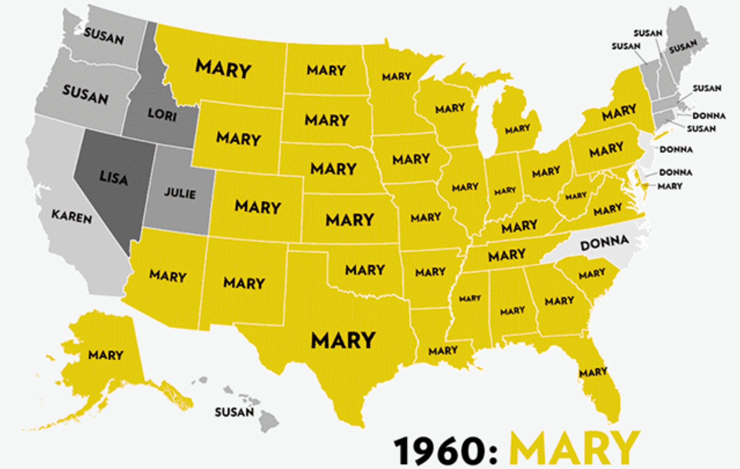

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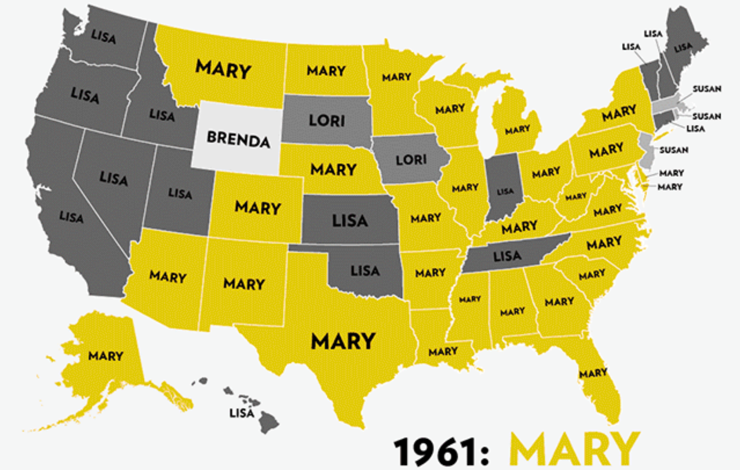

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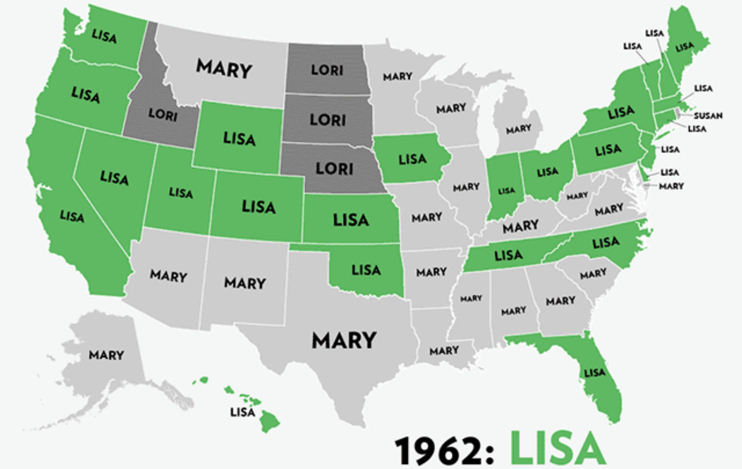

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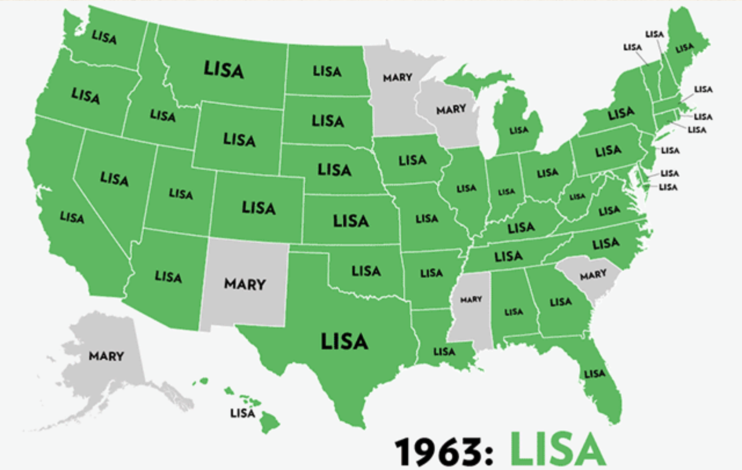

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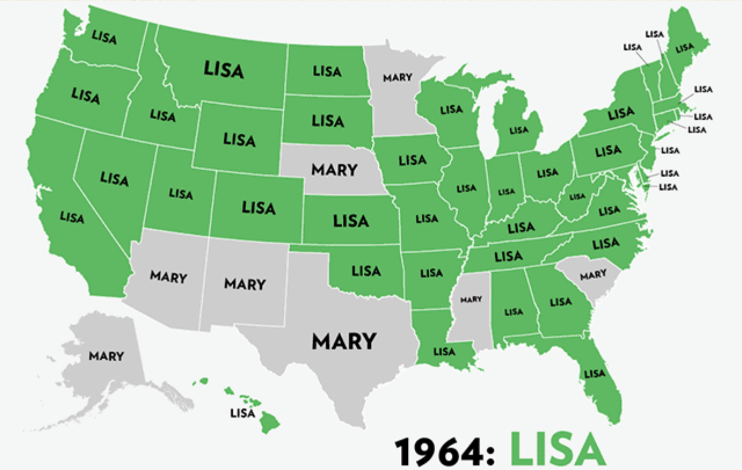

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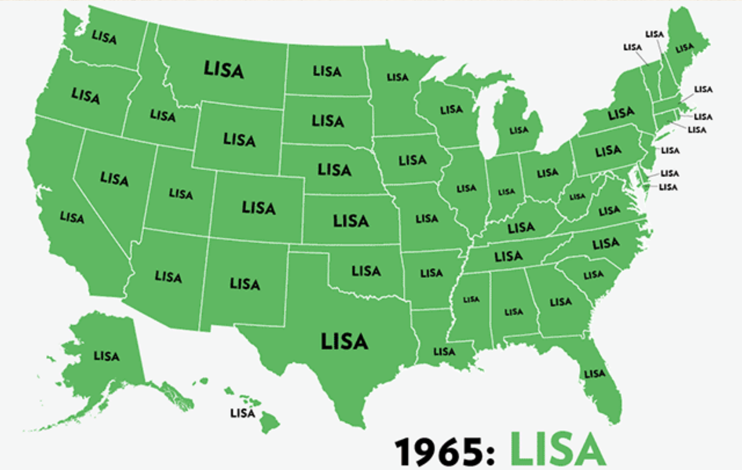

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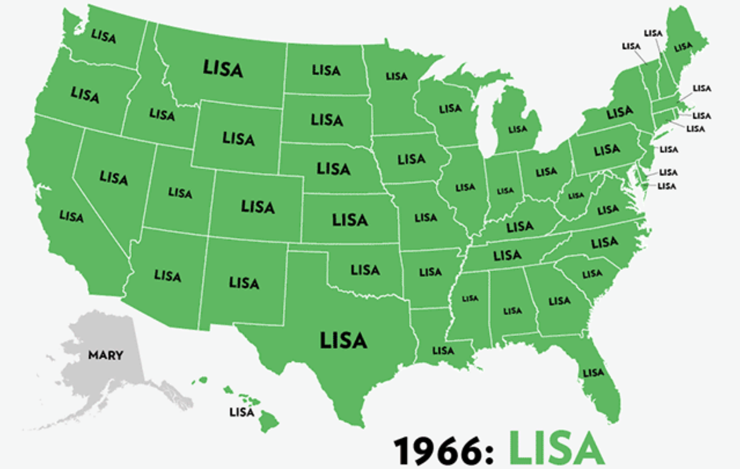

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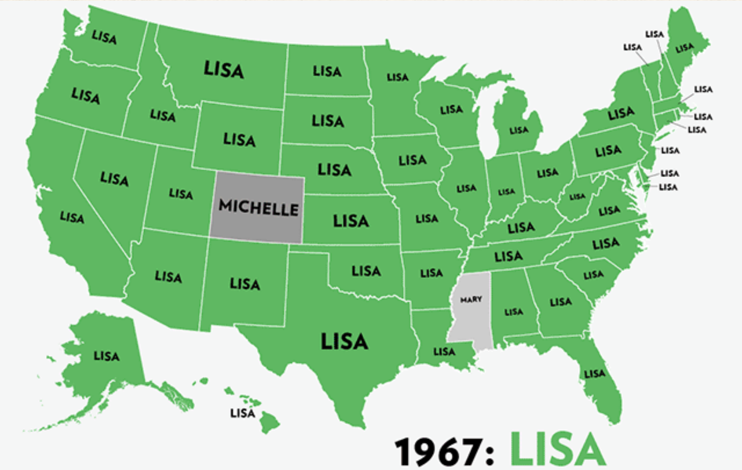

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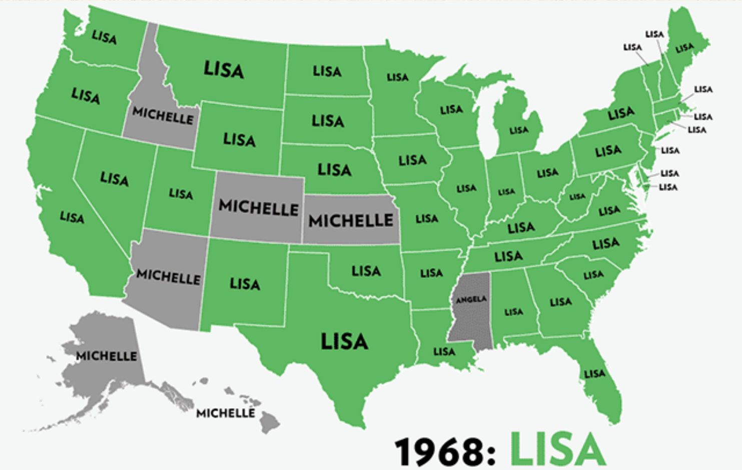

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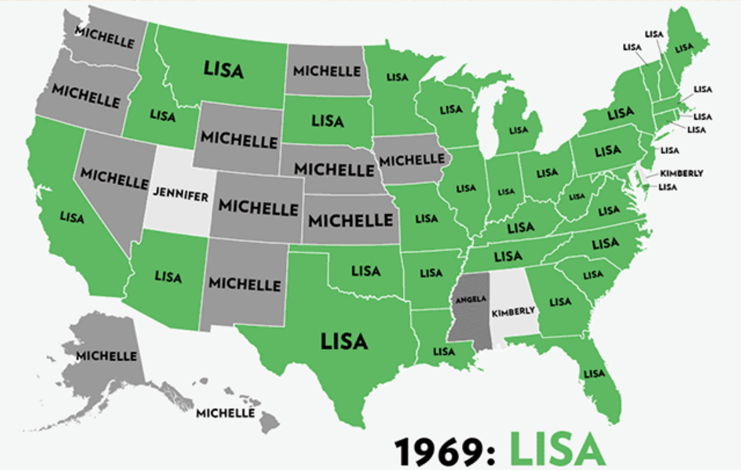

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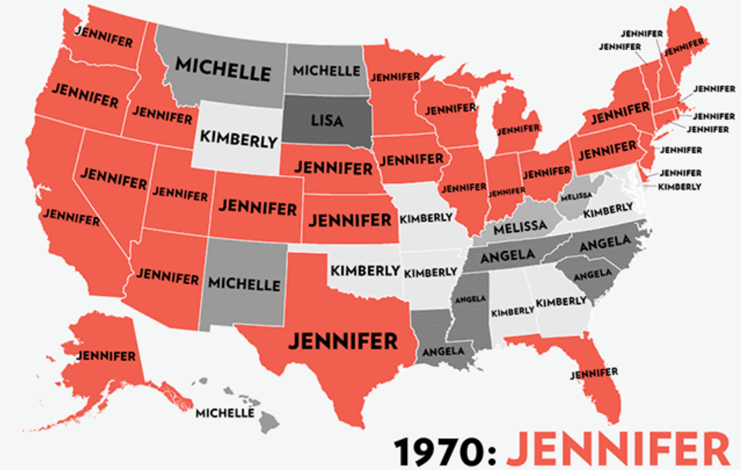

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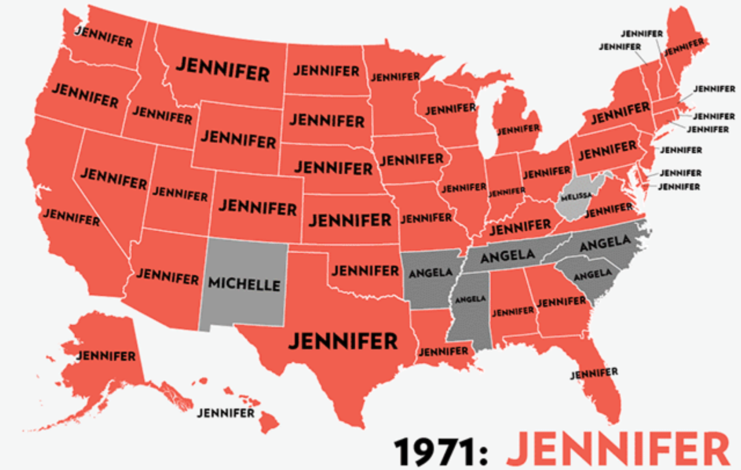

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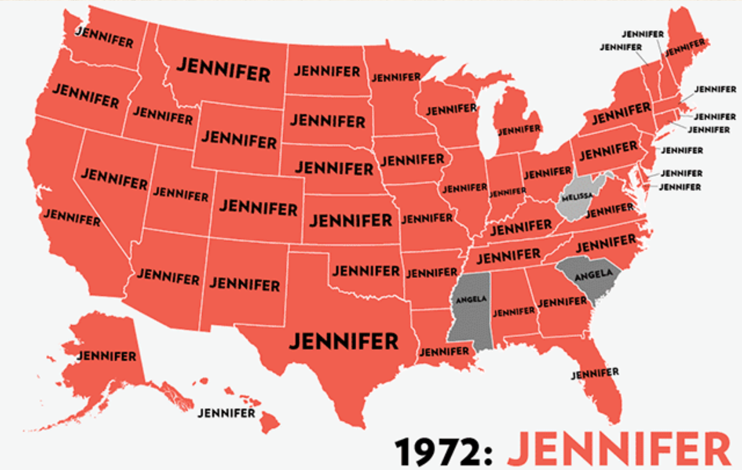

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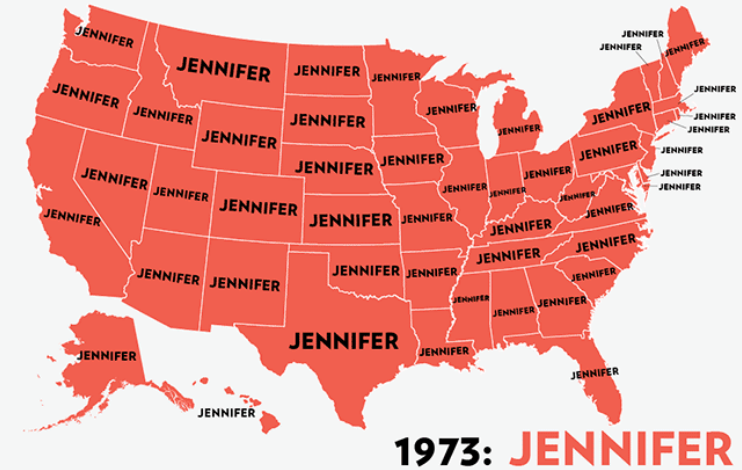

