P o C S What's The Story?

**Principles of Complex Systems, Vols. 1, 2, & 3D**
**CSYS/MATH 300, 303, & 394**
**University of Vermont, Fall 2022**
**Assignment 02**
**Curry with Named Meat 15p**

**Due:** Friday, September 9, by 11:59 pm
https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse/assignments/02/
*Some useful reminders:*
**Deliverator:** Prof. Peter Sheridan Dodds (contact through Teams)
**Assistant Deliverator:** Dylan Casey (contact through Teams)
**Office:** The Ether
**Office hours:** See Teams calendar
**Course website:** https://pdodds.w3.uvm.edu/teaching/courses/2022-2023pocsverse
**Overleaf:** LaTeX templates and settings for all assignments are available at
https://www.overleaf.com/project/631238b0281a33de67fc1c2b.

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All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you ~~conspired~~ collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The (evil) Deliverator uses (evil) Matlab.

Graduate students are requested to use LaTeX (or related TeX variant). If you are new to LaTeX, please endeavor to submit at least $n$ questions per assignment in LaTeX, where $n$ is the assignment number.

**Assignment submission:**

Via Blackboard.

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1. Use a back-of-an-envelope scaling argument to show that maximal rowing speed $V$ increases as the number of oarspeople $N$ as $V \propto N^{1/9}$.

   Assume the following:

   (a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner [1] shows that shell width is roughly proportional to shell length $\ell$.

Shell dimensions and performances.

| No. of oarsmen | Modifying description | Length, $l$ (m) | Beam, $b$ (m) | $l/b$ | Boat mass per oarsman (kg) | Time for 2000 m (min) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | I | II | III | IV |
| 8 | Heavyweight | 18.28 | 0.610 | 30.0 | 14.7 | 5.87 | 5.92 | 5.82 | 5.73 |
| 8 | Lightweight | 18.28 | 0.598 | 30.6 | 14.7 | | | | |
| 4 | With coxswain | 12.80 | 0.574 | 22.3 | 18.1 | | | | |
| 4 | Without coxswain | 11.75 | 0.574 | 21.0 | 18.1 | 6.33 | 6.42 | 6.48 | 6.13 |
| 2 | Double scull | 9.76 | 0.381 | 25.6 | 13.6 | | | | |
| 2 | Pair-oared shell | 9.76 | 0.356 | 27.4 | 13.6 | 6.87 | 6.92 | 6.95 | 6.77 |
| 1 | Single scull | 7.93 | 0.293 | 27.0 | 16.3 | 7.16 | 7.25 | 7.28 | 7.17 |

(b) The resistance encountered by a shell is due largely to drag on its wetted surface.

(c) Drag force is proportional to the product of the square of the shell's speed ($V^2$) and the area of the wetted surface ($\propto \ell^2$ due to shell isometry).

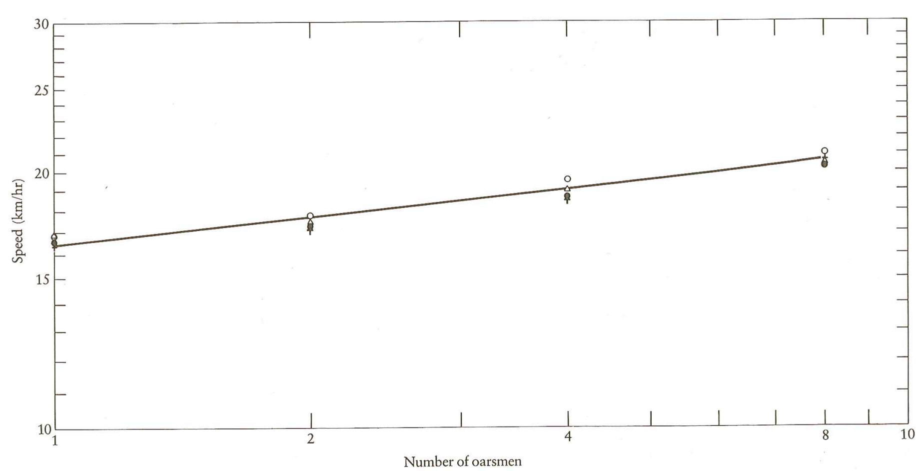
(d) Power $\propto$ drag force $\times$ speed (in symbols: $P \propto D_f \times V$).

