

Overview of Complex Networks

Last updated: 2022/08/28, 03:24:52 EDT

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394, 2022-2023 | @pocsvox

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Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont



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Overview of
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Complex
Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions

Examples of
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Relational networks

References



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Overview of
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Sealie & Lambie Productions



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 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 



Outline

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net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Thesaurus deliciousness:

network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.

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From Keith Briggs's excellent etymological
investigation: 



Opus
reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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Ancestry:






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First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

-  1658–: reticulate structures in animals
-  1839–: rivers and canals
-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

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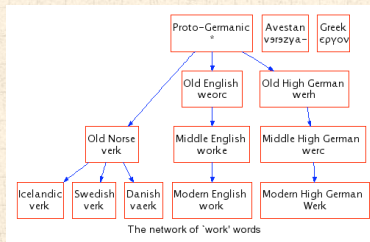
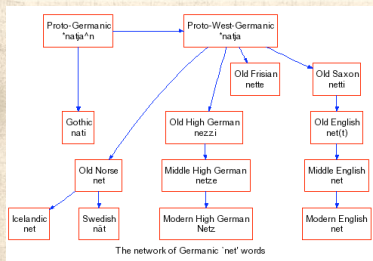



Ancestry:

Net and Work are venerable old words:

 **'Net'** first used to mean spider web (King Ælfréd, 888).

 **'Work'** appear to have long meant purposeful action.



 **'Network'** = something built based on the idea of natural, flexible lattice or web.

 c.f., ironwork, stonework, fretwork.



Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>

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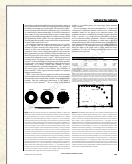
References




Popularity (according to Google Scholar)


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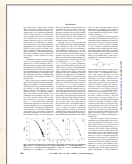
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"Collective dynamics of 'small-world' networks" 


Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[16]

Times cited:  ~ 37,460 (as of October 24, 2018)



"Emergence of scaling in random networks" 

Barabási and Albert,
Science, **286**, 509–511, 1999. ^[2]

Times cited:  ~ 32,093 (as of October 24, 2018)

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Review articles:



“Complex Networks: Structure and Dynamics”

Boccaletti et al.,
Physics Reports, **424**, 175–308, 2006. [3]

Times cited: ~ **8,533** (as of October 24, 2018)



“The structure and function of complex networks”

M. E. J. Newman,
SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: ~ **17,782** (as of October 24, 2018)



“Statistical mechanics of complex networks”

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Times cited: ~ **20,531** (as of October 24, 2018)

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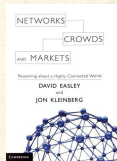
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"Networks" [a](#) [↗](#)
by Mark Newman (2018). ^[11]



"Networks, crowds, and markets: Reasoning
about a highly connected world" [a](#) [↗](#)
by Easley and Kleinberg (2010). ^[7]

<http://cs.cornell.edu/home/kleinber/networks-book/> [↗](#)

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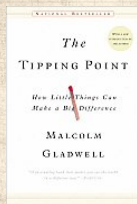
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The Tipping Point: How Little Things can
make a Big Difference—Malcolm
Gladwell [8]

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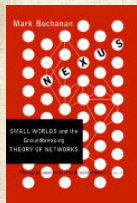
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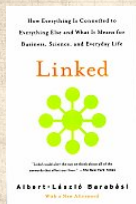
Nexus: Small Worlds and the
Groundbreaking Science of
Networks—Mark Buchanan



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Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[15]

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Numerous others ...

-  Complex Social Networks—F. Vega-Redondo ^[14]
-  Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo ^[13]
-  Random Graph Dynamics—R. Durrette
-  Scale-Free Networks—Guido Caldarelli
-  Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
-  Complex Graphs and Networks—Fan Chung
-  Social Network Analysis—Stanley Wasserman and Kathleen Faust
-  Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
-  Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes ^[6]

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More observations

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- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish **mechanistic explanations**.

**If this is upsetting, maybe string theory is for you ...*

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More observations


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 Web-scale data sets can be overly **exciting**.


Witness:


 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 "The Unreasonable Effectiveness of Data,"
Halevy et al. [9].

 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [17]

But:

 For scientists, description is only part of the battle.

 We still need to **understand**.

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Super Basic definitions


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Overview of
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Nodes = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

 **Links** may be directed or undirected.

 **Links** may be binary or weighted.

Other spiffing words: vertices and edges.

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Node degree = Number of links per node


 Notation: Node i 's degree = k_i .

 $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)

 Connection between number of edges m and
average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

 Defn: \mathcal{N}_i = the set of i 's k_i neighbors

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Adjacency matrix:

 We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)

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So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...

