Branching Networks I

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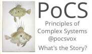
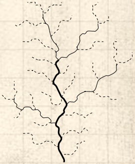
Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394, 2022–2023| @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont





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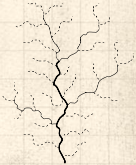

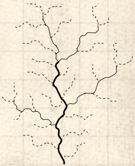

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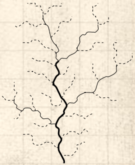
Outline




THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
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THE UNEXPECTED TRANSFIGURATION
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THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
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THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE LANDSCAPES OF FORKING PATHS
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
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THE SUPPLY MOST FAIR
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THE GOLDEN AGE OF REDUCTIONISM
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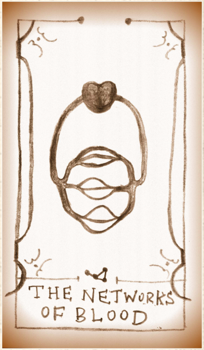
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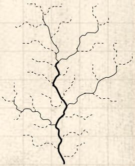
Introduction

Branching networks are useful things:

- Fundamental to material supply and collection



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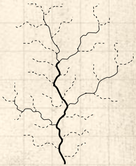

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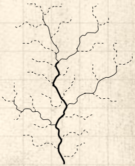

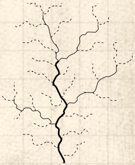

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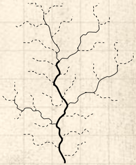

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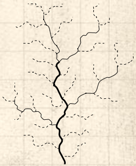

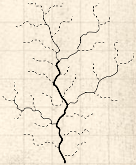

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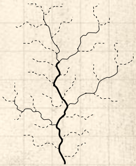

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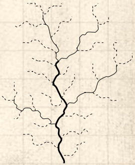

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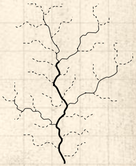
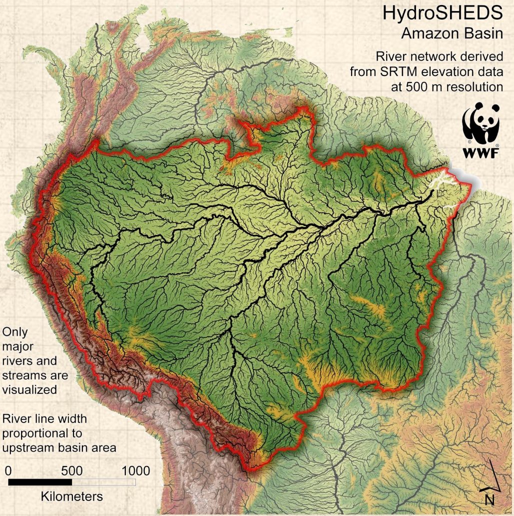
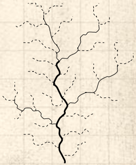
- Evolutionary trees
- Organizations (only in theory ...)



Branching networks are everywhere ...



http://hydrosheds.cr.usgs.gov/



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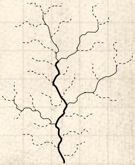
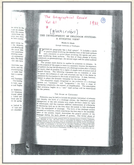
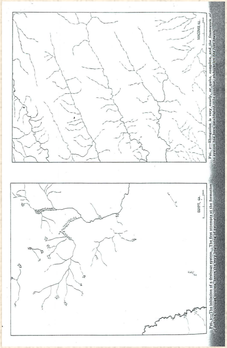
http://en.wikipedia.org/wiki/Image:Applebox.JPG



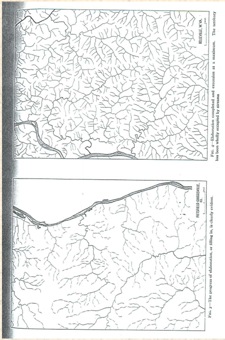
An early thought piece: Extension and Integration



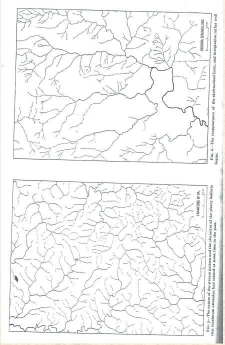
"The Development of Drainage Systems: A Synoptic View"
Waldo S. Glock,
The Geographical Review, **21**, 475–482, 1931. [2]



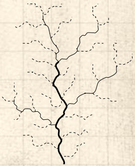
Initiation,
Elongation



Elaboration,
Piracy.



Abstraction,
Absorption.



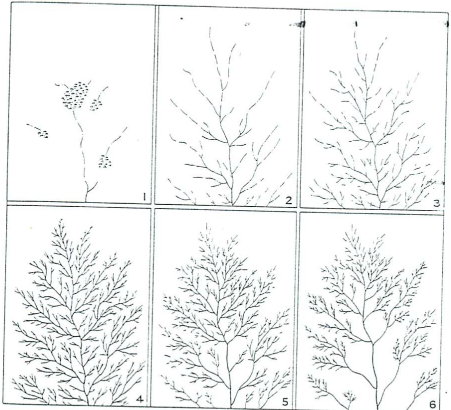
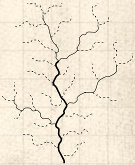
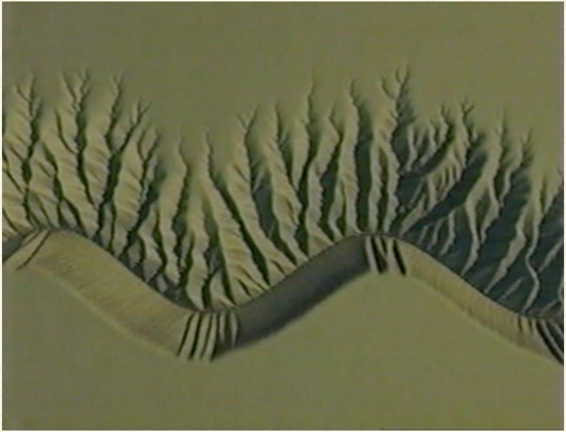


FIG. 8—An ideal diagrammatic summary of the development of a drainage system given for purposes of comparison only. The first four parts show extension, thus: 1, initiation; 2, elongation; 3, elaboration; and 4, maximum extension. Parts 5 and 6 represent steps during integration.

