Principles of Complex Systems, Vols. 1, 2, \& 3D


What's
CSYS/MATH 300, 303, \& 394
University of Vermont, Fall 2022
Assignment 19
Abomination Unto Nuggan

Due: Friday, February 24, by $11: 59$ pm
https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse/assignments/19/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Assistant Deliverator: Dylan Casey (contact through Teams)
Office: The Ether
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at https://www.overleaf.com/project/631238b0281a33de67fc1c2b.

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you conspired collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The (evil) Deliverator uses (evil) Matlab.

Graduate students are requested to use $\operatorname{LA}_{\mathrm{E}} \mathrm{EX}$ (or related $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ variant). If you are new to ${ }^{\Delta A} T_{E X}$, please endeavor to submit at least $n$ questions per assignment in $\Delta T_{E X}$, where $n$ is the assignment number.

## Assignment submission:

Via Blackboard.

1. Surface area of allometrically growing LoveMinecraftian organisms:

Let's consider animals as parallelepipeds (e.g., the well known box cow), with dimensions $L_{1}, L_{2}$, and $L_{3}$ and volume $V=L_{1} \times L_{2} \times L_{3}$.
Let's assume length $L_{i}$ scales with volume as $L_{i}=c_{i}^{-1} V^{\gamma_{i}}$ where the exponents satisfy $\gamma_{1}+\gamma_{2}+\gamma_{3}=1$ and the $c_{i}$ are prefactors such that $c_{1} \times c_{2} \times c_{3}=1$. Let's also arrange our organisms so that $\gamma_{1} \geq \gamma_{2} \geq \gamma_{3}$.
(a) Show that the scalings $L_{i}=c_{i}^{-1} V^{\gamma_{i}}$ mean that indeed $L_{1} \times L_{2} \times L_{3}=V$.
(b) Write down the $\gamma_{i}$ corresponding to isometric scaling.
(c) Calculate the surface area $S$ of our imaginary blockular beings for general allometric scaling of the sides.
(d) Show how $S$ behaves as $V$ becomes large (i.e., which term(s) dominate).
(e) Which sets of $\gamma_{i}$ give the fastest and slowest possible scaling of $S$ as a function of $V$ ?

Relevant tarot cards, for your consideration:

2. Lexical calculus:

Derive the word shift equation for simple additive lexical instruments.
You will have the derivation per class.
The idea is to simply work through it yourself.
There are no advanced mathematics here.
But over and over, people do not understand what's going on.
Word shifts are a kind of discrete derivative (difference) with words on the inside.
Per lectures, the goal is to derive.

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\delta h_{\mathrm{avg}, i}=\frac{100}{\left|h_{\mathrm{avg}}^{(\mathrm{comp})}-h_{\mathrm{avg}}^{(\mathrm{ref})}\right|} \underbrace{\left[h_{\mathrm{avg}}\left(w_{i}\right)-h_{\mathrm{avg}}^{(\mathrm{ref})}\right]}_{+/-} \underbrace{\left[p_{i}^{(\mathrm{comp})}-p_{i}^{(\mathrm{ref})}\right]}_{\uparrow / \downarrow}
$$

Performed in class and in numerous papers $[1,2,3]$.

## References

[1] P. S. Dodds and C. M. Danforth. Measuring the happiness of large-scale written expression: Songs, blogs, and presidents. Journal of Happiness Studies, 11(4):441-456, 2009. pdf ${ }^{\top}$
[2] P. S. Dodds, K. D. Harris, I. M. Kloumann, C. A. Bliss, and C. M. Danforth. Temporal patterns of happiness and information in a global social network:
Hedonometrics and Twitter. PLoS ONE, 6:e26752, 2011. pdf ${ }^{\square}$
[3] P. S. Dodds, E. M. Clark, S. Desu, M. R. Frank, A. J. Reagan, J. R. Williams, L. Mitchell, K. D. Harris, I. M. Kloumann, J. P. Bagrow, K. Megerdoomian, M. T. McMahon, B. F. Tivnan, and C. M. Danforth. Human language reveals a universal positivity bias. Proc. Natl. Acad. Sci., 112(8):2389-2394, 2015. Available online at http://www.pnas.org/content/112/8/2389. pdf [ ${ }^{\top}$