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Examples

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Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids

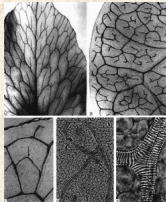
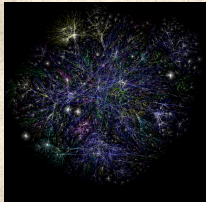
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 **Distribution** (branching) versus **redistribution** (cyclical)





Examples


Interaction networks


 The Blogosphere


 Biochemical networks


 Gene-protein networks

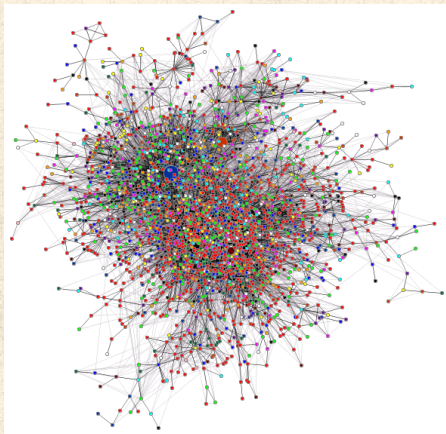
 Food webs: who eats whom


 The World Wide Web (?)

 Airline networks

 Call networks (AT&T)

 The Media



datamining.typepad.com 

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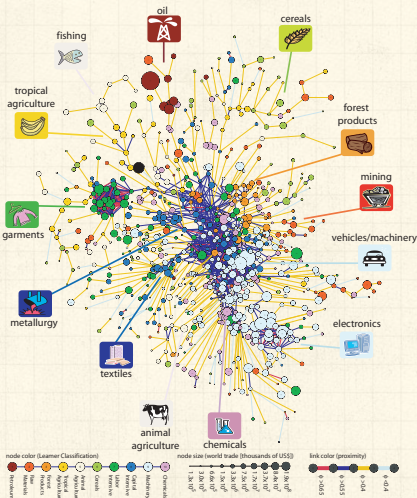


topics:

 Hidalgo et al.'s "The Product Space Conditions the Development of Nations"^[10]

 How do products depend on each other, and how does this network evolve?

 How do countries depend on each other for water, energy, people (immigration), investments?



Examples

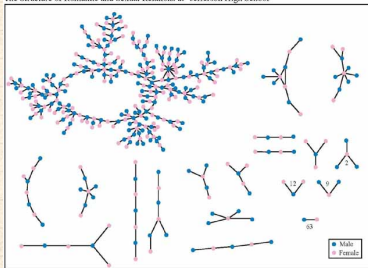
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Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook ↗ twitter ↗,

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

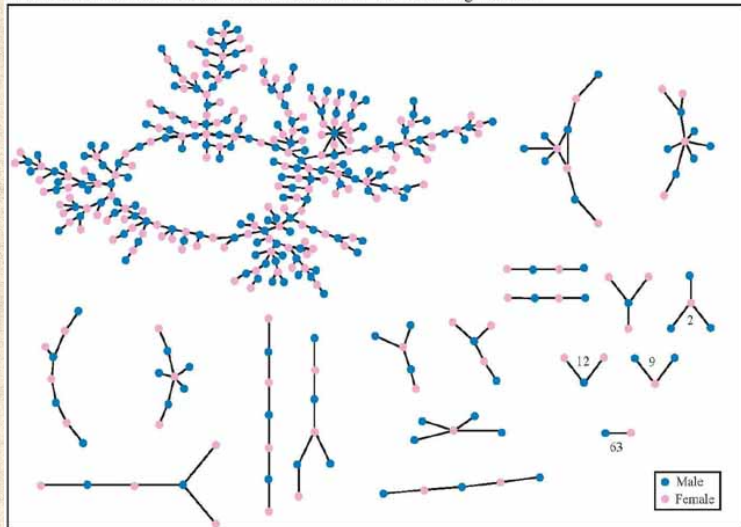


Examples

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Examples

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Consumer purchases
(Walmart, Target, Amazon, ...)





Thesauri: Networks of words generated by meanings



Knowledge/Databases/Ideas



Metadata—Tagging: bit.ly  [flickr](https://www.flickr.com/photos/pocsvox/) 

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

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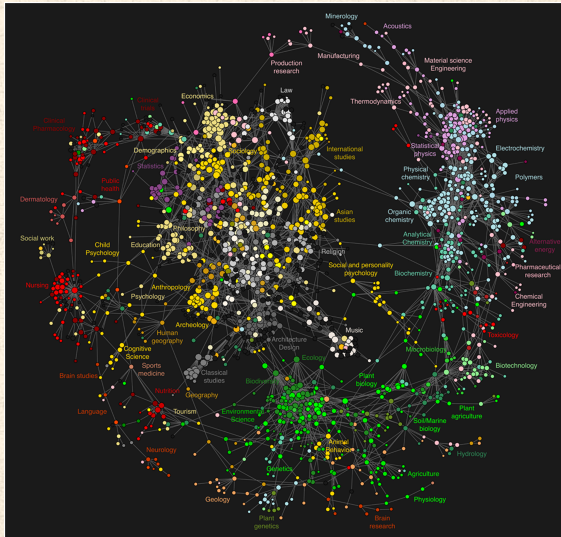
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Clickworthy Science:

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“Clickstream Data Yields High-Resolution Maps of Science”,
Bollen et al. ^[4], 2009.



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The unreasonable effectiveness of data.
IEEE Intelligent Systems, 24:8–12, 2009. [pdf](#) 
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Cambridge University Press, Cambridge, UK, 1997.
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Nature, 393:440–442, 1998. [pdf](#) 
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Communications on Pure and Applied
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