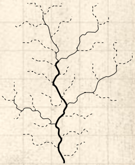


The sequential stages recognized in the evolution of a drainage system are "extension" and "integration"; the first, a stage of increasing complexity; the second, of simplification.

Shaw and Magnasco's beautiful erosion simulations



- Unpublished.
- Though to be destroyed and lost.
- The VHS.



Outline

Geomorphological networks

Definitions

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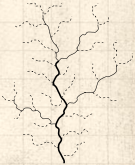

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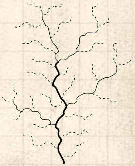

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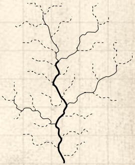

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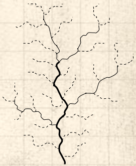

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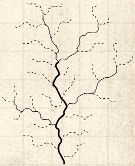

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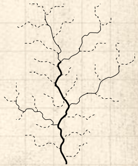

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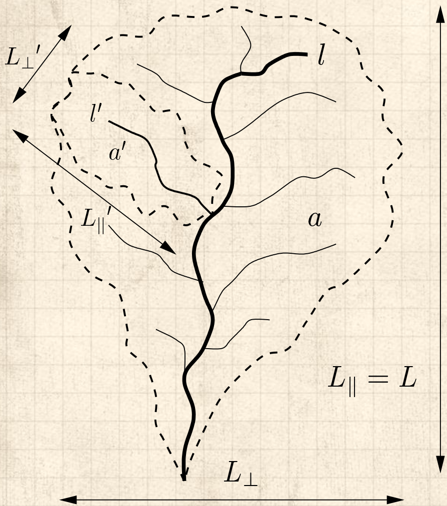

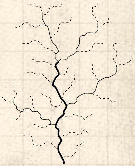

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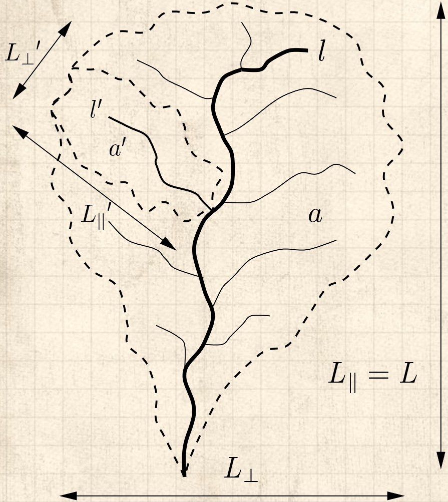
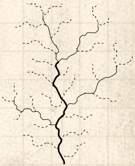


- a = drainage basin area



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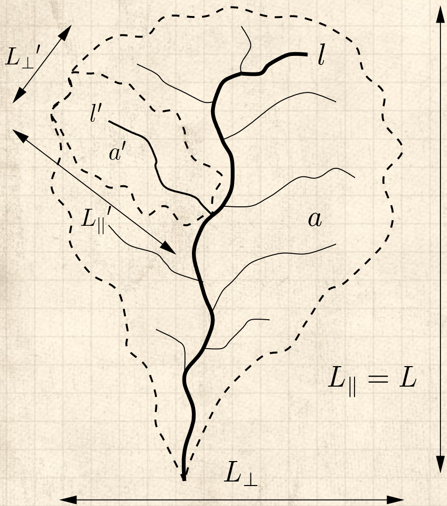

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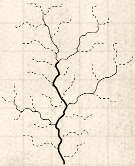

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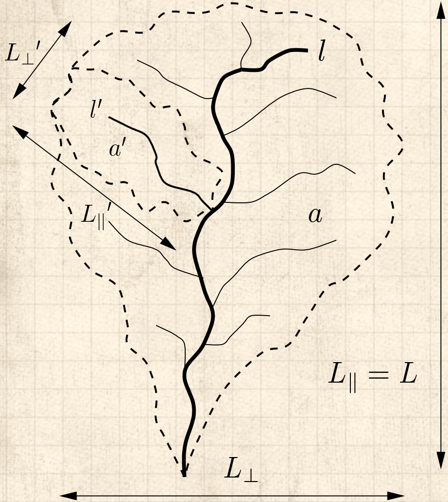

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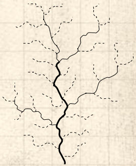

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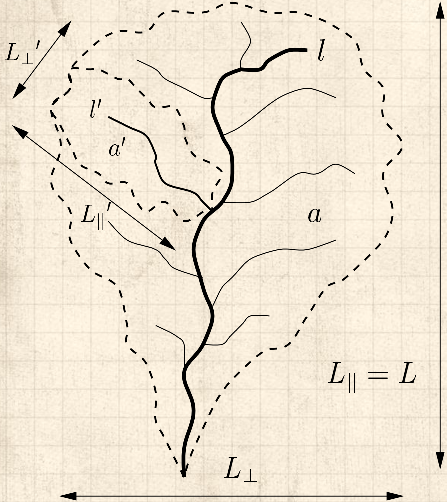