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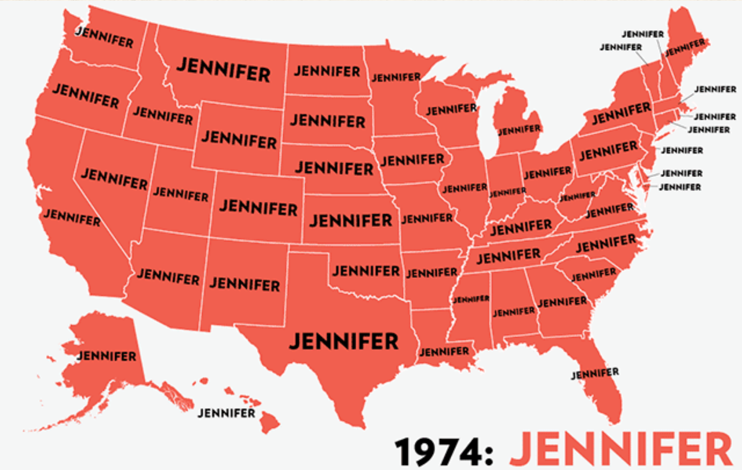

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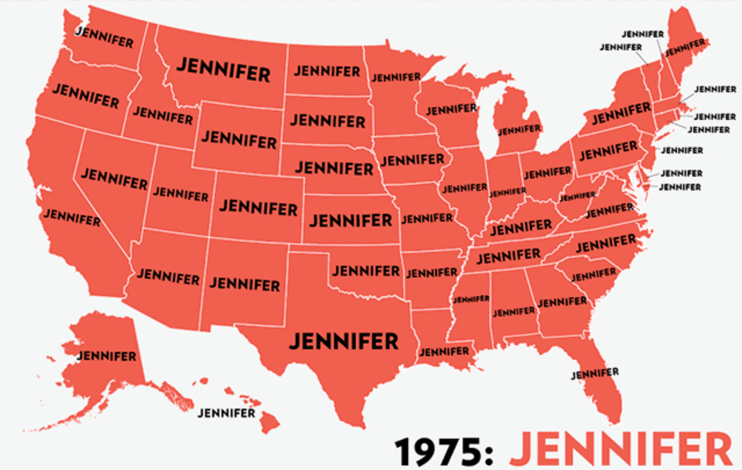

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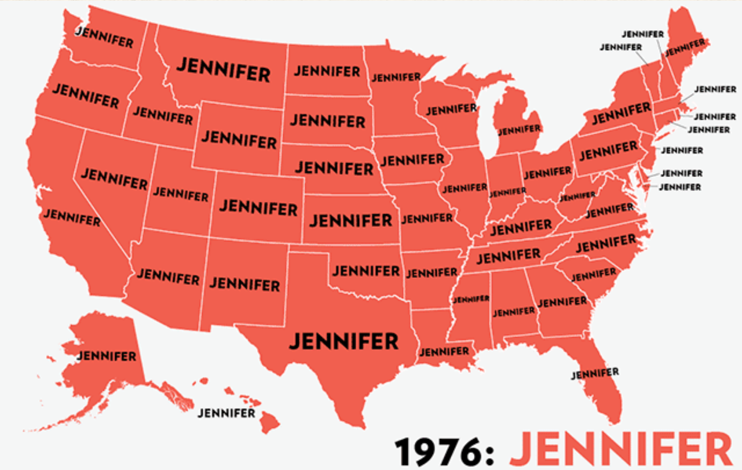

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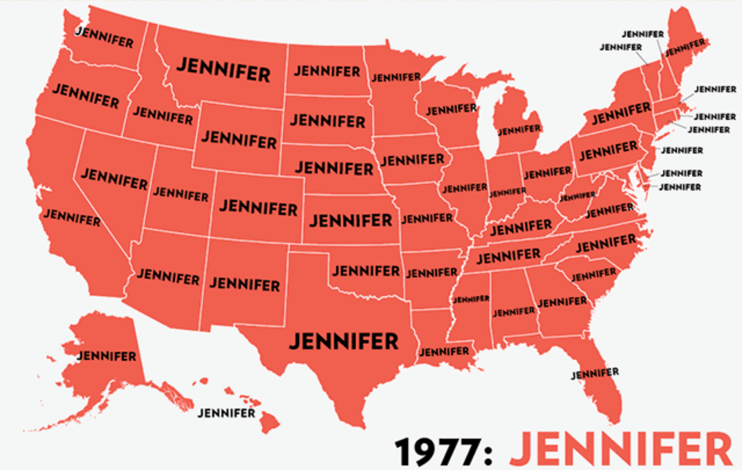

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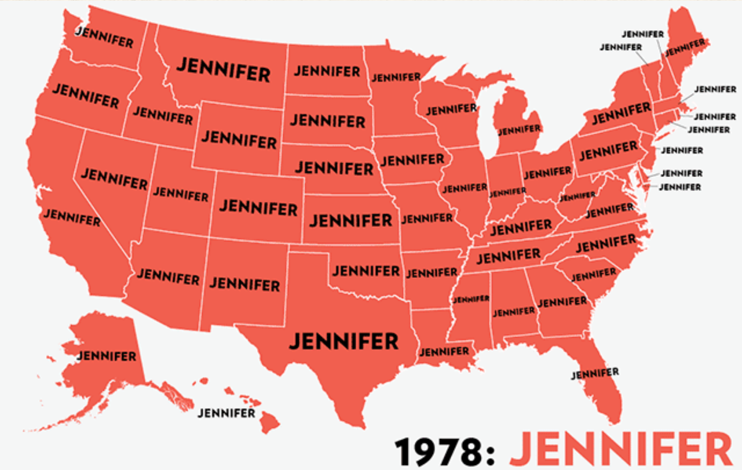

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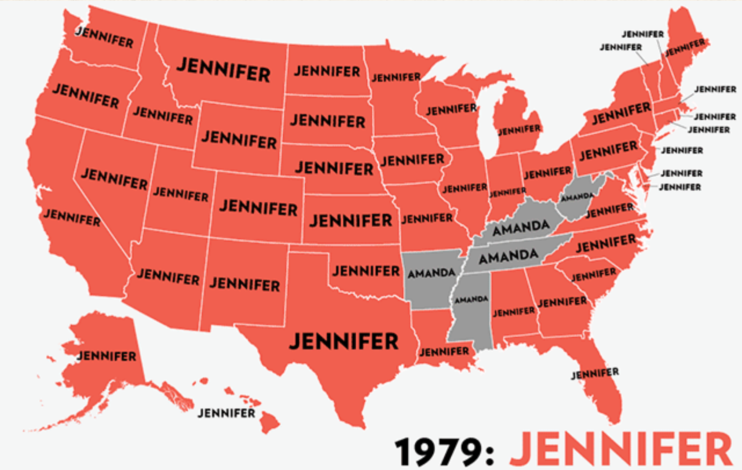

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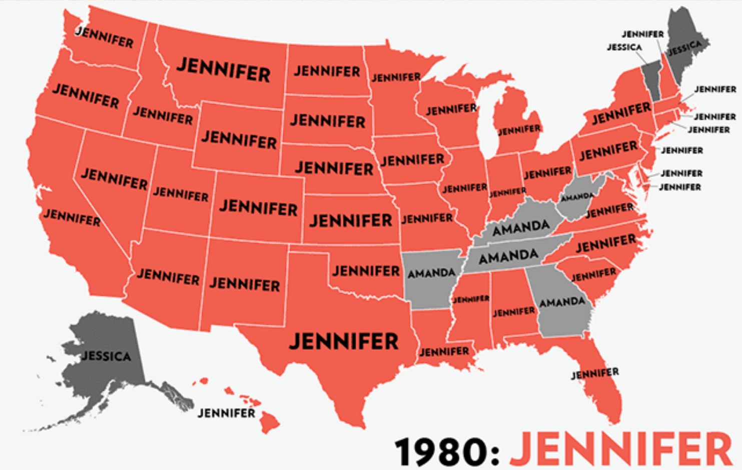

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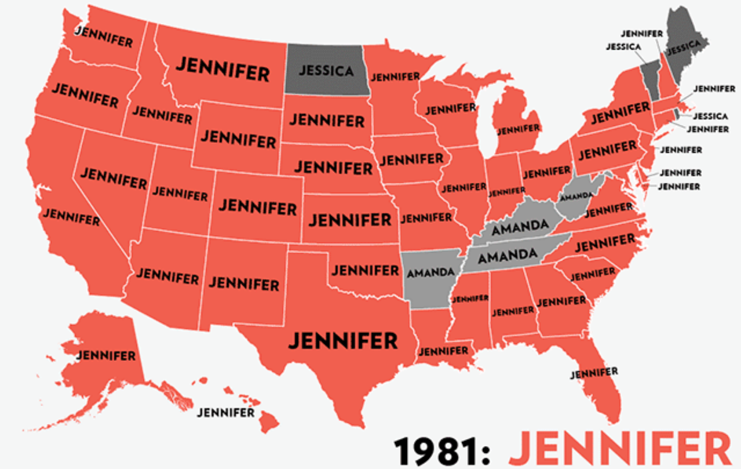

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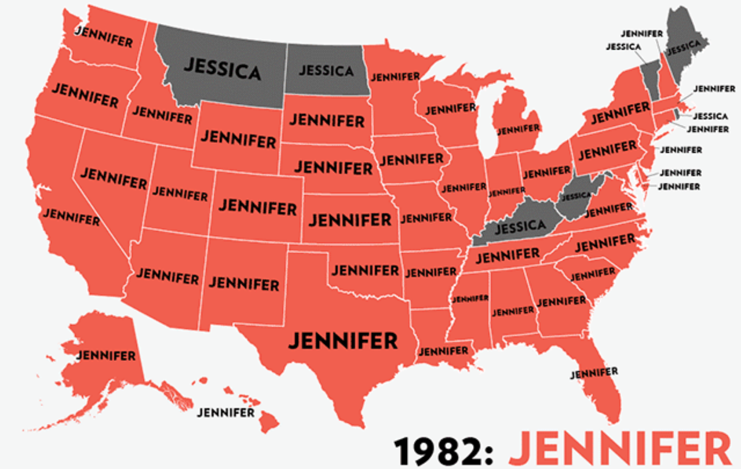

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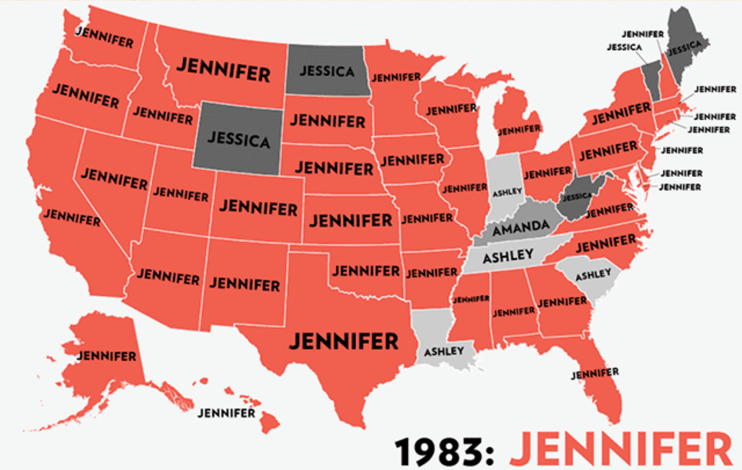

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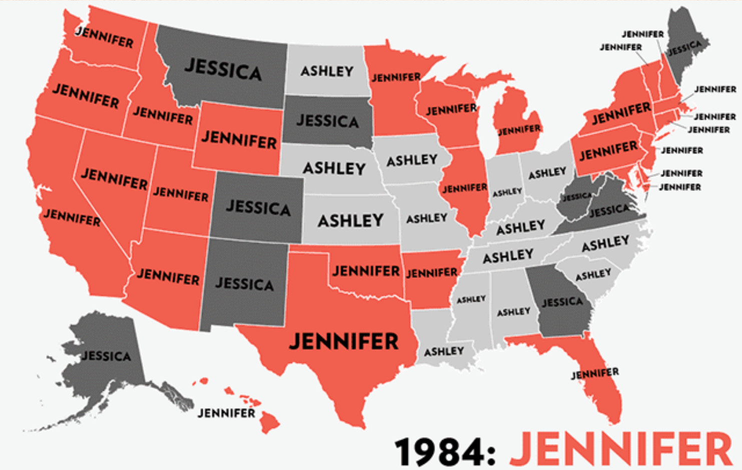

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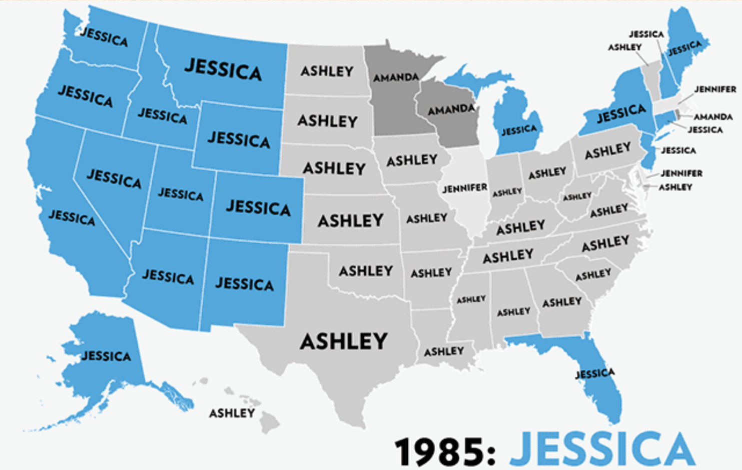

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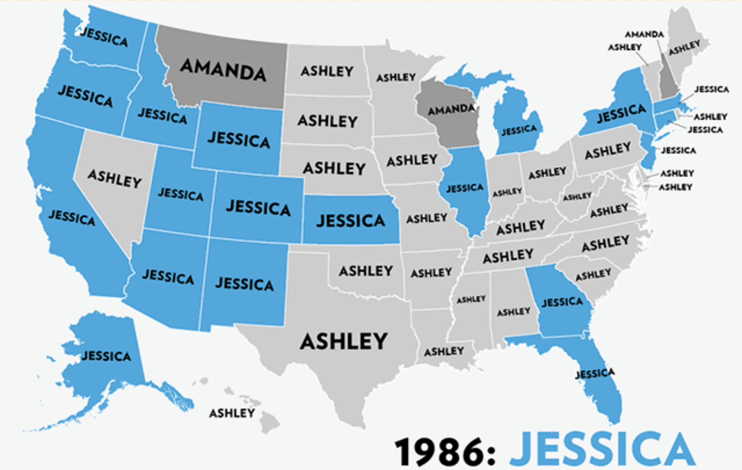

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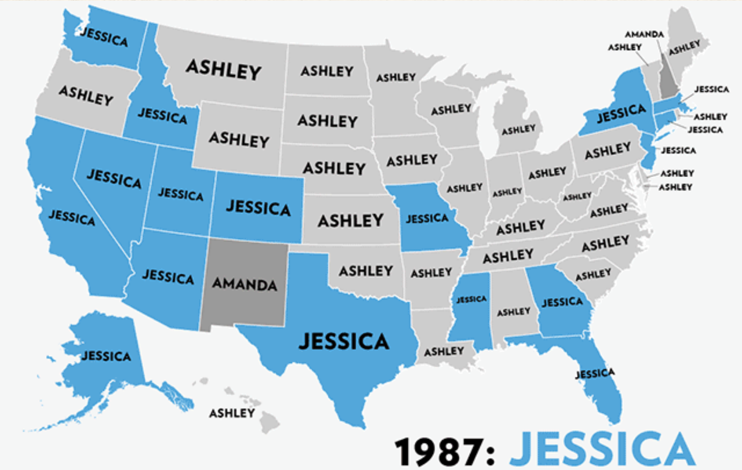

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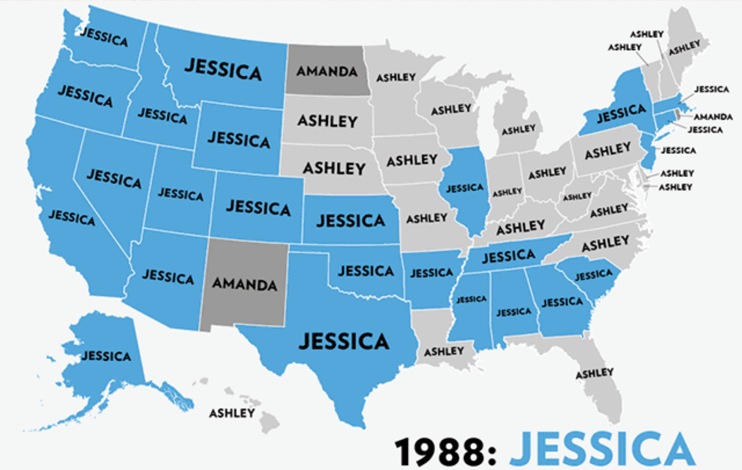