(e) Volume displacement of water by a shell is proportional to the number of oarspeople $N$ (i.e., the team's combined weight).

(f) Assume the depth of water displacement by the shell grows isometrically with boat length $\ell$.

(g) Power is proportional to the number of oarspeople $N$.

2. Find the modern day world record times for 2000 metre races and see if this scaling still holds up. Of course, our relationship is approximate as we have neglected numerous factors, the range is extremely small (1–8 oarspeople), and the scaling is very weak (1/9). But see what you can find. The figure below shows data from McMahon and Bonner.




3. Finish the calculation for the platypus on a pendulum problem so show that a simple pendulum's period $\tau$ is indeed proportional to $\sqrt{\ell/g}$.

Basic plan from lectures: Create a matrix $A$ where $ij$th entry is the power of dimension $i$ in the $j$th variable, and solve by row reduction to find basis null vectors.

In lectures, we arrived at:

$$A\vec{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

You only have to take a few steps from here.

From Lecture 3: the Buckingham $\pi$ theorem (20 minutes).

4. Show that the maximum speed of animals $V_{\max}$ is proportional to their length $L$ [2]. Here are five dimensionful parameters:

   - $V_{\max}$, maximum speed.
   - $\ell$, animal length.
   - $\rho$, organismal density.
   - $\sigma$, maximum applied force per unit area of tissue.
   - $b$, maximum metabolic rate per unit mass ($b$ has the dimensions of power per unit mass).

   And here are the three dimensions: $L$, $M$, and $T$.

   Use a back-of-the-envelope calculation to express $V_{\max}/\ell$ in terms of $\rho$, $\sigma$, and $b$.

   Note: It's argued in [2] that these latter three parameters vary little across all organisms (we're mostly thinking about running organisms here), and so finding $V_{\max}/\ell$ as a function of them indicates that $V_{\max}/\ell$ is also roughly constant.

5. Use the Buckingham $\pi$ theorem to reproduce G. I. Taylor's finding the energy of an atom bomb $E$ is related to the density of air $\rho$ and the radius of the blast wave $R$ at time $t$:

$$E = \text{constant} \times \rho R^5/t^2. \tag{2}$$

   In constructing the matrix, order parameters as $E$, $\rho$, $R$, and $t$ and dimensions as $L$, $T$, and $M$.

6. Use the Buckingham $\pi$ theorem to derive Kepler's third law, which states that the square of the orbital period of a planet is proportional to the cube of its semi-major axis.

Let's shed some enlightenment and assume circular orbits.

Parameters:

- Planet's mass $m$;
- Sun's mass $M_{\text{sun}}$;
- Orbital period $\tau$;
- Orbital radius $r$;
- Graviational constant $G$.

(a) What are the dimensions of these five quantities?

(b) You will find that there are two dimensionless parameters using the Buckingham $\pi$ theorem, and that you can choose one to be $\pi_2 = m/M_{\text{sun}}$. Find the other dimensionless parameter, $\pi_1$.

(c) Now argue that $\tau^2 \propto r^3$.

(d) For our solar system's nine (9) planets (yes, Pluto is on the team here), plot $\tau^2$ versus $r^3$, and using basic linear regression report on how well Kepler's third law holds up.

# References


[1] T. A. McMahon and J. T. Bonner. *On Size and Life*. Scientific American Library, New York, 1983.

[2] N. Meyer-Vernet and J.-P. Rospars. How fast do living organisms move: Maximum speeds from bacteria to elephants and whales. *American Journal of Physics*, pages 719–722, 2015. pdf