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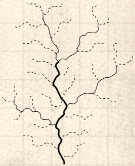
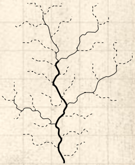


Outline



Allometry

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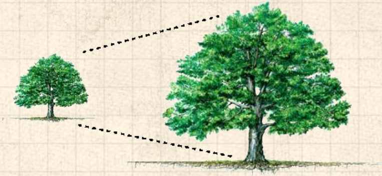

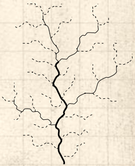

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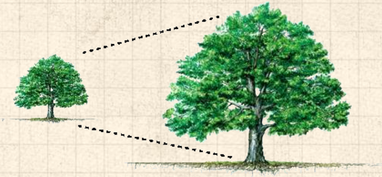

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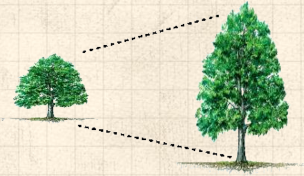
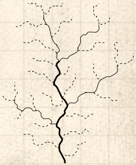
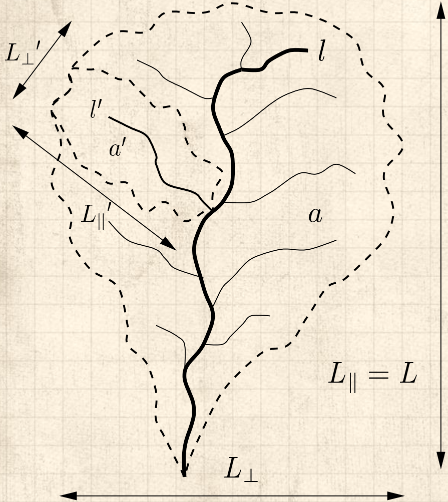




Basin allometry



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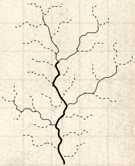

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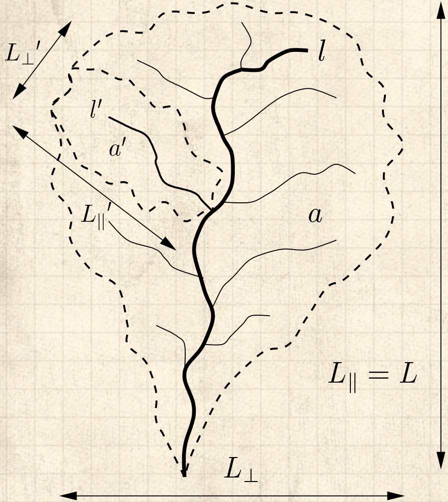

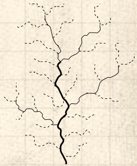

Allometric relationships:

$$\ell \propto a^h$$

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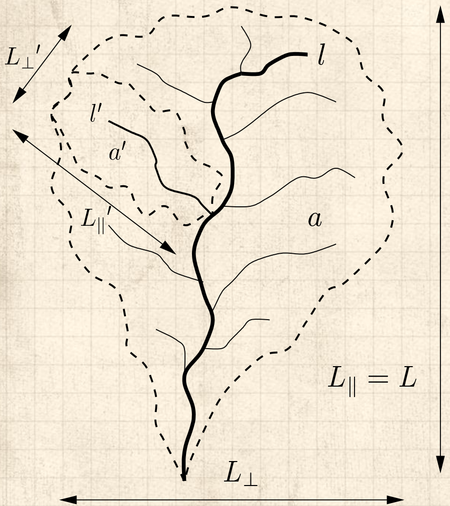
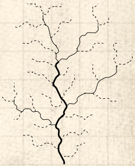


Allometric relationships:

- $$\ell \propto a^h$$
- $$\ell \propto L^d$$



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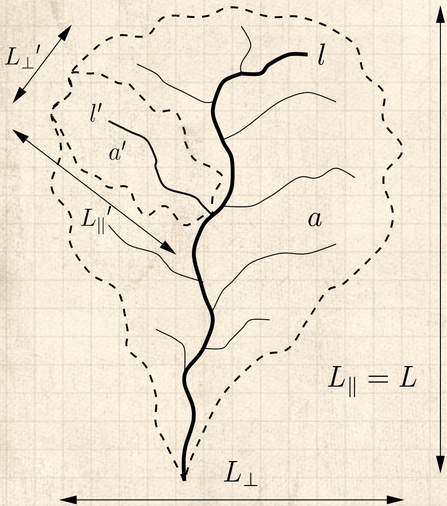
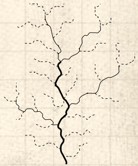


Allometric relationships:

- $$\ell \propto a^h$$
- $$\ell \propto L^d$$
- Combine above:

$$a \propto L^{d/h} \equiv L^D$$



‘Laws’

- Hack’s law (1957) [3]:

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reportedly $0.5 < h < 0.7$

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- Basin allometry:

$$\boxed{L_{\parallel} \propto a^{h/d} \equiv a^{1/D}}$$

$D < 2 \rightarrow$ basins elongate.