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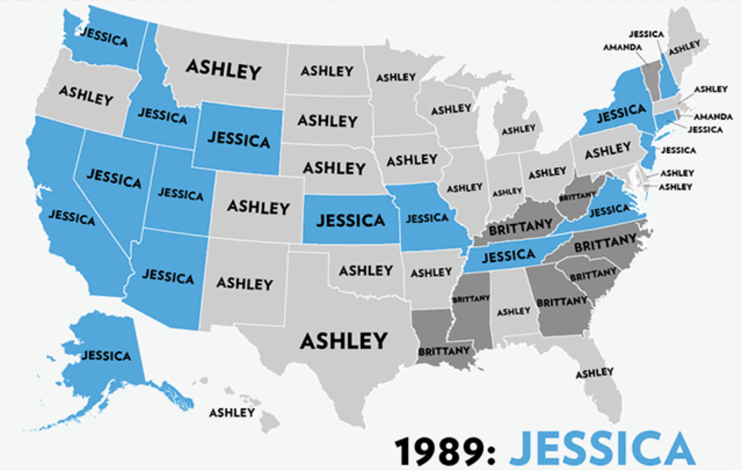

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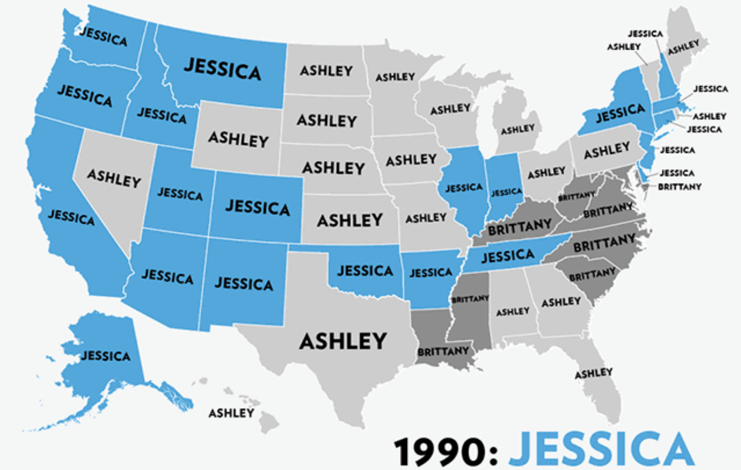

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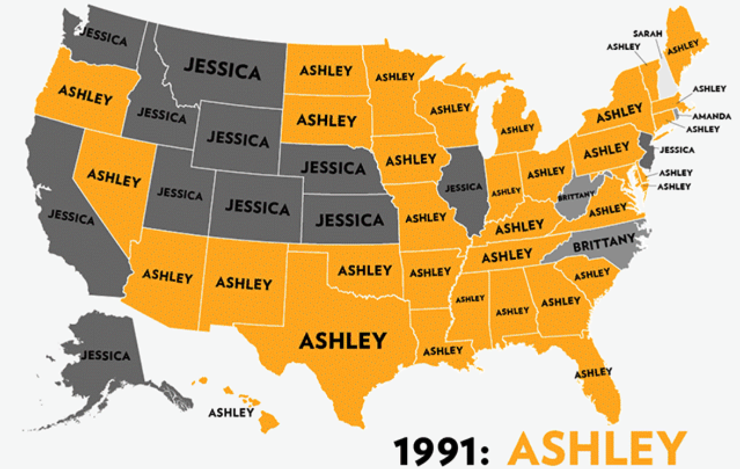

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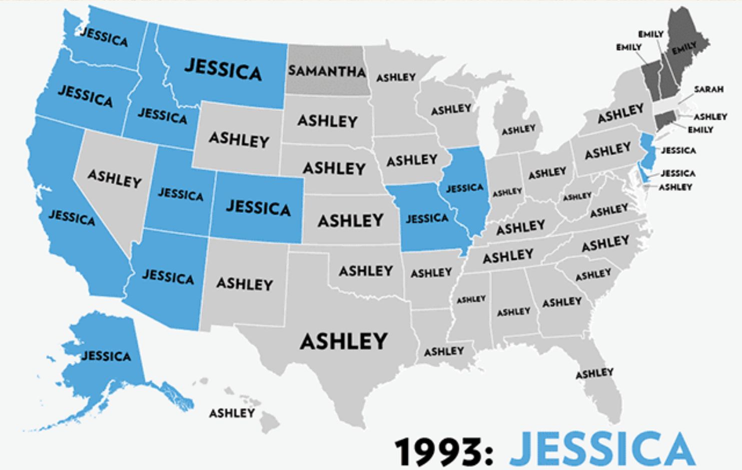

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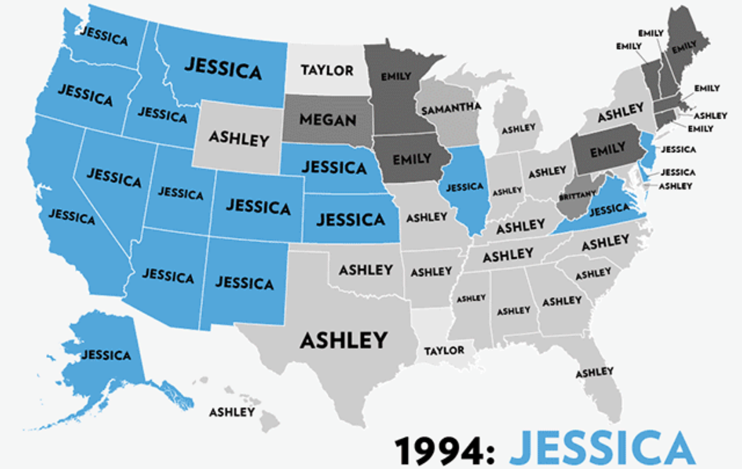

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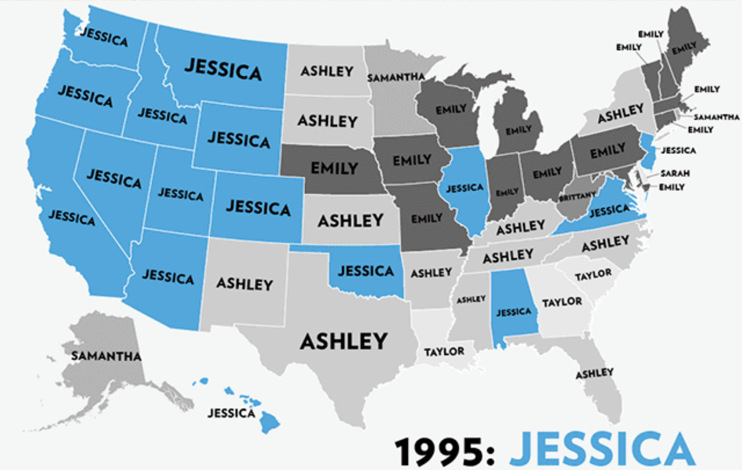

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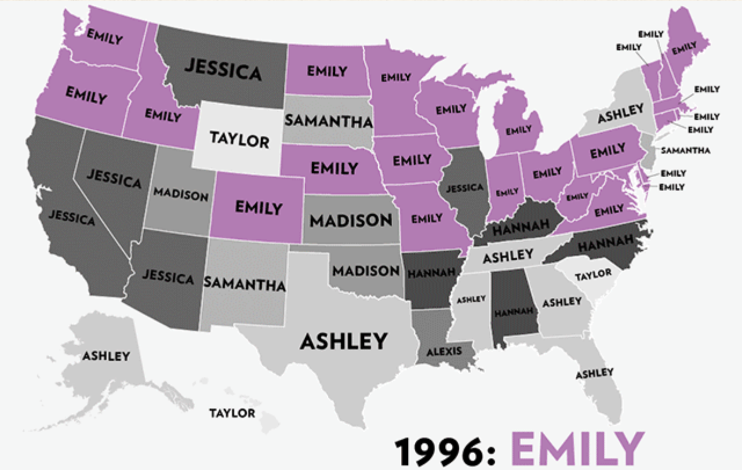

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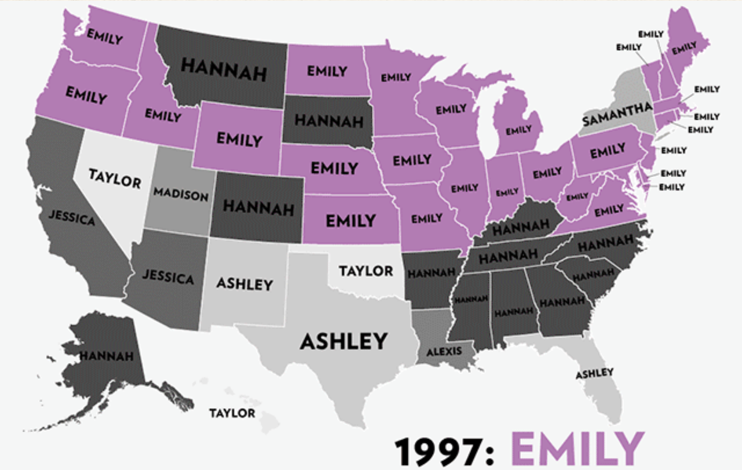

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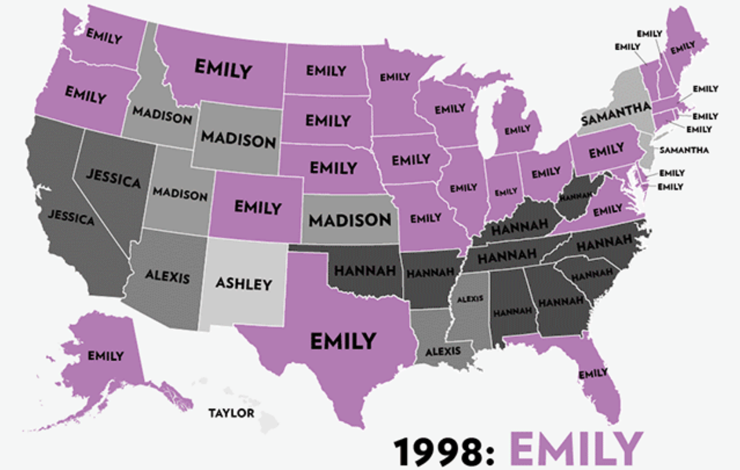

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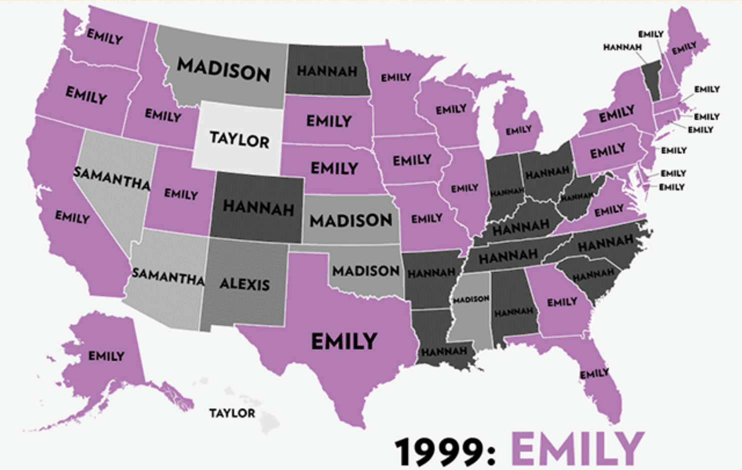

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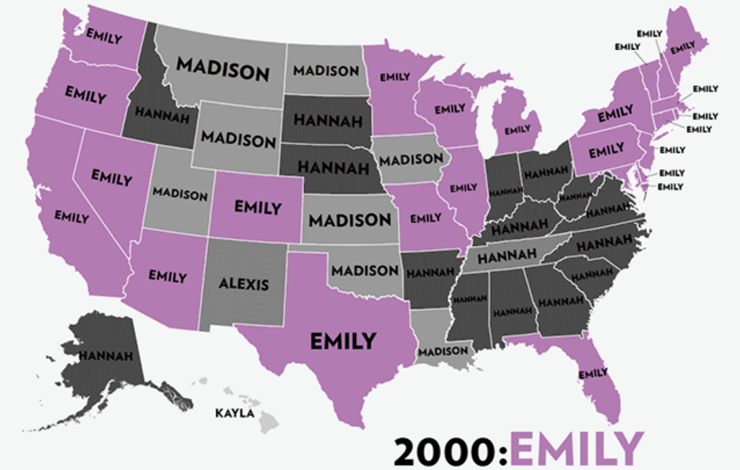

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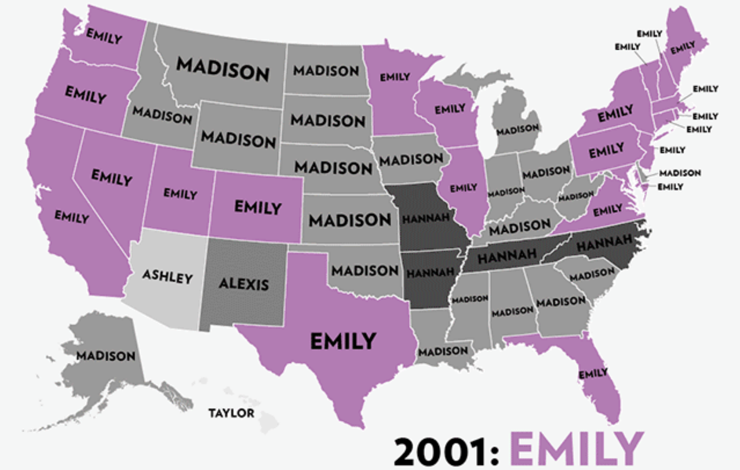

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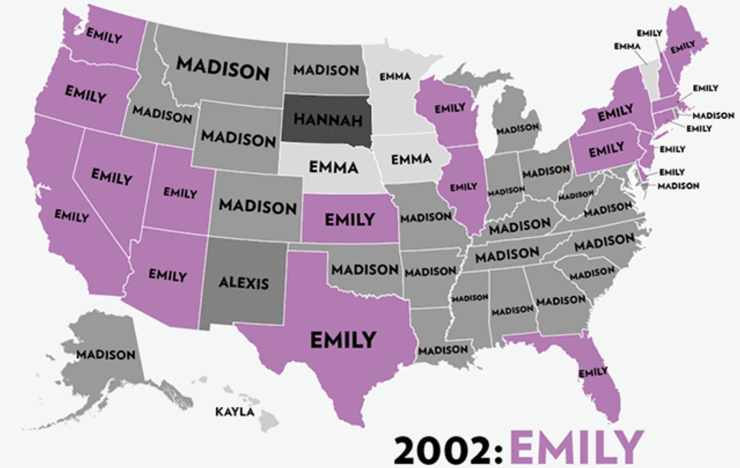

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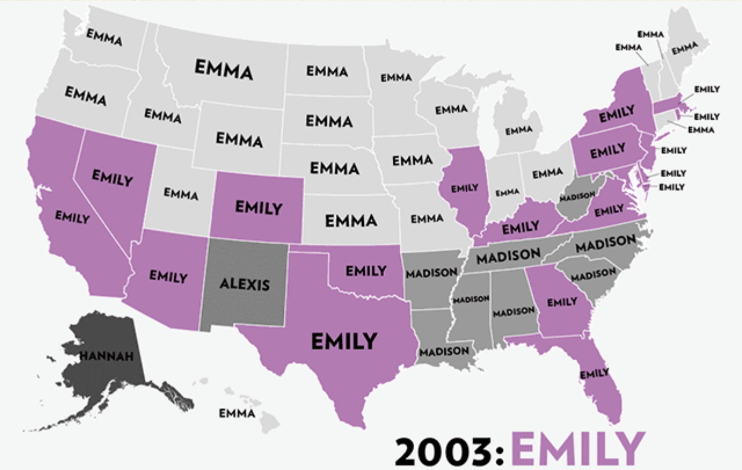

