Chaotic Contagion: The Idealized Hipster Effect

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Outline

Chaotic Contagion

Chaos Invariant densities

References

Chaotic Contagion on Networks:



"Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability"

Dodds, Harris, and Danforth, Phys. Rev. Lett., **110**, 158701, 2013. [1]



"Dynamical influence processes on networks: General theory and applications to social contagion"

Harris, Danforth, and Dodds, Phys. Rev. E, 88, 022816, 2013. [2]

A. Mandel, conference at Urbana-Champaign, 2007:

"If I was a younger man, I would have stolen this from you."

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Chaotic contagion:



- Consider a simple deterministic version:
- Node i has an 'activation threshold' $\phi_{i,1}$
 - ...and a 'de-activation threshold' $\phi_{i,2}$
- Nodes like to imitate but only up to a limit—they don't want to be like everyone else.





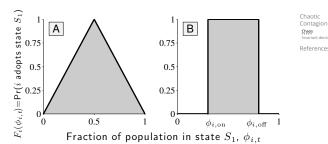




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Definition of the tent map:

$$F(x) = \left\{ \begin{array}{l} rx \text{ for } 0 \leq x \leq \frac{1}{2}, \\ r(1-x) \text{ for } \frac{1}{2} \leq x \leq 1. \end{array} \right.$$

 \clubsuit The usual business: look at how F iteratively maps the unit interval [0,1].

The tent map

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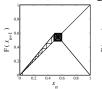
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Effect of increasing r from 1 to 2.

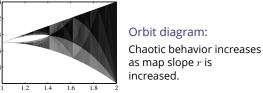














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Chaotic behavior

Take r=2 case:



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- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- information

Two population examples:

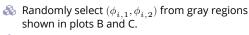


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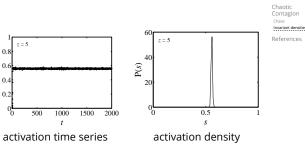
- Insets show composite response function averaged over population.
- & We'll consider plot C's example: the tent map.

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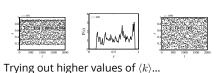
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Invariant densities—stochastic response functions



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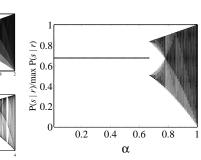
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Bifurcation diagram: Asynchronous updating



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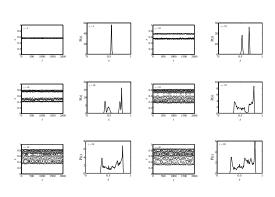
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Invariant densities—stochastic response functions



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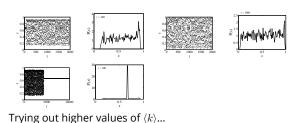
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Invariant densities—deterministic response functions



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Bifurcation diagram: Asynchronous updating

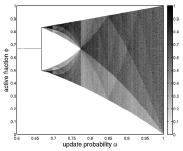
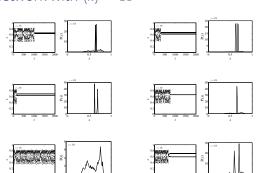


FIG. 3. Bifurcation diagram for the dense map $\Phi(\phi; \alpha)$ Eqn. (18). This was generated by iterating the map at 1000 α values between 0 and 1. The iteration was carried out with 3 random initial conditions for 10000 time steps each, discarding the first 1000. The ϕ -axis contains 1000 bins and the invariant density, shown by the grayscale value, is normalized by the maximum for each α . With $\alpha < 2/3$, all trajectories

go to the fixed point at $\phi = 2/3$.

Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



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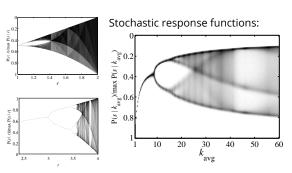
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Connectivity leads to chaos:



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https://www.youtube.com/watch?v=7JHrZyyq870?rel=0

How the bifurcation diagram changes with increasing average degree $\langle k \rangle$ as a function of the synchronicity parameter α for the stochastic response (tent map) case.



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References I

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[1] P. S. Dodds, K. D. Harris, and C. M. Danforth. Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability. Phys. Rev. Lett., 110:158701, 2013. pdf

[2] K. D. Harris, C. M. Danforth, and P. S. Dodds. Dynamical influence processes on networks: General theory and applications to social

contagion. Phys. Rev. E, 88:022816, 2013. pdf ☑



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