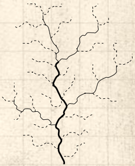
Outline

There are a few more 'laws': [1]

Relation:	Name or description:
$T_k = T_1 (R_T)^{k-1}$	Tokunaga's law
$\ell \sim L^d$	self-affinity of single channels
$n_\omega / n_{\omega+1} = R_n$	Horton's law of stream numbers
$\bar{\ell}_{\omega+1} / \bar{\ell}_\omega = R_\ell$	Horton's law of main stream lengths
$\bar{a}_{\omega+1} / \bar{a}_\omega = R_a$	Horton's law of basin areas
$\bar{s}_{\omega+1} / \bar{s}_\omega = R_s$	Horton's law of stream segment lengths
$L_\perp \sim L^H$	scaling of basin widths
$P(a) \sim a^{-\tau}$	probability of basin areas
$P(\ell) \sim \ell^{-\gamma}$	probability of stream lengths
$\ell \sim a^h$	Hack's law
$a \sim L^D$	scaling of basin areas
$\Lambda \sim a^\beta$	Langbein's law
$\lambda \sim L^\varphi$	variation of Langbein's law

Reported parameter values: [1]

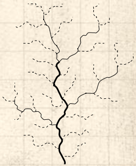
Parameter:	Real networks:
R_n	3.0–5.0
R_a	3.0–6.0
$R_\ell = R_T$	1.5–3.0
T_1	1.0–1.5
d	1.1 ± 0.01
D	1.8 ± 0.1
h	0.50–0.70
τ	1.43 ± 0.05
γ	1.8 ± 0.1
H	0.75–0.80
β	0.50–0.70
φ	1.05 ± 0.05



Kind of a mess ...

Order of business:

Kind of a mess ...



Order of business:

1. Find out how these relationships are connected.

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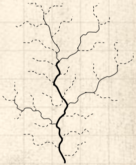
Order of business:

1. Find out how these relationships are connected.
2. Determine most fundamental description.



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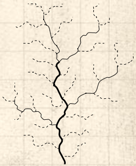

Order of business:

1. Find out how these relationships are connected.
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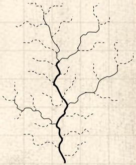
3. Explain origins of these parameter values



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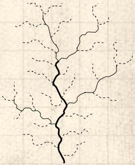
For (3): Many attempts: not yet sorted out ...


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
BIO HAZARD
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE ORDERS OF STREAMS
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING SHAPES FOR THE THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END

Stream Ordering:

Method for describing network architecture:



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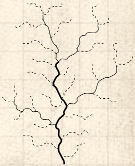

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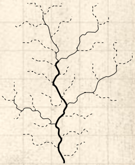

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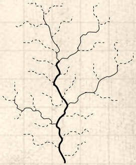

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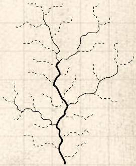

Method for describing network architecture:

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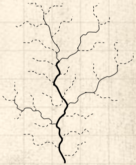
- Term: Horton-Strahler Stream Ordering [5]
- Can be seen as iterative trimming of a network.

Stream Ordering:



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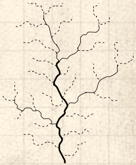

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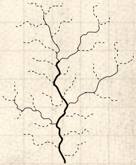

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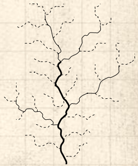

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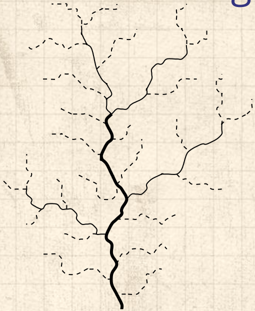
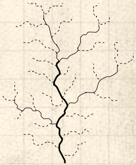
- Use symbol $\omega = 1, 2, 3, ...$ for stream order.

Stream Ordering:





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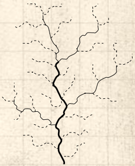

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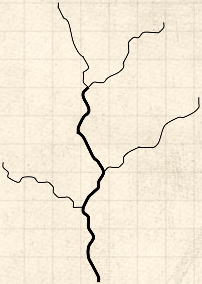

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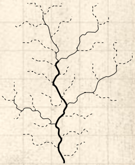

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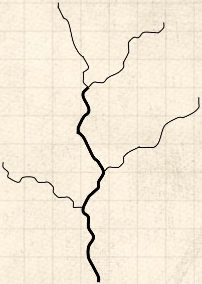

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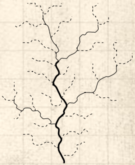
2. Label all new source streams as order $\omega = 2$ and remove.



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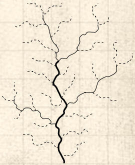

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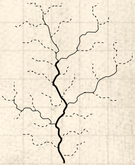

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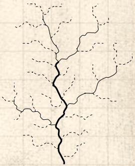

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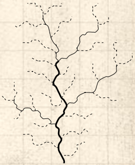
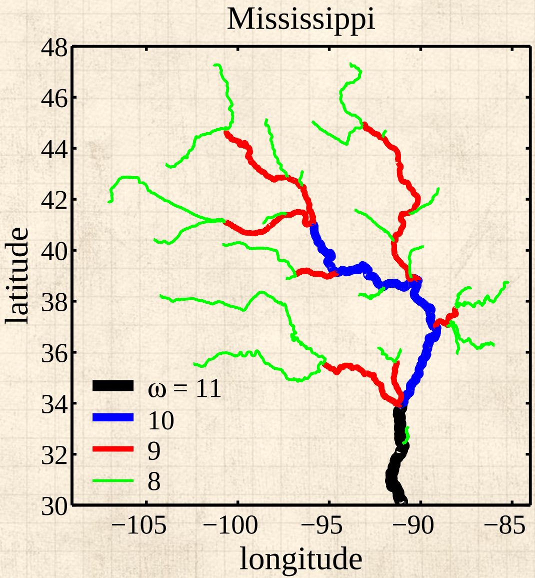
5. Example above is a basin of order $\Omega = 3$.