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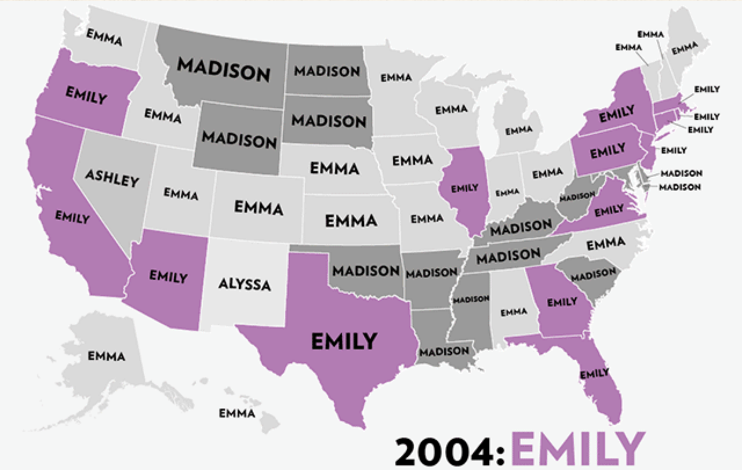

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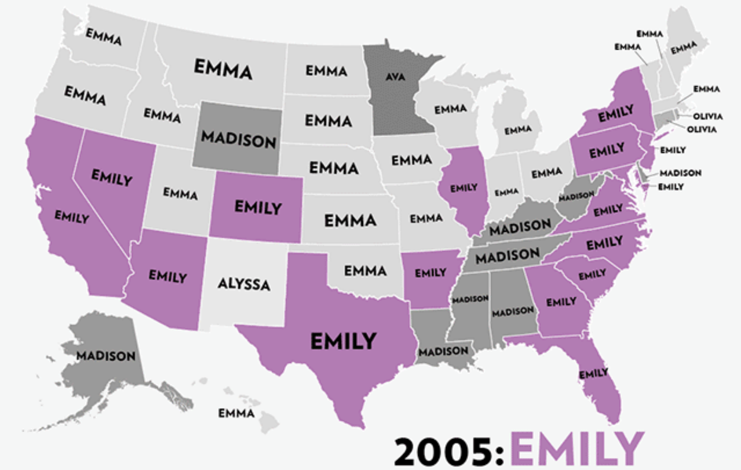

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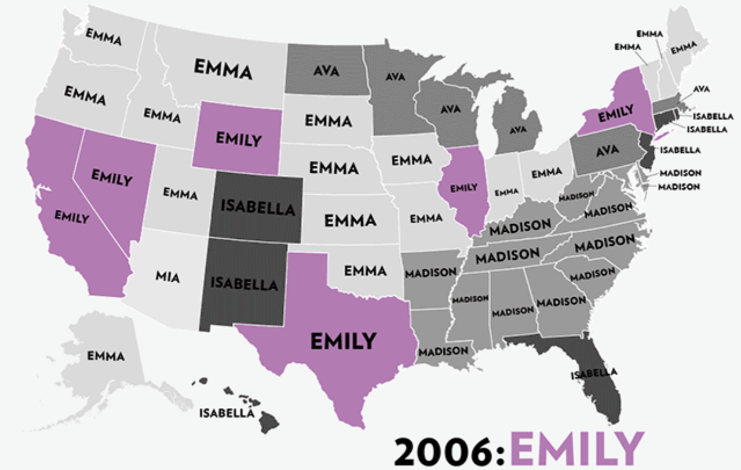

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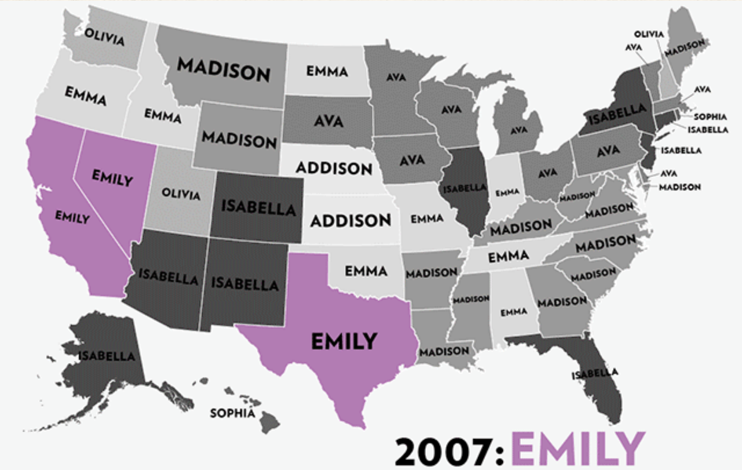

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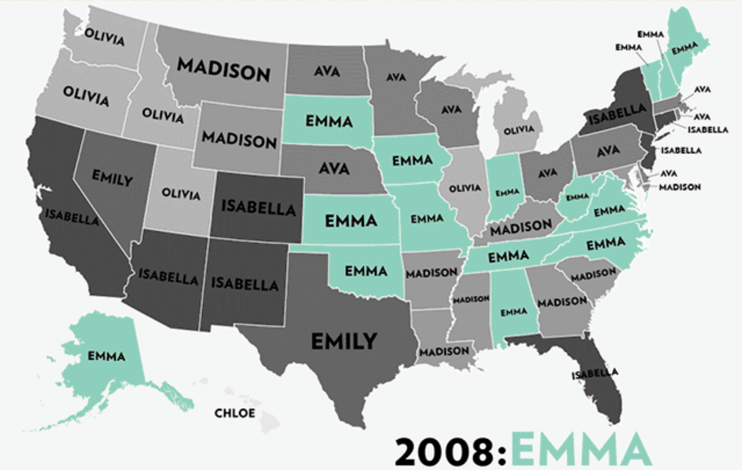

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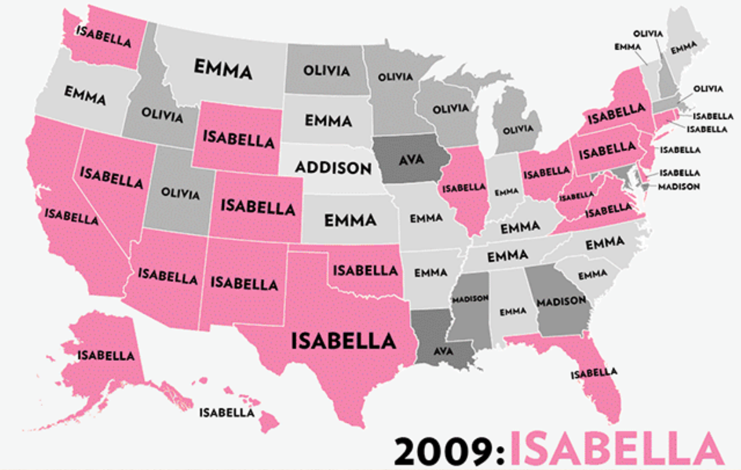

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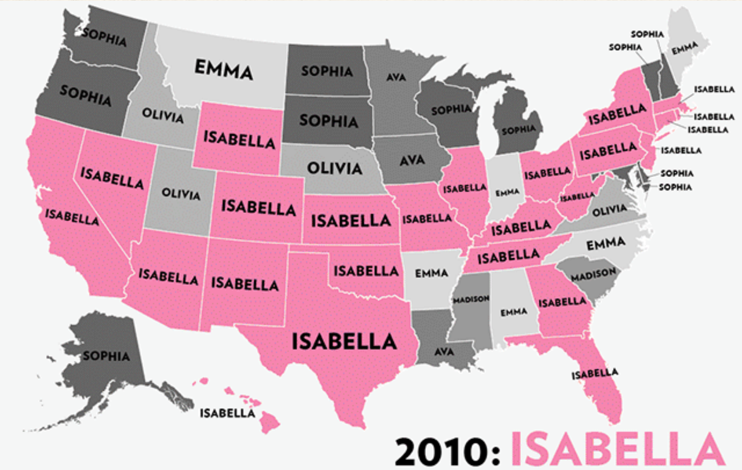

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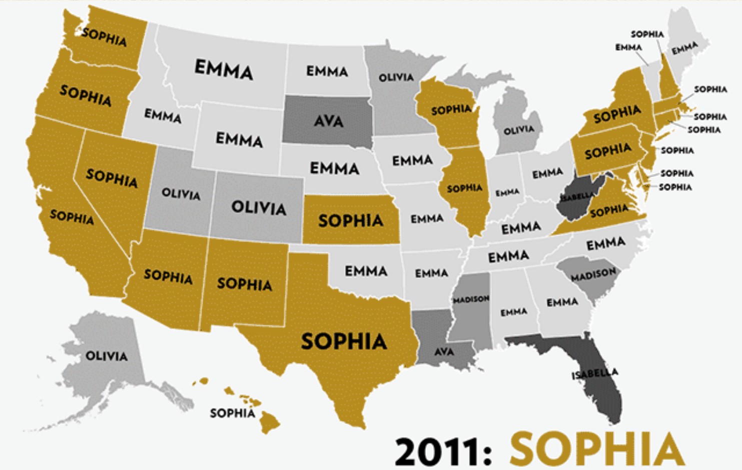
THE CHURCH OF QUARTEROLOGY



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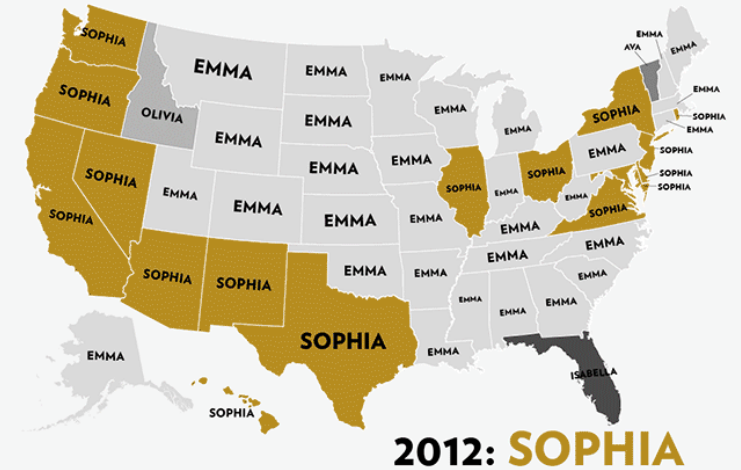

From the Atlantic


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE THRESHOLDS OF THE MIND
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
D-1
D
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
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THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
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THE OVERCOMING OF CATASTROPHE
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THE CHAOS OF FETTERED CONTAGION
THE END

Social Contagion

Some important models:

- Tipping models—Schelling (1971) [85, 86, 87]

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  - Social learning theory, Informational cascades,...

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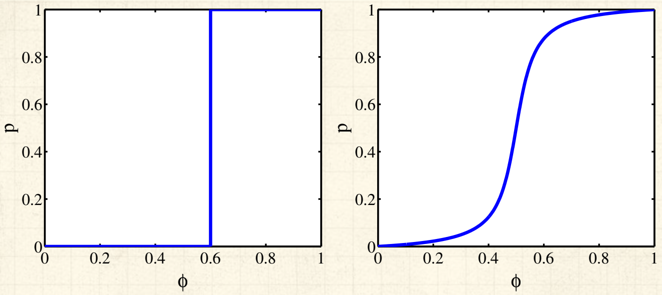
  - An individual's utility increases with the adoption level among peers and the population in general

Threshold models—response functions



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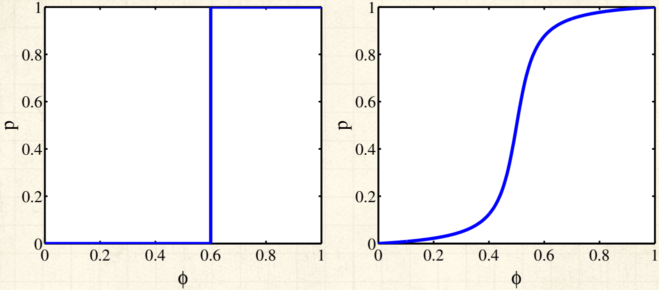

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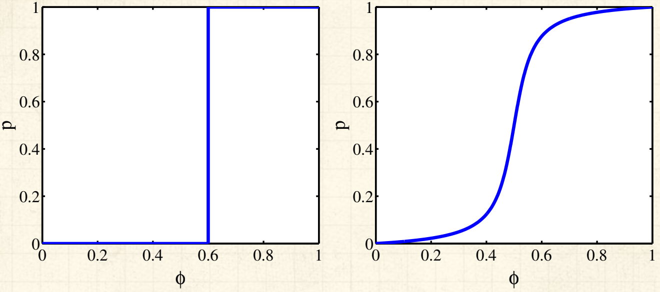

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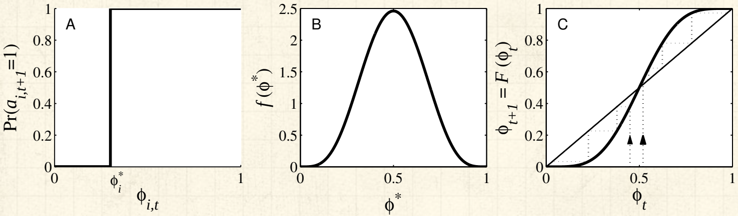
Action based on perceived behavior of others:



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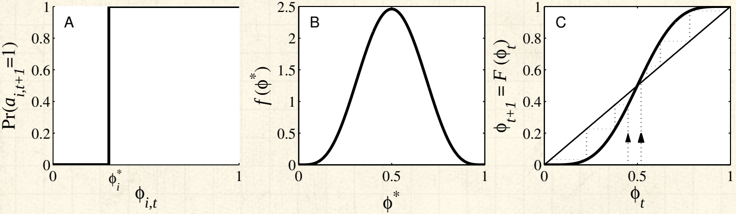

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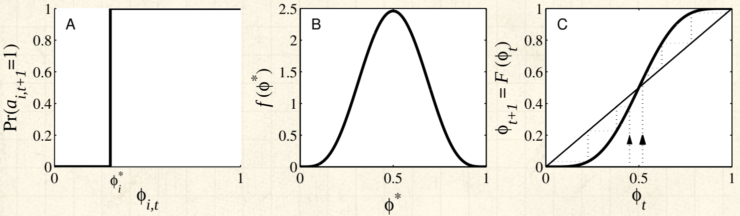
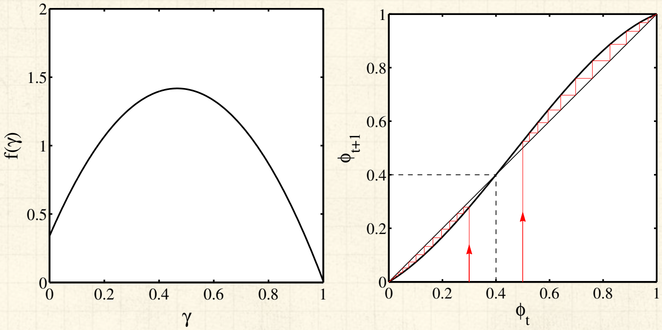


- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

Threshold models

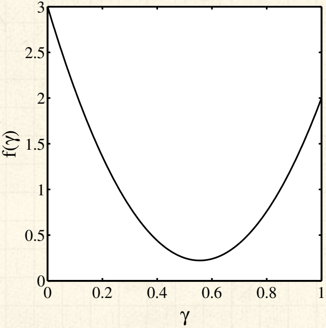
Another example of critical mass model:

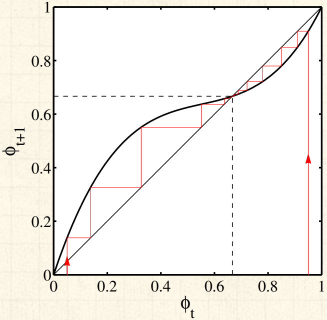


- Fragility of fixed point at $\phi = 0$.
- Critical slope = 1.

Threshold models

Example of single stable state model:





Threshold models—Nutshell

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4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves.

Many years after Granovetter and Soong's work:

- "A simple model of global cascades on random networks"
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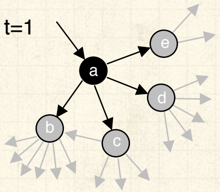
Also consider:

- "Seed size strongly affects cascades on random networks" [44]
  Gleeson and Cahalane, Phys. Rev. E, 2007.
- "Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks" [30] Dodds, Harris, and Payne, Phys. Rev. E, 2011
- "Influentials, Networks, and Public Opinion Formation" [108]
  Watts and Dodds, J. Cons. Res., 2007.

