

# Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D  
CSYS/MATH 300, 303, & 394, 2022-2023 | @pocsvox

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## Outline

### Complex Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

### Examples of Complex Networks

- Physical networks
- Interaction networks
- Relational networks

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### net•work |ˈnetwɜːk|

noun

- an arrangement of intersecting horizontal and vertical lines.
  - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- a group or system of interconnected people or things : *a trade network.*
  - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
  - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
  - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
  - a system of connected electrical conductors.

verb [ trans. ]

- connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*
- link (machines, esp. computers) to operate interactively : [as adj.] ( **networked** ) *networked workstations.*
  - [ intrans. ] [often as n. ] ( **networking** ) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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## Thesaurus deliciousness:

network  
noun

- a *network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- a *network of lanes* MAZE, labyrinth, warren, tangle.
- a *network of friends* SYSTEM, complex, nexus, web, webwork.

## Ancestry:

From Keith Briggs's excellent etymological investigation:

- Opus reticulatum:
- A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network/>]

## Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

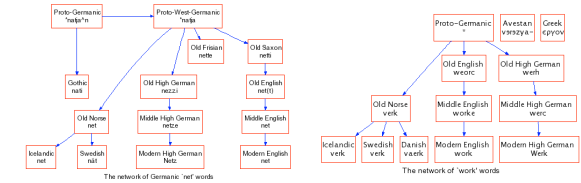


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## Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

## Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mech** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>

## Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"  
Watts and Strogatz,  
Nature, **393**, 440–442, 1998. [16]

Times cited: ~ 37,460 (as of October 24, 2018)



"Emergence of scaling in random networks"  
Barabási and Albert,  
Science, **286**, 509–511, 1999. [2]

Times cited: ~ 32,093 (as of October 24, 2018)

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## Review articles:



“Complex Networks: Structure and Dynamics”  
Boccaletti et al.,  
Physics Reports, **424**, 175–308, 2006. [3]

Times cited: ~ 8,533 (as of October 24, 2018)



“The structure and function of complex networks”  
M. E. J. Newman,  
SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: ~ 17,782 (as of October 24, 2018)



“Statistical mechanics of complex networks”  
Albert and Barabási,  
Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited: ~ 20,531 (as of October 24, 2018)

## Popularity according to textbooks:



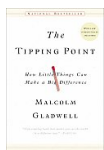
“Networks”  
by Mark Newman (2018). [11]



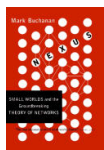
“Networks, crowds, and markets: Reasoning about a highly connected world”  
by Easley and Kleinberg (2010). [7]

<http://cs.cornell.edu/home/kleinber/networks-book/>

## Popularity according to books:

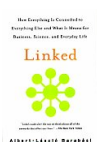


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

## Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [15]

## Numerous others ...

- Complex Social Networks—F. Vega-Redondo [14]
- Fractal River Basins: Chance and Self-Organization—I. Rodriguez-Iturbe and A. Rinaldo [13]
- Random Graph Dynamics—R. Durrett
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

## More observations

- But surely **networks aren't new** ...
  - Graph theory is well established ...
  - Study of social networks started in the 1930's ...
  - So why all this 'new' research on networks?
  - Answer:** Oodles of Easily Accessible Data.
  - We can now inform (alas) our theories with a much more measurable reality.\*
  - A worthy goal: establish mechanistic explanations.
- \*If this is upsetting, maybe string theory is for you ...

## More observations

Web-scale data sets can be overly **exciting**.

## Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) [8]
- “The Unreasonable Effectiveness of Data,” Halevy et al. [9].
- c.f. Wigner’s “The Unreasonable Effectiveness of Mathematics in the Natural Sciences” [17]

## But:

- For scientists, description is only part of the battle.
- We still need to **understand**.

## Super Basic definitions

**Nodes** = A collection of entities which have properties that are somehow related to each other

e.g., people, forks in rivers, proteins, webpages, organisms, ...

**Links** = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

## Super Basic definitions

**Node degree** = Number of links per node

- Notation: Node  $i$ 's degree =  $k_i$ .
- $k_i = 0, 1, 2, \dots$
- Notation: the average degree of a network =  $\langle k \rangle$  (and sometimes  $z$ )
- Connection between number of edges  $m$  and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

Defn:  $\mathcal{N}_i$  = the set of  $i$ 's  $k_i$  neighbors

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# Super Basic definitions

## Adjacency matrix:

We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .

e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(n.b., for numerical work, we always use sparse matrices.)

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# Examples

## So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...

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# Examples

## Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids



**Distribution** (branching) versus **redistribution** (cyclical)

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# Examples

## Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media



datamining.typepad.com

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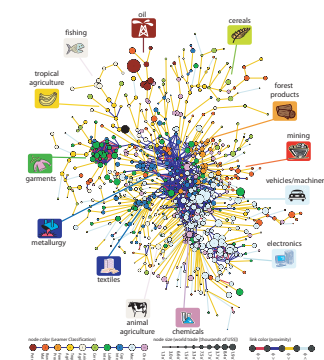
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# topics:

- Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [10]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



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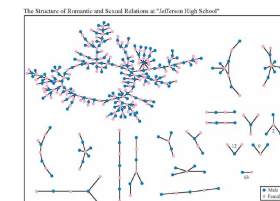


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# Examples

## Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook twitter



(Bearman et al., 2004)

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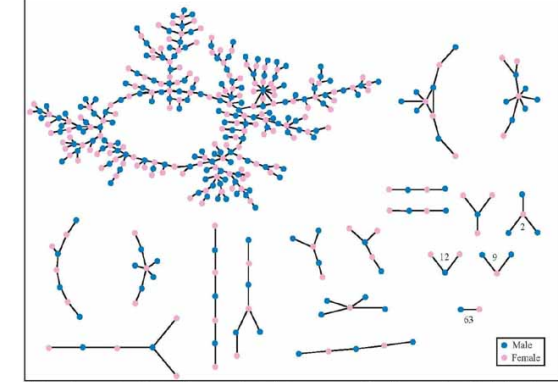


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'Remotely sensed' by: email activity, instant messaging, phone logs (\*cough\*).

# Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else)

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# Examples

## Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: [bit.ly](#) [flickr](#)

common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

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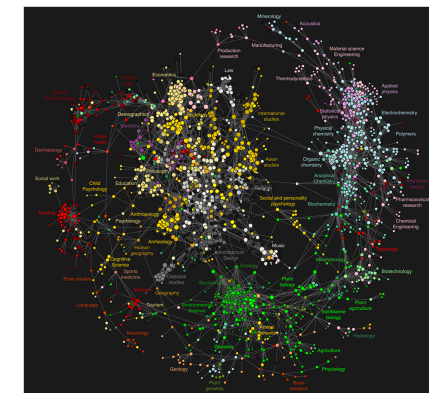
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# Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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- [5] S. Bornholdt and H. G. Schuster, editors. Handbook of Graphs and Networks. Wiley-VCH, Berlin, 2003.
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- [8] M. Gladwell. The Tipping Point. Little, Brown and Company, New York, 2000.
- [9] A. Halevy, P. Norvig, and F. Pereira. The unreasonable effectiveness of data. *IEEE Intelligent Systems*, 24:8–12, 2009. [pdf](#)
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