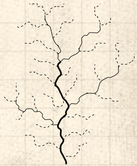


Stream Ordering—A large example:



[source=/data6/dodds/work/rivers/dems/mississippi/figures/figorder_paths_mispi10.ps]

[21−Mar−2000 peter dodds]



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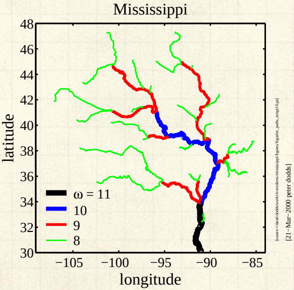

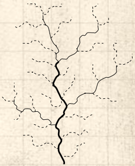

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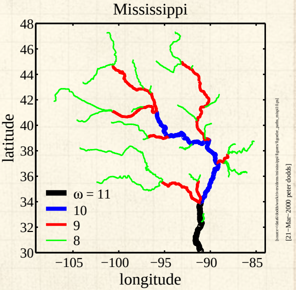

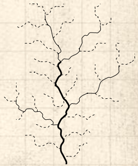

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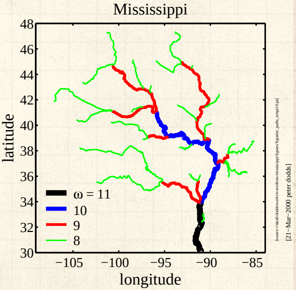

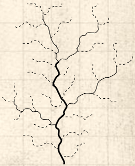

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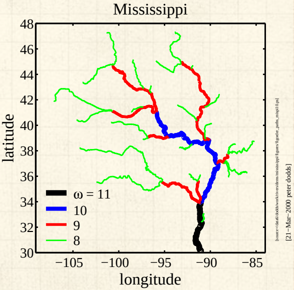

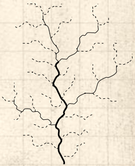

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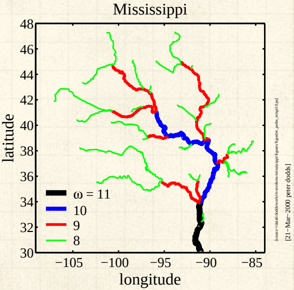

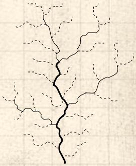

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- Simple rule:

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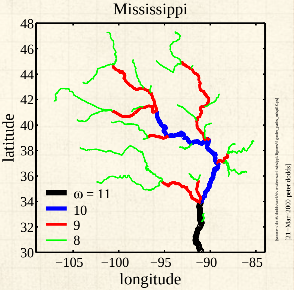
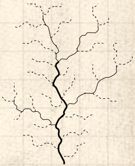
where δ is the Kronecker delta.





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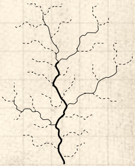

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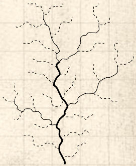

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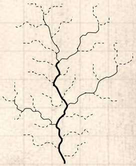

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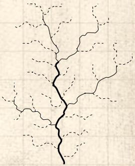

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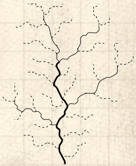

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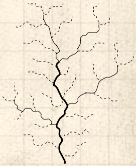
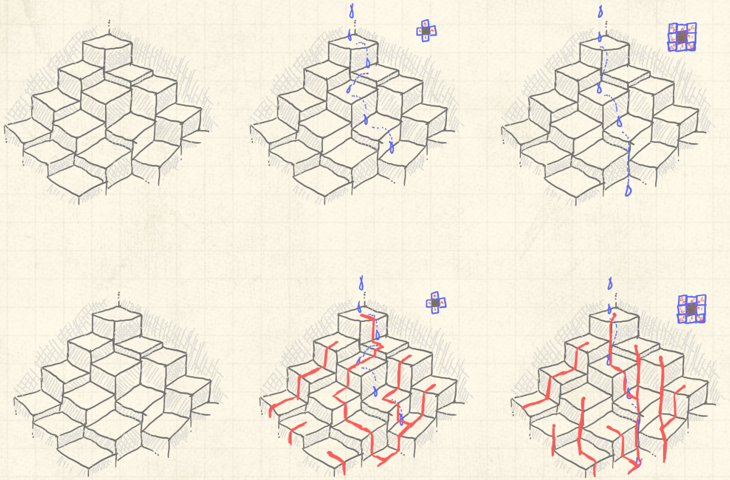
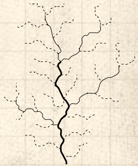


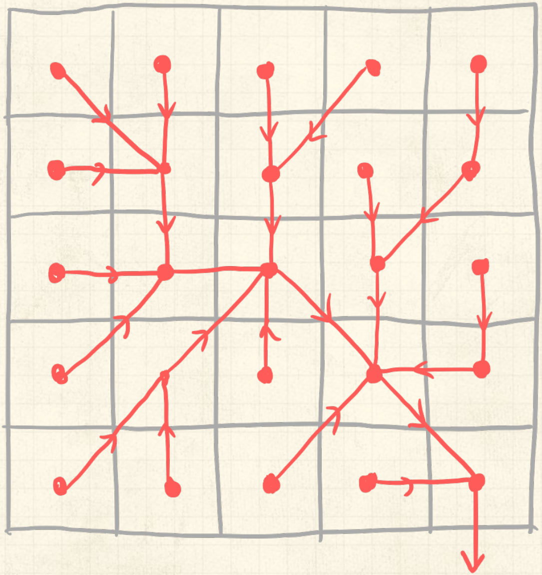
Basic algorithm for extracting networks from Digital Elevation Models (DEMs):