Threshold model on a network



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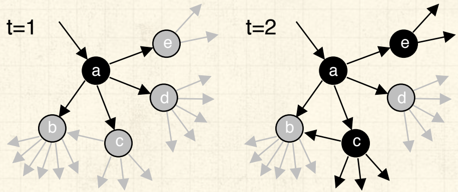

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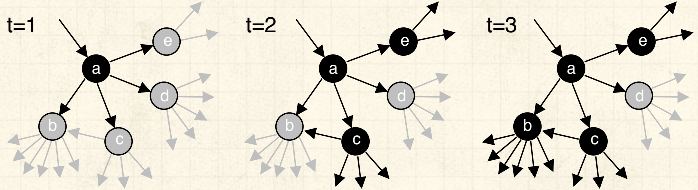

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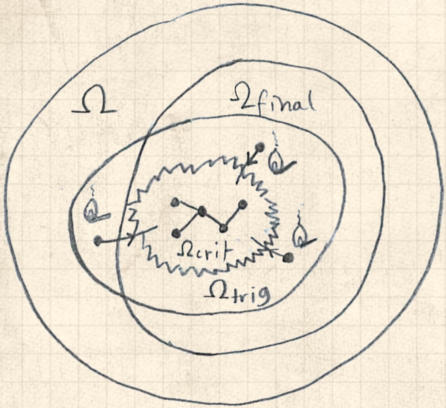
Example random network structure:



- $\Omega_{crit} = \Omega_{vuln}$ = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

$\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and $\Omega_{trig}, \Omega_{final} \subset \Omega$.

Cascade condition

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THE CHURCH
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$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

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$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

Cascade condition

So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty}(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle} > 1.$$

- $\beta_k =$ probability a degree k node is vulnerable.
- $P_k =$ probability a node has degree k.

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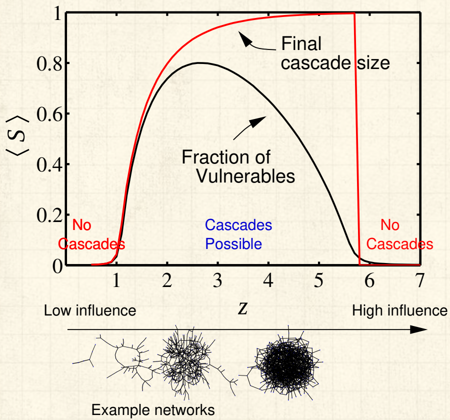
Cascades on random networks



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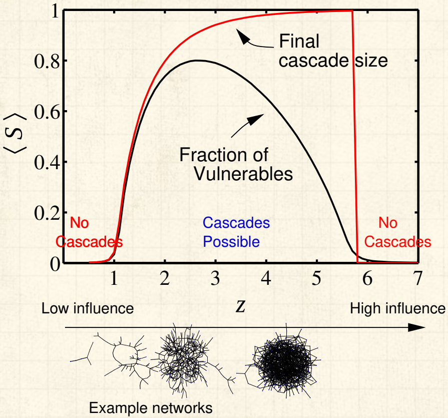

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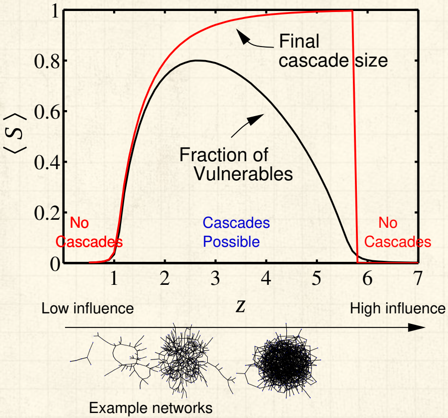

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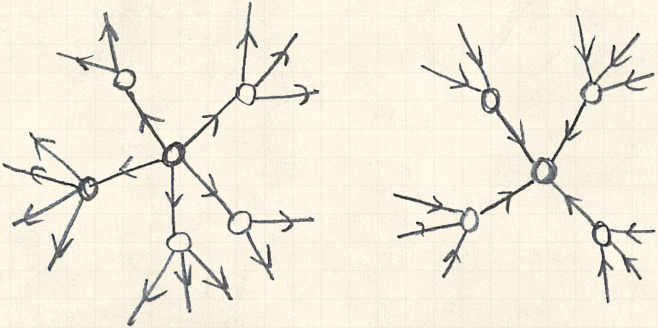
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.



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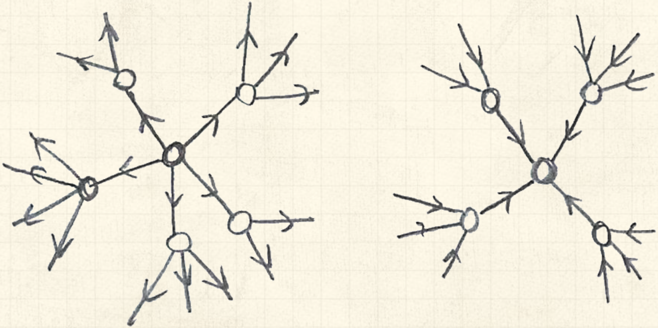
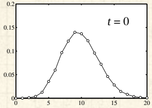
- Extent of spreading story is about contraction at a node.



Early adopters—degree distributions

$t = 0$



$P_{k,t}$ versus k

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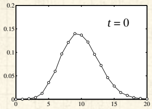

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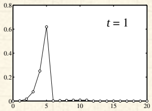

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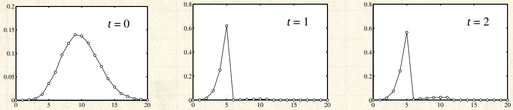
$t = 0$ $t = 1$ $t = 2$



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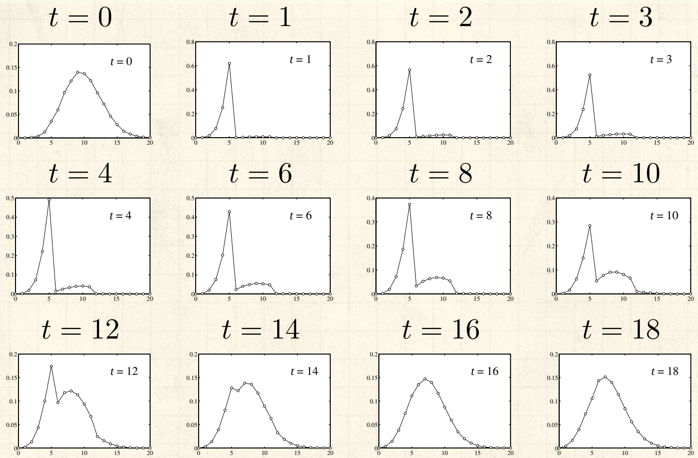

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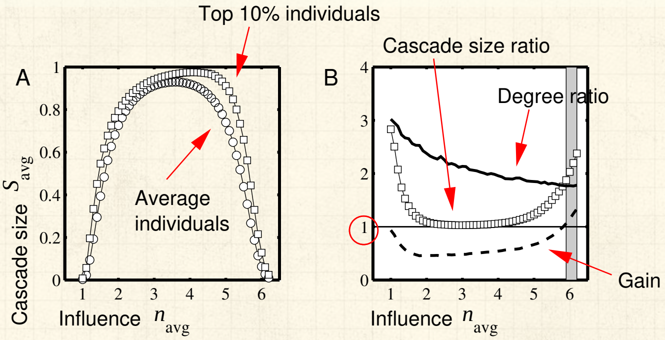
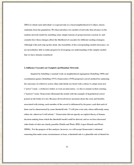
The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

Extensions



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Watts and Dodds,
The Oxford Handbook of Analytical Sociology, **63**, 475–497, 2009. [109]

- Assumption of sparse interactions is good

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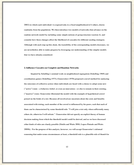

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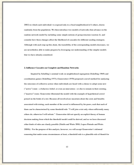

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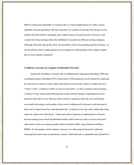

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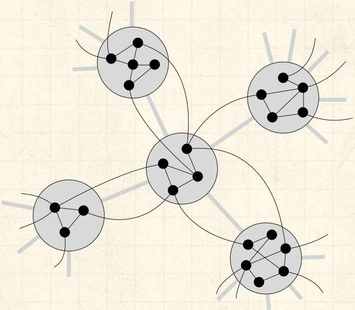
- Still, random networks don’t represent all networks
- Major element missing: group structure

Group structure—Ramified random networks



p = intergroup connection probability
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Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto e^{-\alpha d}$
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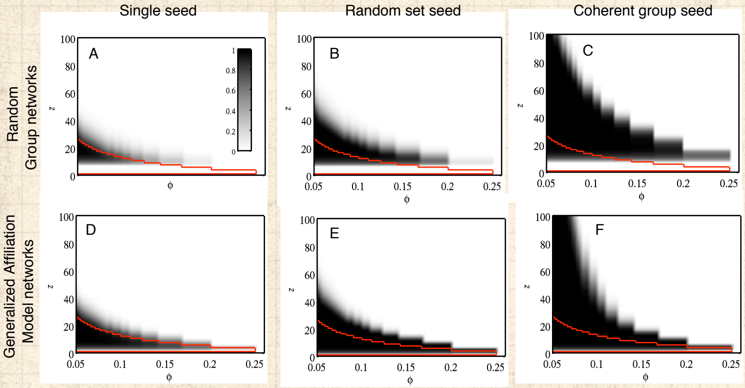
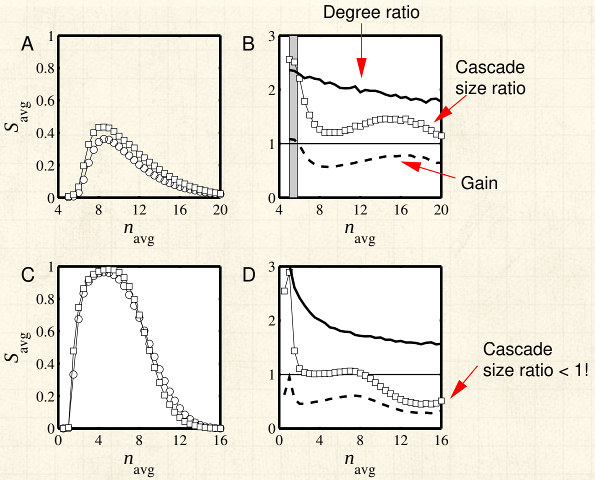
Cascade windows for group-based networks

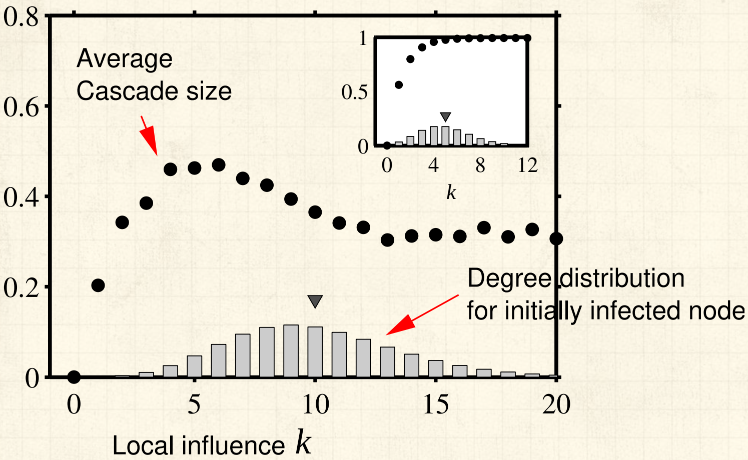


Multiplier effect for group-based networks:



- Multiplier almost always below 1.

Assortativity in group-based networks



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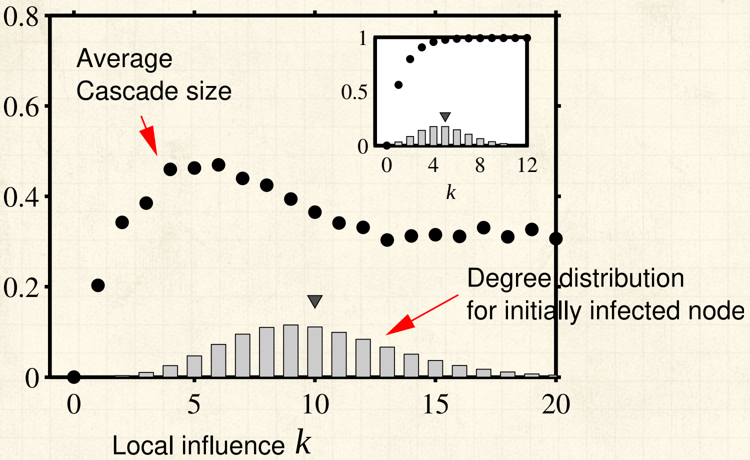

- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

Social contagion

“Without followers, evil cannot spread.” –Leonard Nimoy

Summary

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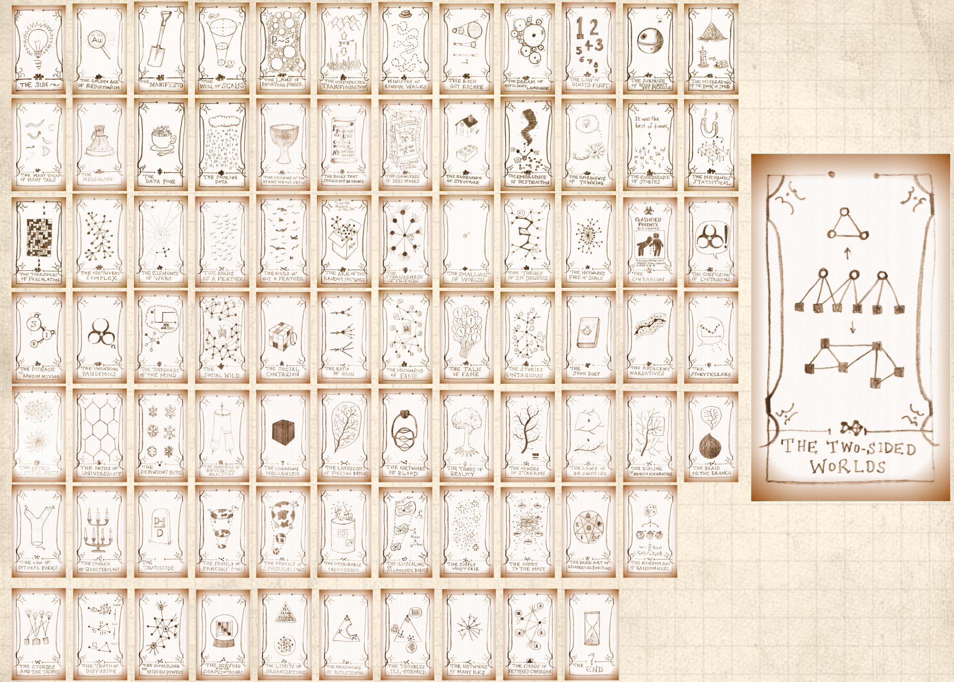
- Entities can be novel or designed to combine with others, e.g. block another one.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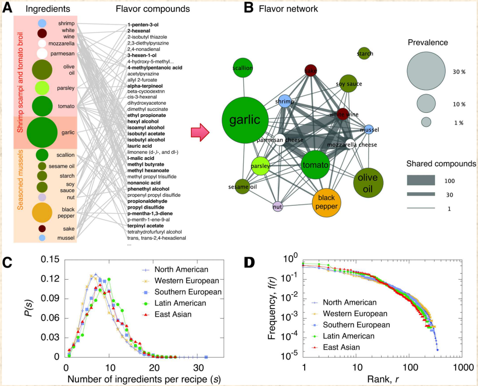

Figure 1 | Flavor network. (A) The ingredients contained in two recipes (left column), together with the flavor compounds that are known to be present in the ingredients (right column). Each flavor compound is linked to the ingredients that contain it, forming a bipartite network. Some compounds (shown in boldface) are shared by multiple ingredients. (B) If we project the ingredient-compound bipartite network into the ingredient space, we obtain the *flavor network*, whose nodes are ingredients, linked if they share at least one flavor compound. The thickness of links represents the number of flavor compounds two ingredients share and the size of each circle corresponds to the prevalence of the ingredients in recipes. (C) The distribution of recipe size, capturing the number of ingredients per recipe, across the five cuisines explored in our study. (D) The frequency-rank plot of ingredients across the five cuisines show an approximately invariant distribution across cuisines.

