PoCS What's The Story?

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 300, 303, & 394
University of Vermont, Fall 2022
Assignment 16
Don't eat anything that glows

Due: Friday, February 3, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse/assignments/16/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Assistant Deliverator: Dylan Casey (contact through Teams)
Office: The Ether
Office hours: See Teams calendar
Course website: https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse
Overleaf: LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/project/631238b0281a33de67fc1c2b.

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The (evil) Deliverator uses (evil) Matlab.

Graduate students are requested to use LaTeX (or related TeX variant). If you are new to LaTeX, please endeavor to submit at least n questions per assignment in LaTeX, where n is the assignment number.

Assignment submission:

Via Blackboard.

Please submit your project's current draft in pdf format via Blackboard by the same time specified for this assignment. For teams, please list all team member names clearly at the start.

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1. Tokunaga's law implies Horton's laws:

   In lectures, we established the following:

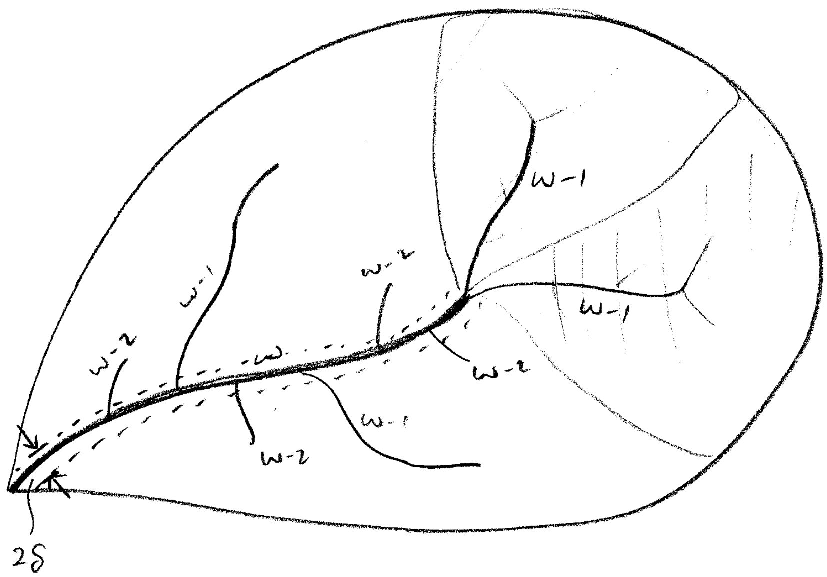
   $$n_\omega = \underbrace{2\, n_{\omega+1}}_{\text{generation}} + \sum_{\omega'=\omega+1}^{\Omega} \underbrace{T_{\omega'-\omega}\, n_{\omega'}}_{\text{absorption}}$$

   From here, derive Horton's law for stream numbers: $n_\omega / n_{\omega+1} = R_n$, where $R_n > 1$ and is independent of ω, and find R_n in terms of Tokunaga's two parameters T_1 and R_T.

2. Show $R_n = R_a$ by using Tokunaga's law to find the average area of an order ω basin, $\langle a \rangle_\omega$, in terms of the average area of basins of order 1 to $\omega - 1$.

   (In lectures, we use Horton's laws to roughly demonstrate this result.)

   Here's the set up:



   Using the Tokunaga picture, we see a basin of order ω can be broken down into non-overlapping sub-basins.

   Connect $\langle a \rangle_\omega$ to the average areas of basins of lower orders as follows:

$$\langle a \rangle_\omega = 2 \langle a \rangle_{\omega-1} + \sum_{\omega'=1}^{\omega-1} T_{\omega,\omega'} \langle a \rangle_{\omega'} + 2\delta \langle s \rangle_\omega .$$

   The first term on the right hand side corresponds to the two 'generating' streams of order $\omega - 1$. The second term (the sum) accounts for side streams entering the sole order ω stream segment in the basin. And the last term gives the contribution of 'overland flow,' i.e., flow that does not arrive in the main stream segment through a stream. The length scale δ is the typical distance from stream to ridge.

3. For river networks, basin areas are distributed according to $P(a) \propto a^{-\tau}$.

   Determine the exponent τ in terms of the Horton ratios R_n and R_s.

   Guide:

   Follow the same procedure shown in lectures for $P(\ell) \propto \ell^{-\gamma}$.

   In class, we derived $P(\ell) \propto \ell^{-\gamma}$ starting from Horton's laws (see the section of scaling relations in the slides on Branching Networks II. In doing so, we started with the following observation:

$$P_>(\ell_\omega) = \frac{N_>(\ell_\omega; \Delta)}{N_>(0; \Delta)}$$

where $N_>(\ell_\omega; \Delta)$ was the number of sites with main stream length $> \ell_\omega$.

Now, we can equally well use the right hand side to count the number of sites with drainage area exceeding a_ω. So,

$$P_>(a_\omega) \propto \left(\frac{R_n}{R_s}\right)^{-\omega} = e^{-\omega \ln(R_n/R_s)}.$$

Our task is now to wrangle the right hand side so that we see it in terms of a_ω.