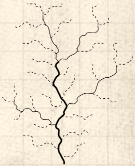


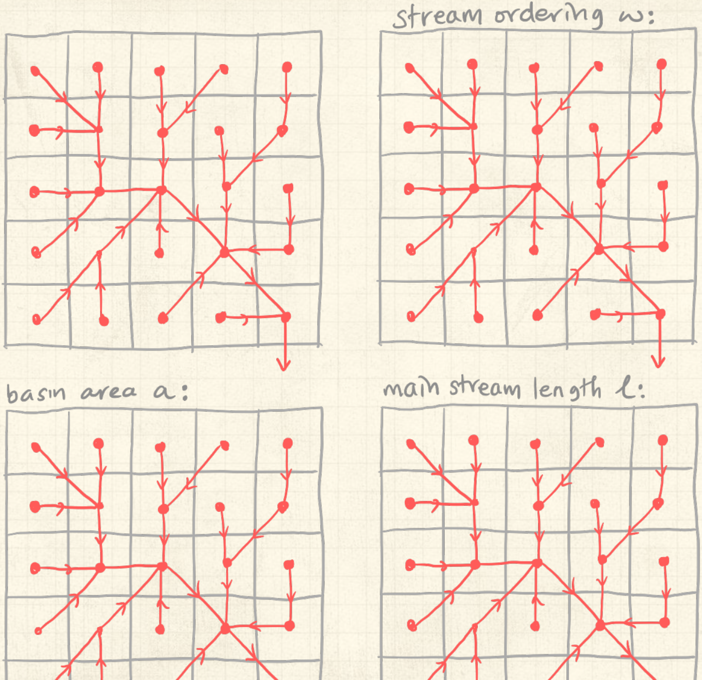


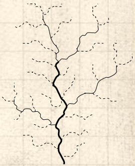
Also:

/Users/dodds/work/rivers/1998dems/kevinlakewaster.c






stream ordering ω:
basin area a:
main stream length ℓ:



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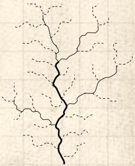

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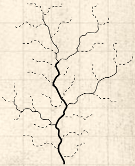

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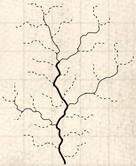

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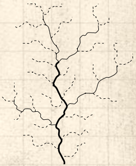

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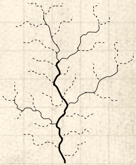

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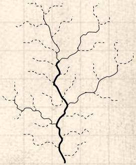

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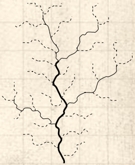


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
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THE KNOWLEDGE OF DEAD MONKS
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THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
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THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
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THE NETWORKS FREE OF SCALE
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THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
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THE LANDSCAPES OF FORKING PATHS
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THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
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THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
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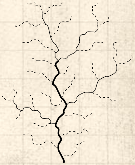
THE END

Horton's laws

Self-similarity of river networks



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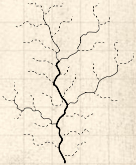
- First quantified by Horton (1945) [4], expanded by Schumm (1956) [6]



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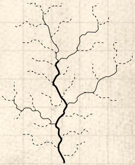

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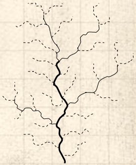

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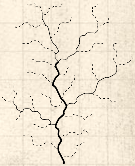

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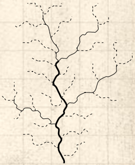

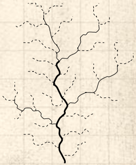

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Horton's Ratios:

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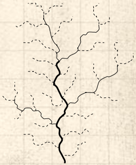

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- Horton's laws describe exponential decay or growth:

$$\begin{aligned} n_\omega &= n_{\omega-1}/R_n \\ &= n_{\omega-2}/R_n^2 \\ &\ \vdots \\ &= n_1/R_n^{\omega-1} \\ &= n_1 e^{-(\omega-1)\ln R_n} \end{aligned}$$

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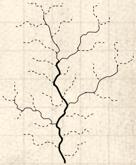

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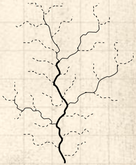

Similar story for area and length:

- $$\bar{a}_\omega = \bar{a}_1 e^{(\omega-1)\ln R_a}$$

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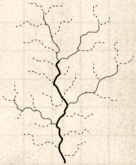
- As stream order increases, number drops and area and length increase.

Horton's laws

A few more things:



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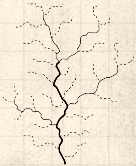

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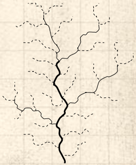

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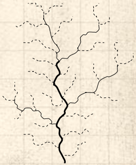

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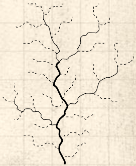
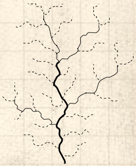
- Horton's ratios go a long way to defining a branching network ...
- But we need one other piece of information ...