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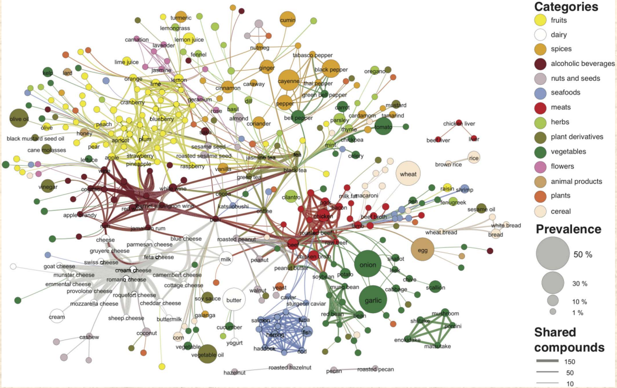
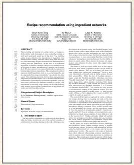


Figure 2 | The backbone of the flavor network. Each node denotes an ingredient, the node color indicates food category, and node size reflects the ingredient prevalence in recipes. Two ingredients are connected if they share a significant number of flavor compounds, link thickness representing the number of shared compounds between the two ingredients. Adjacent links are bundled to reduce the clutter. Note that the map shows only the statistically significant links, as identified by the algorithm of Refs.[28,29] for *p*-value 0.04. A drawing of the full network is too dense to be informative. We use, however, the full network in our subsequent measurements.



"Recipe recommendation using ingredient networks"

Teng, Lin, and Adamic,
Proceedings of the 3rd Annual ACM Web Science Conference, **1**, 298–307, 2012. [97]

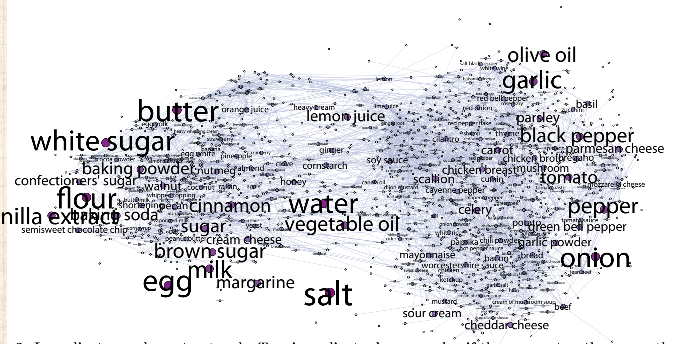
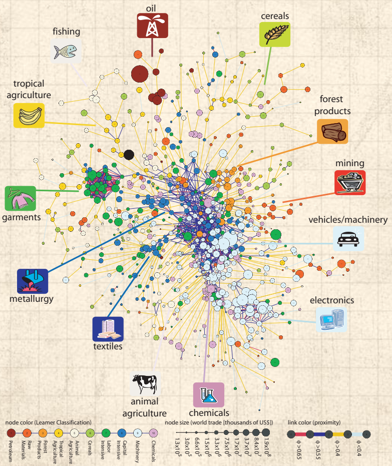


Figure 2: Ingredient complement network. Two ingredients share an edge if they occur together more than would be expected by chance and if their pointwise mutual information exceeds a threshold.

"The Product Space Conditions the Development of Nations"

Hidalgo et al.,
Science, **317**, 482–487, 2007. [52]



Networks and creativity:

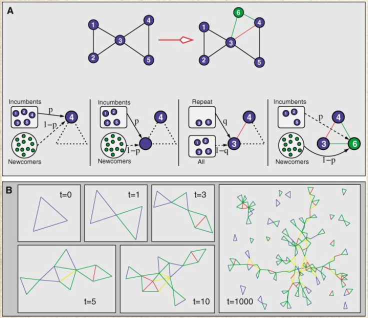
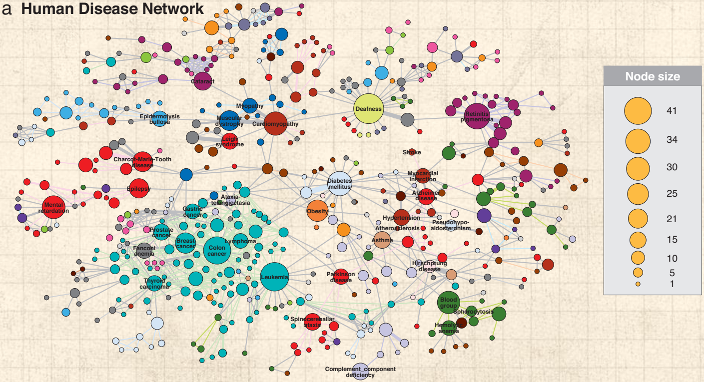


Fig. 2. Modeling the emergence of collaboration networks in creative enterprises. **(A)** Creation of a team with $m = 3$ agents. Consider, at time zero, a collaboration network comprising five agents, all incumbents (blue circles). Along with the incumbents, there is a large pool of newcomers (green circles) available to participate in new teams. Each agent in a team has a probability p of being drawn from the pool of incumbents and a probability $1 - p$ of being drawn from the pool of newcomers. For the second and subsequent agents selected from the incumbents' pool: (i) with probability q, the new agent is randomly selected from among the set of collaborators of a randomly selected incumbent already in the team; (ii) otherwise, he or she is selected at random among all incumbents in the network. For concreteness, let us assume that incumbent 4 is selected as the first agent in the new team (leftmost box). Let us also assume that the second agent is an incumbent, too (center-left box). In this example, the second agent is a past collaborator of agent 4, specifically agent 3 (center-right box). Lastly, the third agent is selected from the pool of newcomers; this agent becomes incumbent 6 (rightmost box). In these boxes and in the following panels and figures, blue lines indicate newcomer-newcomer collaborations, green lines indicate newcomer-incumbent collaborations, yellow lines indicate new incumbent-incumbent collaborations, and red lines indicate repeat collaborations. **(B)** Time evolution of the network of collaborations according to the model for $p = 0.5$, $q = 0.5$, and $m = 3$.

- Guimerà et al., Science 2005: [48] "Team Assembly Mechanisms Determine Collaboration Network Structure and Team Performance"
- Broadway musical industry
- Scientific collaboration in Social Psychology, Economics, Ecology, and Astronomy.

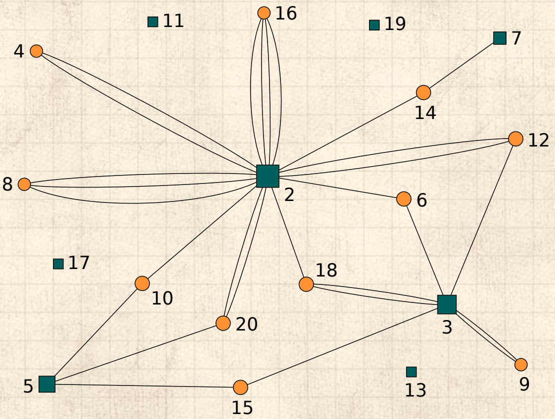
“The human disease network”

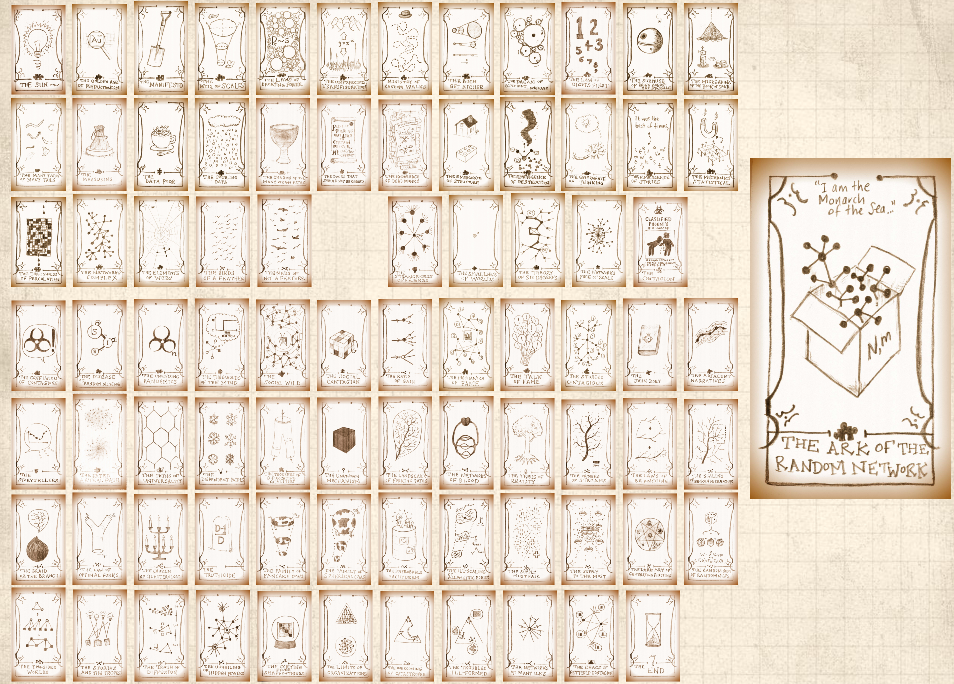
Goh et al.,
Proc. Natl. Acad. Sci., **104**, 8685–8690, 2007. [46]



“The complex architecture of primes and natural numbers”

García-Pérez, Serrano, and Boguñá, https://arxiv.org/abs/1402.3612, 2014. [39]

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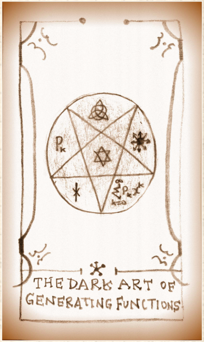
THE DARK ART OF GENERATING FUNCTIONS

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- Related to Fourier, Laplace, Mellin, ...

Simple examples:

Rolling dice and flipping coins:

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- We'll come back to these simple examples as we derive various delicious properties of generating functions.

Useful pieces for probability distributions:

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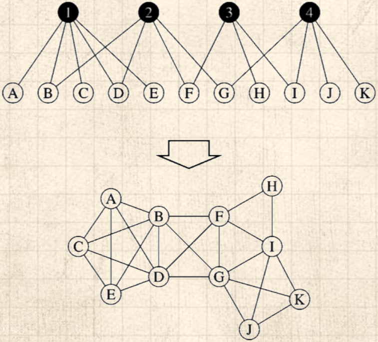
- kth element of sequence (general):

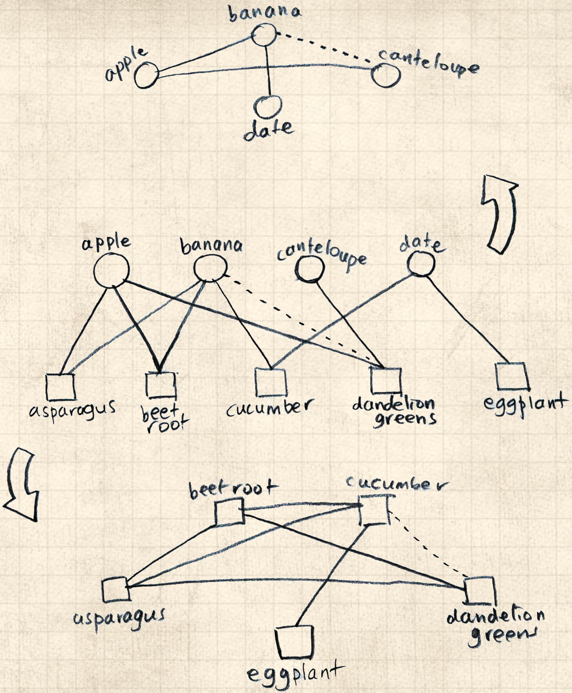
$$P_k = \frac{1}{k!} \frac{\mathrm{d}^k}{\mathrm{d}x^k} F(x) \bigg|_{x=0}$$

Random bipartite networks:

We'll follow this rather well cited paper:

"Random graphs with arbitrary degree distributions and their applications"
Newman, Strogatz, and Watts,
Phys. Rev. E, **64**, 026118, 2001. [80]




banana
apple
canteloupe
date
apple
banana
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date
asparagus
beet root
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eggplant
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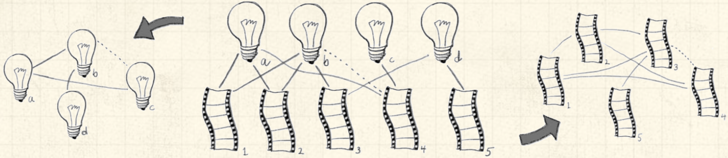
THE CHURCH OF QUARTEROLOGY

Example of a bipartite affiliation network and the induced networks:



- Center: A small story-trope bipartite graph. [28]
- Induced trope network and the induced story network are on the left and right.
- The dashed edge in the bipartite affiliation network indicates an edge added to the system, resulting in the dashed edges being added to the two induced networks.


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
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THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
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THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
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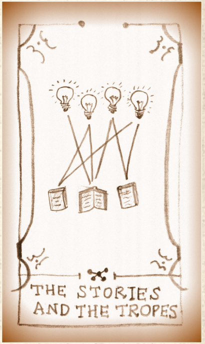
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END


THE STORIES AND THE TROPES

Basic story:

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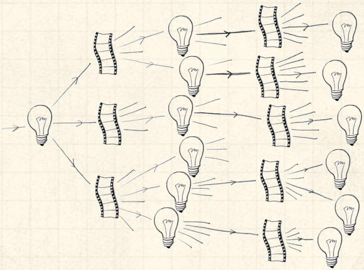
  - $\langle k \rangle_{\text{🎞}}$ = average number of tropes per story.
  - $\langle k \rangle_{\text{💡}}$ = average number of stories containing a given trope.
- Must have balance: $N_{\text{🎞}} \cdot \langle k \rangle_{\text{🎞}} = m_{\text{🎞},\text{💡}} = N_{\text{💡}} \cdot \langle k \rangle_{\text{💡}}$.

Spreading through bipartite networks:



- View as bouncing back and forth between the two connected populations. [28]
- Actual spread may be within only one population (ideas between between people) or through both (failures in physical and communication networks).
- The gain ratio for simple contagion on a bipartite random network = product of two gain ratios.

Usual helpers for understanding network's structure:

- Randomly select an edge connecting a 🎞 to a 💡.
- Probability the 🎞 contains k other tropes:

$$R_k^{(\text{🎞})} = \frac{(k+1)P_{k+1}^{(\text{🎞})}}{\sum_{j=0}^{N_{\text{🎞}}}(j+1)P_{j+1}^{(\text{🎞})}} = \frac{(k+1)P_{k+1}^{(\text{🎞})}}{\langle k \rangle_{\text{🎞}}}.$$

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- Probability the 💡 is in k other stories:

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- Unrelated goal: be 10% happier/weep less.

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- $happiness++;

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$$\langle k \rangle_{R,\text{🎞},\text{ind}} = \frac{\langle k(k-1) \rangle_{\text{🎞}}}{\langle k \rangle_{\text{🎞}}} \frac{\langle k(k-1) \rangle_{\text{💡}}}{\langle k \rangle_{\text{💡}}}$$

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- See this as the product of two gain ratios.