Horton's laws



A bonus law:

- Horton's law of stream segment lengths:

$$\boxed{\bar{s}_{\omega+1}/\bar{s}_{\omega} = R_s > 1}$$

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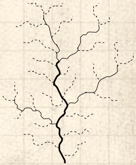

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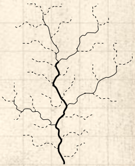

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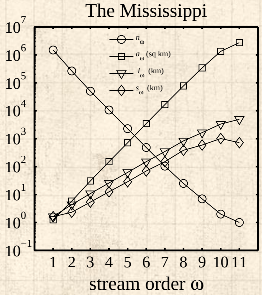
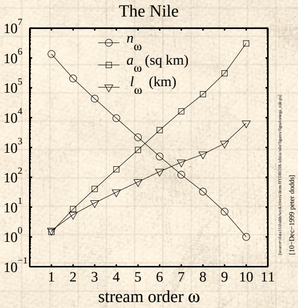
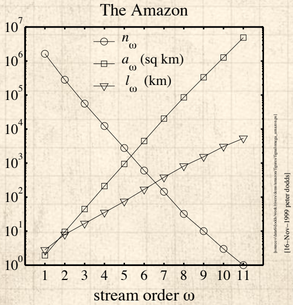
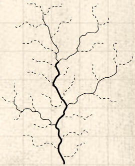
- Can show that $R_s = R_\ell$.
- Insert question from assignment 1

Horton's laws in the real world:





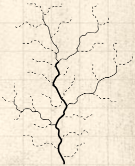




Horton's laws-at-large

Blood networks:

Horton's laws-at-large



Blood networks:

- Horton's laws hold for sections of cardiovascular networks

Horton's laws-at-large

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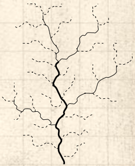
- Measuring such networks is tricky and messy ...

Horton's laws-at-large

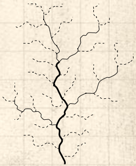
Blood networks:

- Horton's laws hold for sections of cardiovascular networks
- Measuring such networks is tricky and messy ...
- Vessel diameters obey an analogous Horton's law.



Data from real blood networks

Network	R_n	R_r	R_ℓ	$-\frac{\ln R_r}{\ln R_n}$	$-\frac{\ln R_\ell}{\ln R_n}$	α
West *et al.*	–	–	–	1/2	1/3	3/4
rat (PAT)	2.76	1.58	1.60	0.45	0.46	0.73
cat (PAT) [11]	3.67	1.71	1.78	0.41	0.44	0.79
dog (PAT)	3.69	1.67	1.52	0.39	0.32	0.90
pig (LCX)	3.57	1.89	2.20	0.50	0.62	0.62
pig (RCA)	3.50	1.81	2.12	0.47	0.60	0.65
pig (LAD)	3.51	1.84	2.02	0.49	0.56	0.65
human (PAT)	3.03	1.60	1.49	0.42	0.36	0.83
human (PAT)	3.36	1.56	1.49	0.37	0.33	0.94



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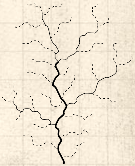
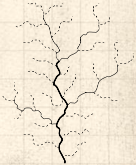


Observations:

- Horton's ratios vary:

R_n	3.0–5.0
R_a	3.0–6.0
R_ℓ	1.5–3.0

Horton's laws



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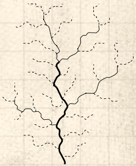

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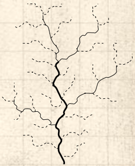
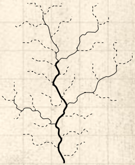
- Horton's laws tell us how quantities vary from level to level ...
- ...but they don't explain how networks are structured.



Tokunaga’s law

Delving deeper into network architecture:

Tokunaga's law



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- Tokunaga (1968) identified a clearer picture of network structure [8, 9, 10]

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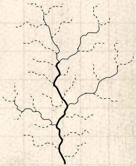

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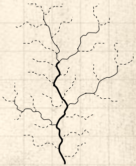

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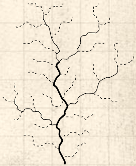

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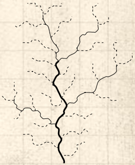
- Tokunaga's law is also a law of averages.

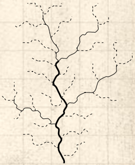
Network Architecture



Definition:

- $T_{\mu,\nu}$ = the average number of side streams of order ν that enter as tributaries to streams of order μ

Network Architecture



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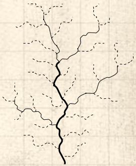

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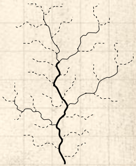

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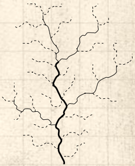

Network Architecture

Tokunaga's law

Network Architecture

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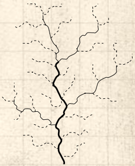
- Property 1: Scale independence—depends only on difference between orders:



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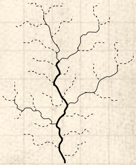

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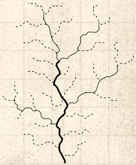

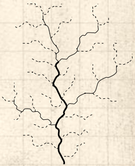

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- We usually write Tokunaga's law as:

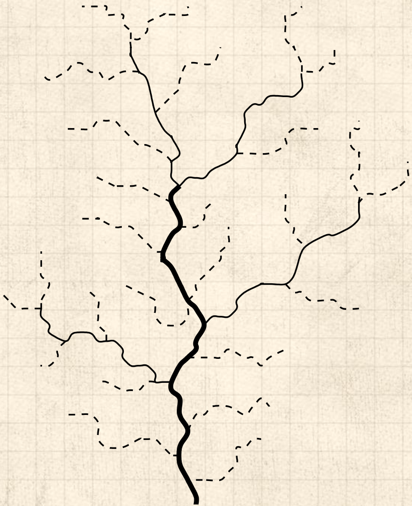
$$\boxed{T_k = T_1 (R_T)^{k-1}} \quad \text{where } R_T \simeq 2$$

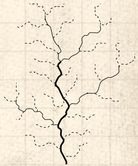
.