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- We can mess with this condition to make it mathematically pleasant and pleasantly inscrutable:

$$\sum_{k=0}^{\infty} \sum_{k'=0}^{\infty} kk'(kk' - k - k') P_k^{(\text{🎞})} P_{k'}^{(\text{💡})} = 0.$$

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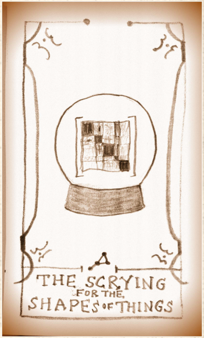
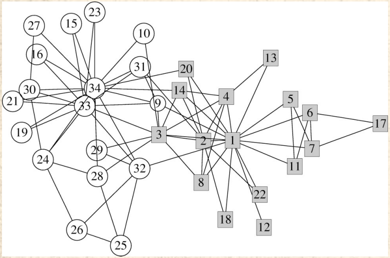
THE CHAOS OF FETTERED CONTAGION
THE END


THE SCRYING FOR THE SHAPES OF THINGS

Structure detection



▲ Zachary's karate club [119, 79]

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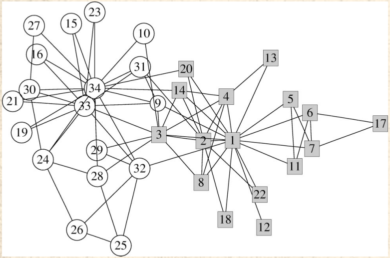

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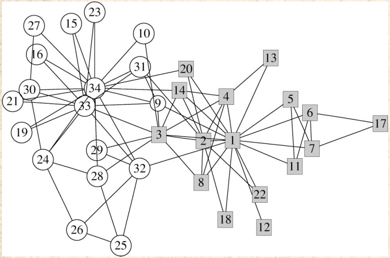

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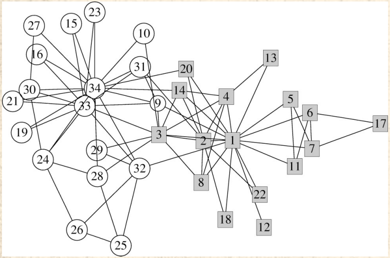

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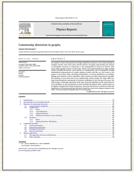
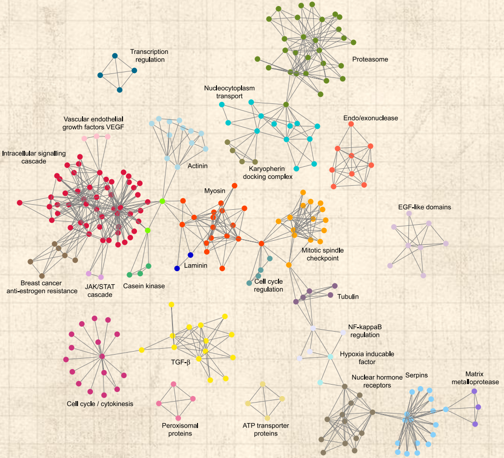
- Much focus on hierarchies (pyramids) ......



“Community detection in graphs”

Santo Fortunato,
Physics Reports, **486**, 75–174, 2010. [38]



Hierarchy by division

Top down:

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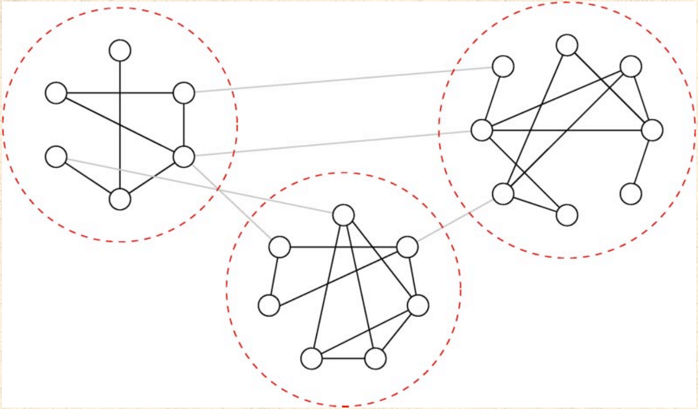

- Idea: Edges that connect communities have higher betweenness than edges within communities.

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One class of structure-detection algorithms:

1. Compute edge betweenness for whole network.

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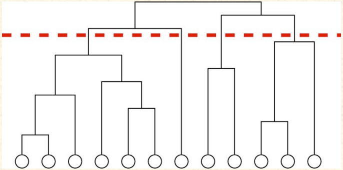

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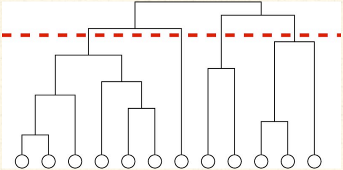
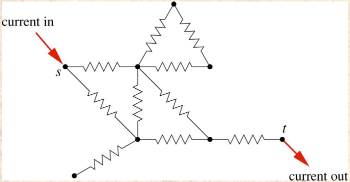


Red line indicates appearance of four (4) components at a certain level.

Betweenness for electrons:



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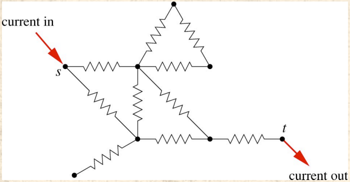

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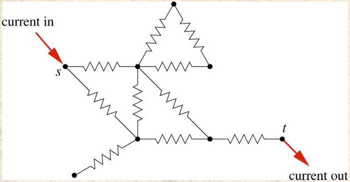

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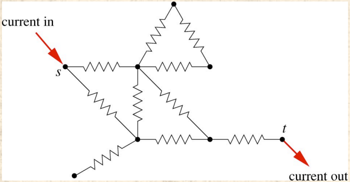

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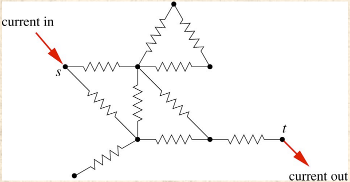

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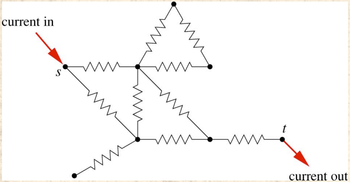

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- In fact, $\mathcal{N}(\mathbf{K} - \mathbf{A}) = \{c\vec{1}, c \in R\}$ since $(\mathbf{K} - \mathbf{A})\vec{1} = \vec{0}$.

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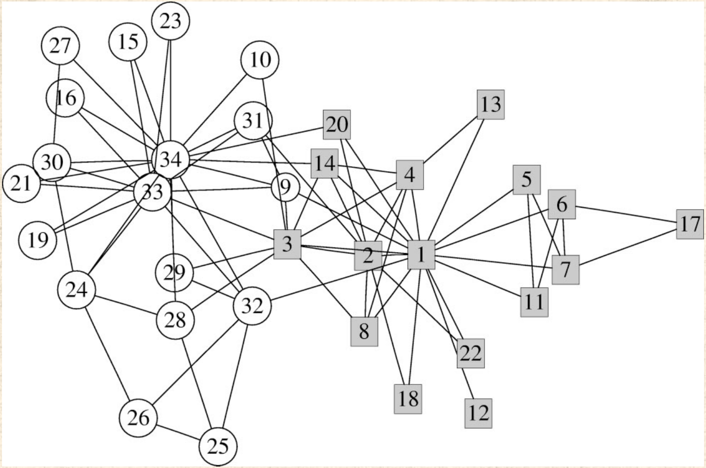
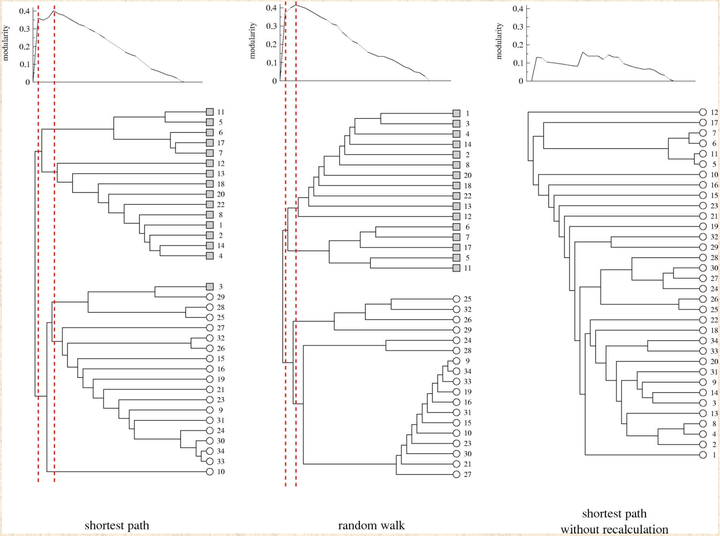
- Equivalent to electronic betweenness (see also diffusion).

Hierarchy by division



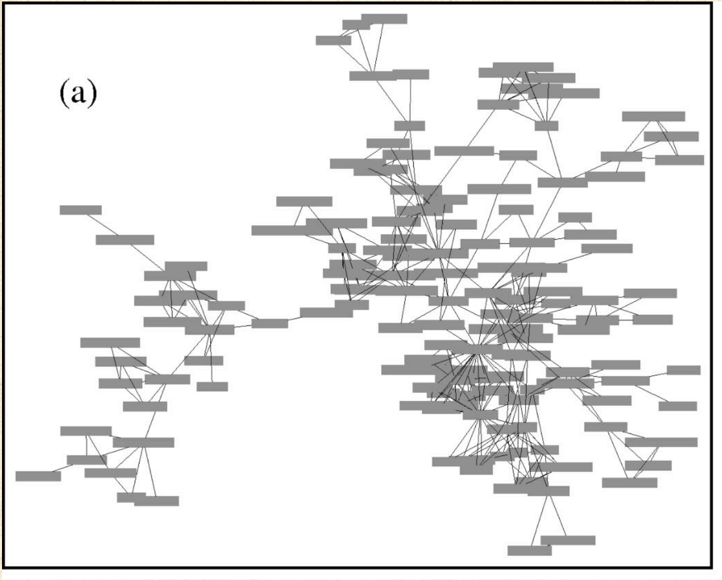
- Factions in Zachary's karate club network. [119]

Hierarchy by division



- Third column shows what happens if we don't recompute betweenness after each edge removal.

Scientists working on networks (2004)



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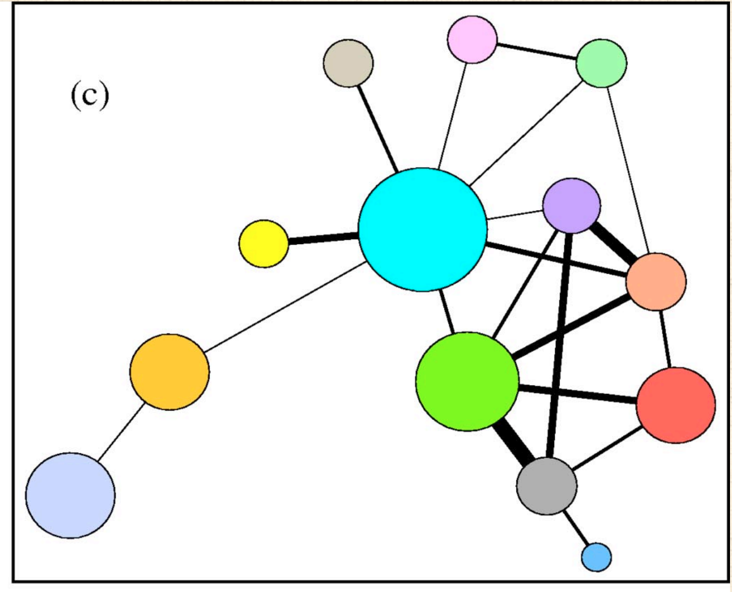

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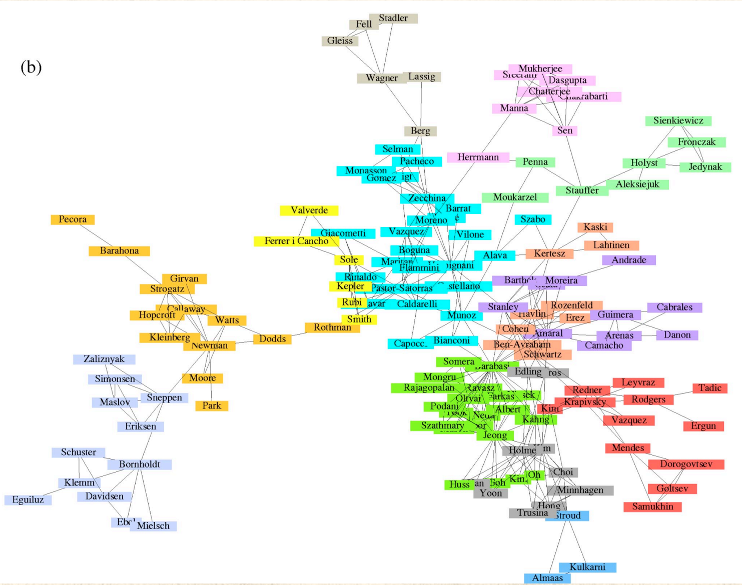
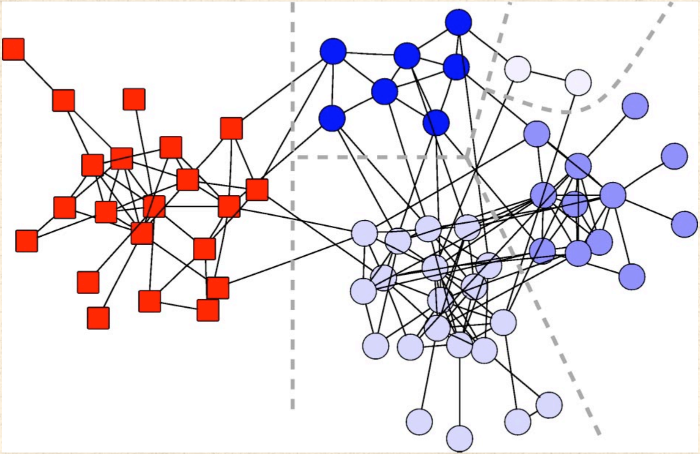
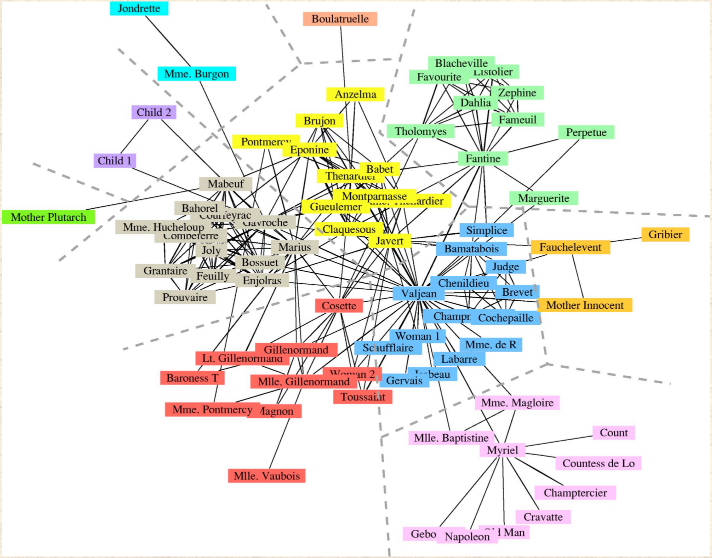


Dolphins!



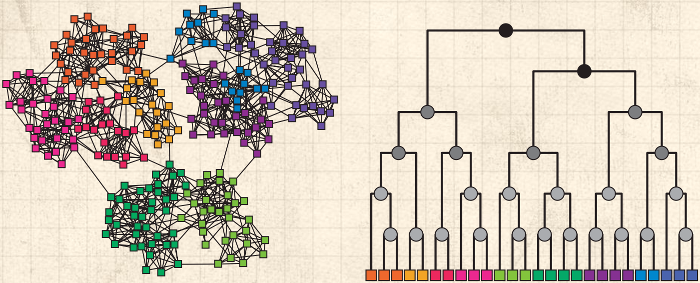
Les Miserables



More network analyses for Les Miserables here and here.

Hierarchies and missing links

Clauset *et al.*, Nature (2008) [25]



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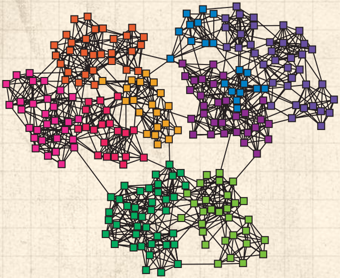

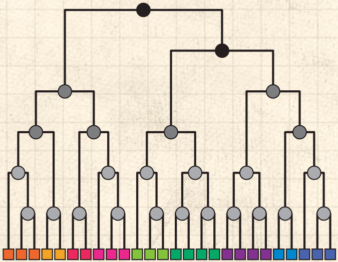

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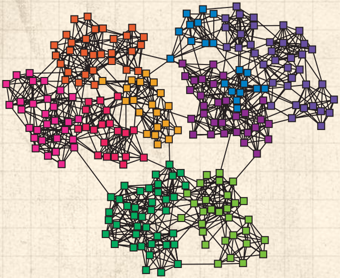

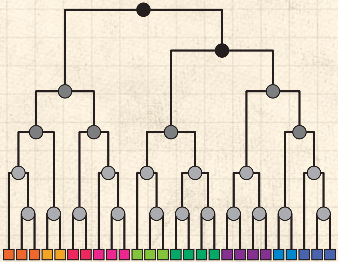

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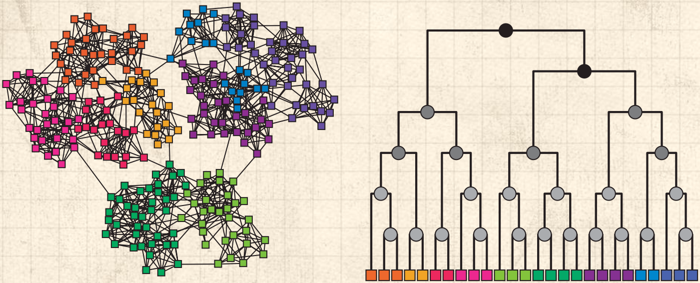

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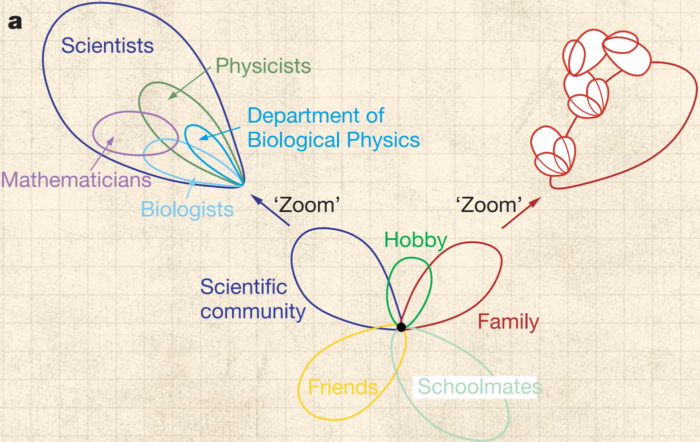
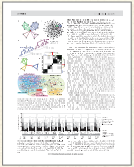
- Obtain probability that links are missing (big problem...).

"Uncovering the overlapping community structure of complex networks in nature and society"

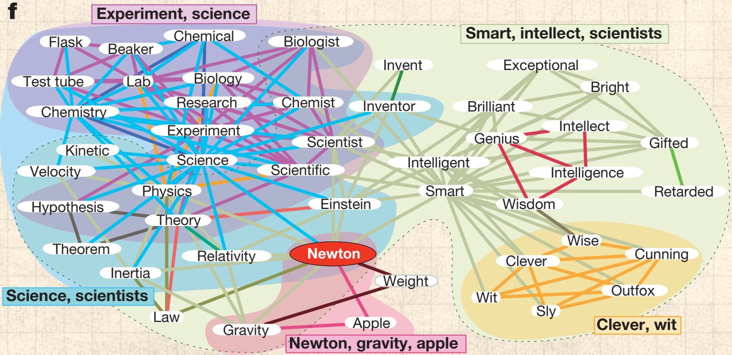
Palla et al.,
Nature, **435**, 814–818, 2005. [81]





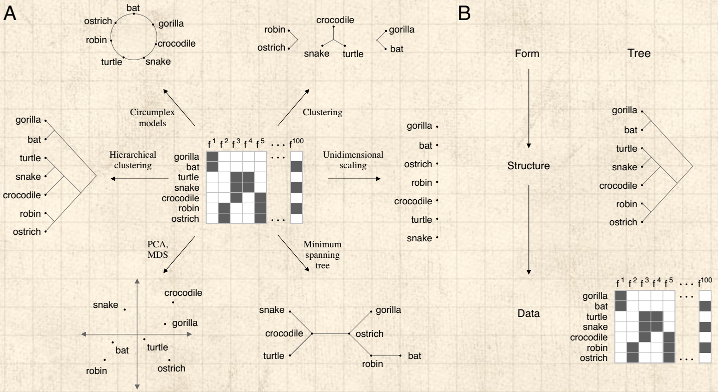
“Link communities reveal multiscale complexity in networks”

Ahn, Bagrow, and Lehmann, Nature, **466**, 761–764, 2010. [2]

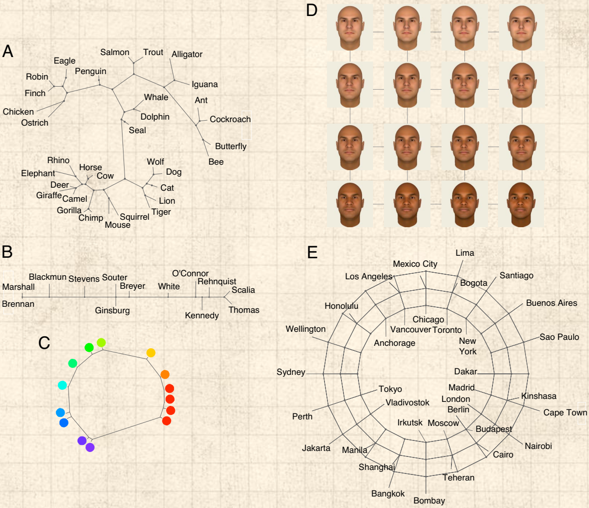


General structure detection

- "The discovery of structural form"
  Kemp and Tenenbaum, PNAS (2008) [54]



Example learned structures:



- Biological features; Supreme Court votes; perceived color differences; face differences; & distances between cities.

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Overview Key Points:

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- Specific focus on networks that are large-scale, sparse, natural or people-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).
- To solve network problems: “Follow the edges.”

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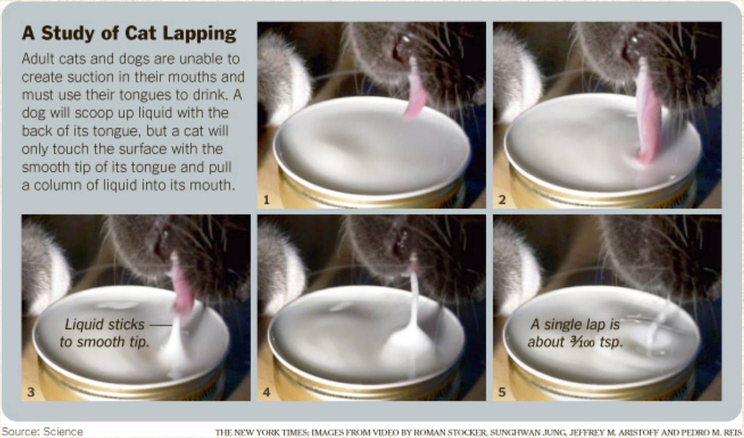
- (Scientific truths shouldn't be named after individuals.)

Rather silly but great example of real science:

"How Cats Lap: Water Uptake by *Felis catus*"
Reis et al., *Science*, 2010.



Amusing interview here

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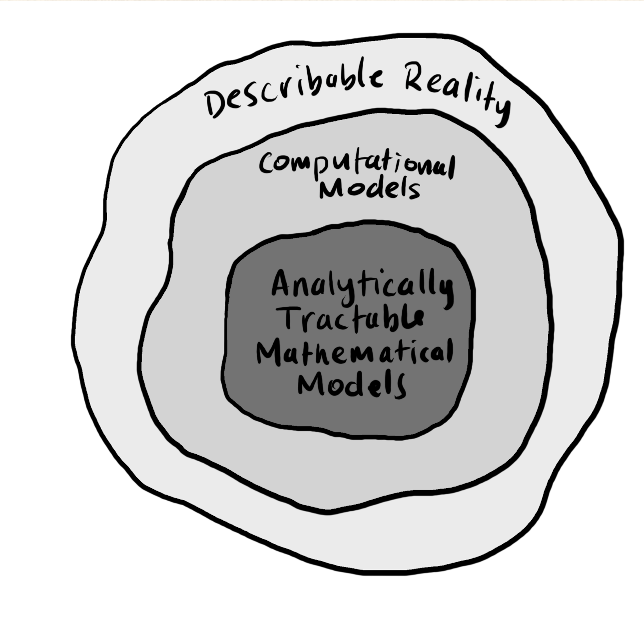
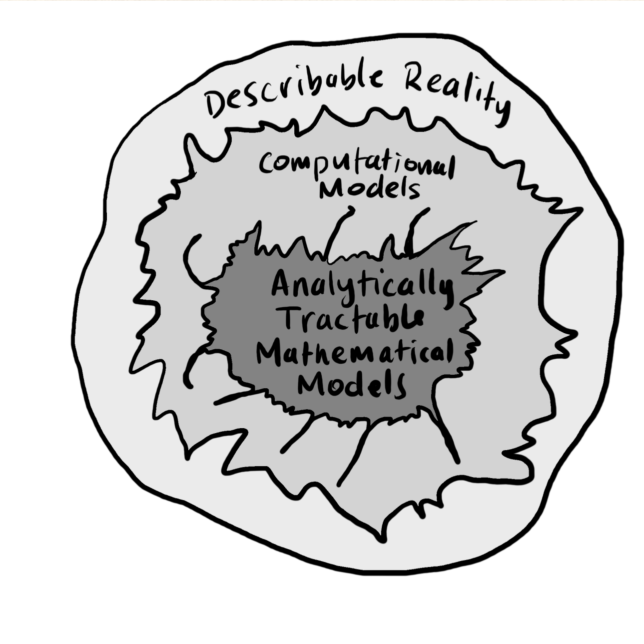
- And pyramids ($\sim$ hierarchies)


Describable Reality
Computational Models
Analytically Tractable Mathematical Models


THE CHURCH OF QUARTEROLOGY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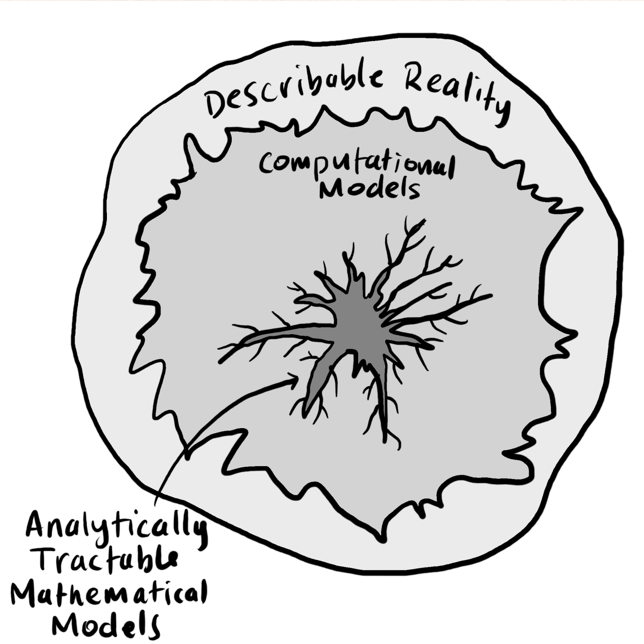
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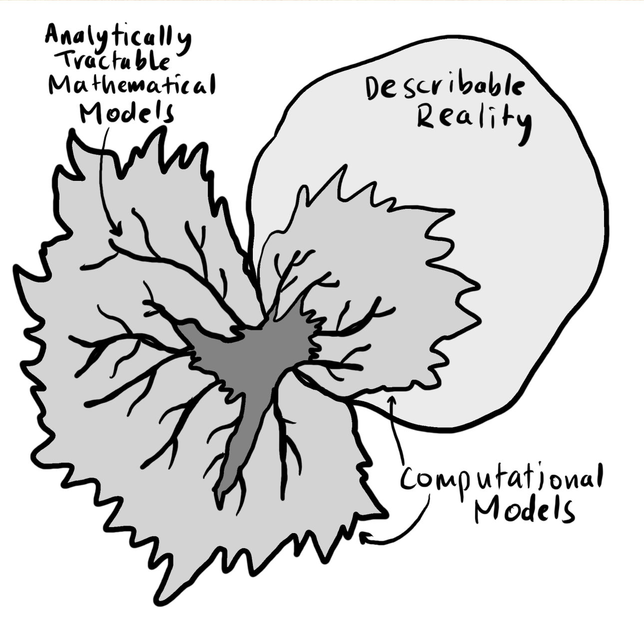

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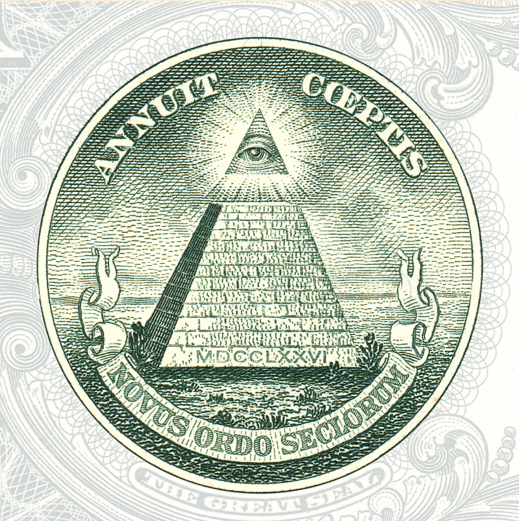
- "Beards will always be cool."

The Pyramid knows what you did.



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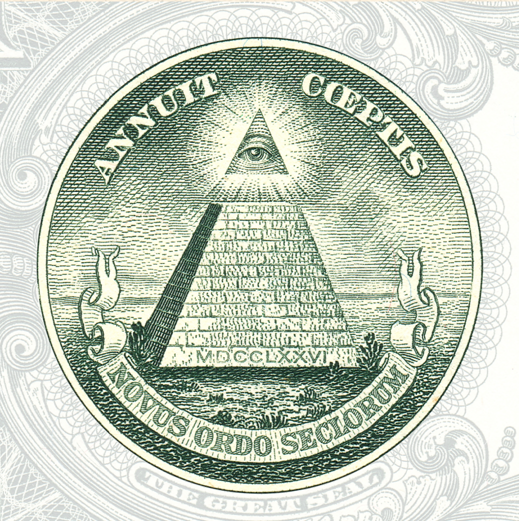

Mass surveillance by story.

The absolute basics:

Modern basic science in three steps:

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Always:

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2. Describe what you see.
3. Explain it.

If you succeed at 1–3:

4. Create.
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Always:

6. Be good people.

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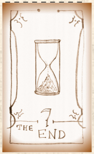

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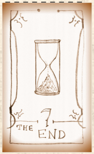

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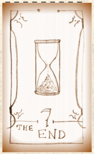

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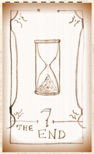

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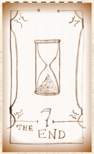

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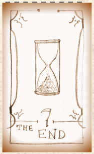

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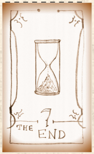

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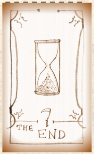

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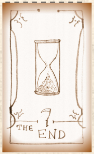

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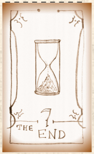

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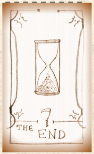

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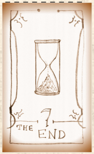

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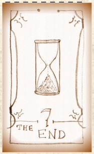

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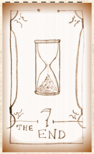

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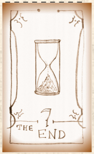

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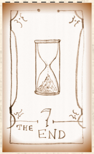

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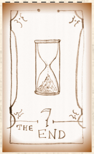

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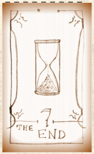

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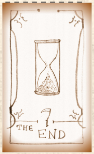

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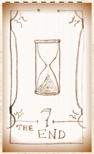

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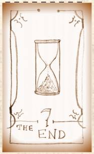

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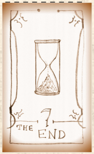

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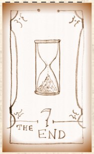

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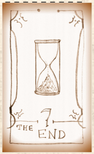

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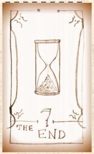

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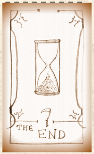

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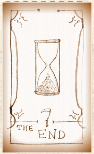

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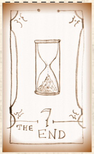

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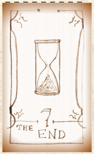

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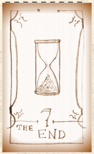

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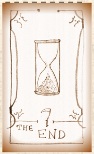

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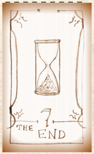

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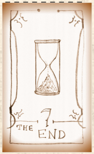

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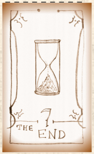

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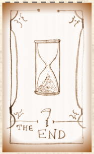

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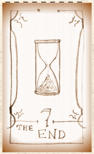

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