Tokunaga's law—an example:

$T_1 \simeq 2$

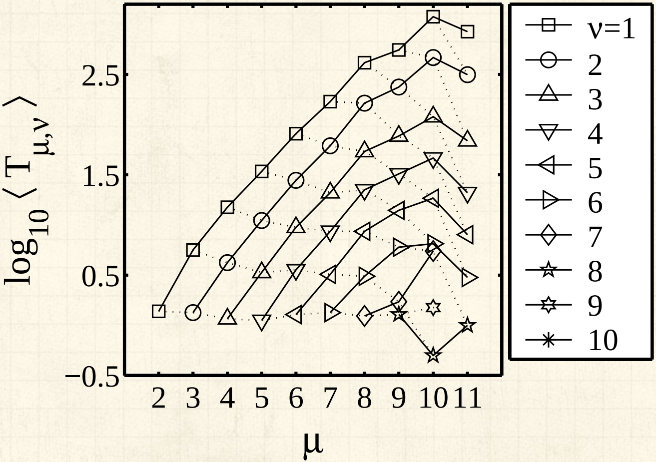
$R_T \simeq 4$

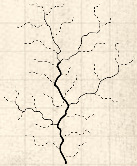




The Mississippi

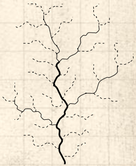
A Tokunaga graph:





Nutshell:

- Branching networks show remarkable self-similarity over many scales.



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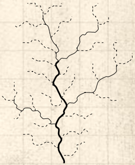

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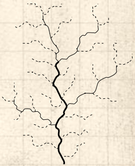

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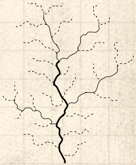

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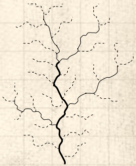

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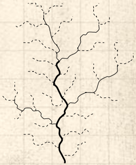

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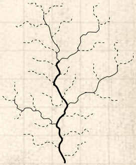

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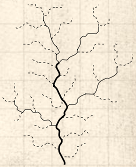

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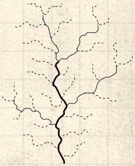
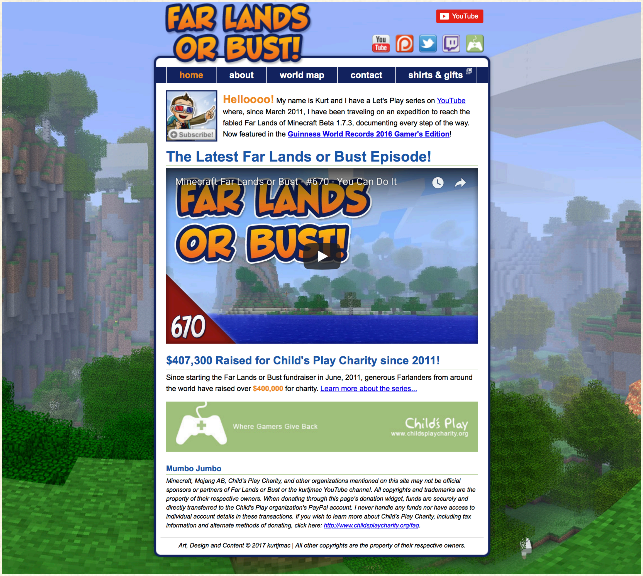
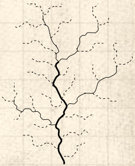
- Surprisingly:

$$R_n = \frac{(2 + R_T + T_1) + \sqrt{(2 + R_T + T_1)^2 - 8R_T}}{2}$$



Crafting landscapes—Far Lands or Bust:



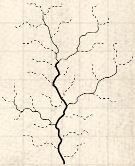


References I

[1] P. S. Dodds and D. H. Rothman.
Unified view of scaling laws for river networks.
Physical Review E, 59(5):4865–4877, 1999. pdf

[2] W. S. Glock.
The development of drainage systems: A synoptic view.
The Geographical Review, 21:475–482, 1931. pdf

[3] J. T. Hack.
Studies of longitudinal stream profiles in Virginia and Maryland.
United States Geological Survey Professional Paper, 294-B:45–97, 1957. pdf

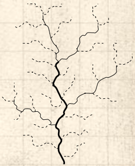


References II

[4] R. E. Horton.
Erosional development of streams and their drainage basins; hydrophysical approach to quatitative morphology.
Bulletin of the Geological Society of America, 56(3):275–370, 1945. pdf

[5] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambrigde, UK, 1997.

[6] S. A. Schumm.
Evolution of drainage systems and slopes in badlands at Perth Amboy, New Jersey.
Bulletin of the Geological Society of America, 67:597–646, 1956. pdf

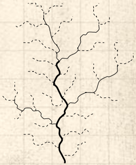


References III

[7] A. N. Strahler.
Hypsometric (area altitude) analysis of erosional topography.
Bulletin of the Geological Society of America, 63:1117–1142, 1952.

[8] E. Tokunaga.
The composition of drainage network in Toyohira River Basin and the valuation of Horton's first law.
Geophysical Bulletin of Hokkaido University, 15:1–19, 1966. pdf

[9] E. Tokunaga.
Consideration on the composition of drainage networks and their evolution.
Geographical Reports of Tokyo Metropolitan University, 13:G1–27, 1978. pdf



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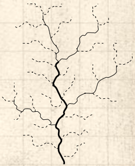

[10] E. Tokunaga.
Ordering of divide segments and law of divide segment numbers.
Transactions of the Japanese Geomorphological Union, 5(2):71–77, 1984.

[11] D. L. Turcotte, J. D. Pelletier, and W. I. Newman.
Networks with side branching in biology.
Journal of Theoretical Biology, 193:577–592, 1998.
pdf