### Small-world networks

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Principles of Complex Systems, Vols. 1 & 2 CSYS/MATH 300 and 303, 2021–2022 |@pocsvox

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References

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Computational Story Lab | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont





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# Outline

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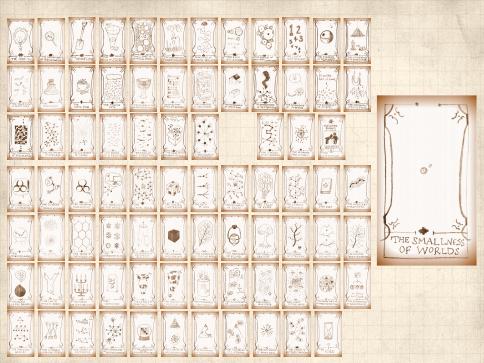
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#### References

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# Outline

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# People thinking about people: How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

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# People thinking about people: How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

#### What about the dynamics of social networks?

- How do social networks/movements begin & evolve?
- How does collective problem solving work?
- How does information move through social networks?
- Which rules give the best 'game of society?'

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# People thinking about people: How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

#### What about the dynamics of social networks?

- How do social networks/movements begin & evolve?
- How does collective problem solving work?
- How does information move through social networks?
- Which rules give the best 'game of society?'

#### Sociotechnical phenomena and algorithms:

What can people and computers do together? (google)
 Use Play + Crunch to solve problems. Which problems?

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### Social Search

### A small slice of the pie:

Q. Can people pass messages between distant individuals using only their existing social connections? The PoCSverse Small-world networks 8 of 68

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### Social Search

### A small slice of the pie:

- Q. Can people pass messages between distant individuals using only their existing social connections?
- \lambda A. Apparently yes ...

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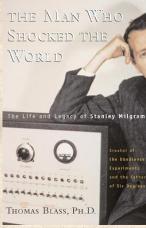
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# Milgram's social search experiment (1960s)



http://www.stanleymilgram.com

 Target person = Boston stockbroker.
 296 senders from Boston and Omaha. The PoCSverse Small-world networks 9 of 68

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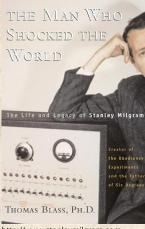
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# Milgram's social search experiment (1960s)



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- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- 🚳 chain length  $\simeq$  6.5.

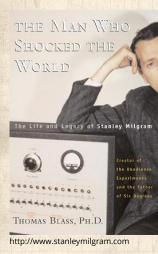
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# Milgram's social search experiment (1960s)



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Popular terms:

- The Small World Phenomenon;
- 🗞 "Six Degrees of Separation."



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### From Frigyes Karinthy's "Chain-links" C in "Everything is Different", 1929:

'A fascinating game grew out of this discussion. One of us suggested performing the following experiment to prove that the population of the Earth is closer together now than they have ever been before. We should select any person from the 1.5 billion inhabitants of the Earth-anyone, anywhere at all. He bet us that, using no more than five individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the network of personal acquaintances. For example, "Look, you know Mr. X.Y., please ask him to contact his friend Mr. Q.Z., whom he knows, and so forth."

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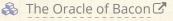
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It's a game C: "Kevin Bacon is the Center of the Universe"



### Six Degrees of Paul Erdös:



Academic papers.
Erdös Number C
Erdös Number Project C

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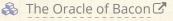
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🗞 So naturally we must have the Erdös-Bacon Number 🗹.



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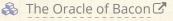
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& So naturally we must have the Erdös-Bacon Number  $\mathbb{Z}$ . & One Story Lab alum has EB#  $< \infty$ .



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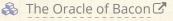
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Natalie Hershlag's (Portman's) EB# = 5 + 2 = 7.

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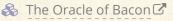
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The EBS# is also a thing: erdosbaconsabbath.com .



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# Good Will Hunting:

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Boardwork by Dan Kleitman C, EB# = 1 + 2 = 3.

See Kleitman's sidebar in Mark Saul's Movie Review (Notices of the AMS, Vol. 45, 1998.)



# You may already be a winner in NSA's "three-degrees" surveillance sweepstakes! NSA's probes could cover hundreds of millions of Americans. Thanks, Kevin Bacon.

by Sean Gallagher - July 18 2013, 4:00pm EDT





Aurich Lawson



\lambda Many people 🗹 are within three degrees from a random person ...

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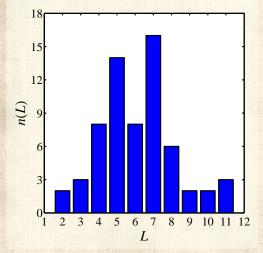
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### Lengths of successful chains:



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From Travers and Milgram (1969) in Sociometry: <sup>[9]</sup> "An Experimental Study of the Small World Problem."



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### Two features characterize a social 'Small World':



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### Two features characterize a social 'Small World':

1. Short paths exist, and



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References

### Two features characterize a social 'Small World':

1. Short paths exist,

and

2. People are good at finding them.



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References

#### Two features characterize a social 'Small World':

- 1. Short paths exist, (= Geometric piece) and
- 2. People are good at finding them.



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References

### Two features characterize a social 'Small World':

- 1. Short paths exist, (= Geometric piece) and
- People are good at finding them. (= Algorithmic piece)



# Social Search

#### Milgram's small world experiment with email:



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"An Experimental study of Search in Global Social Networks" Dodds, Muhamad, and Watts, Science, **301**, 827–829, 2003. <sup>[4]</sup>

60,000+ participants in 166 countries

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60,000+ participants in 166 countries 18 targets in 13 countries including

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left for the second sec 18 targets in 13 countries including a professor at an Ivy League university, The PoCSverse Small-world networks 17 of 68

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60,000+ participants in 166 countries
 18 targets in 13 countries including
 a professor at an Ivy League university,
 an archival inspector in Estonia,

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60,000+ participants in 166 countries
 18 targets in 13 countries including
 a professor at an Ivy League university,
 an archival inspector in Estonia,
 a technology consultant in India,

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- 60,000+ participants in 166 countries
- 🗞 18 targets in 13 countries including
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  - a veterinarian in the Norwegian army.

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  - a policeman in Australia, and
  - a veterinarian in the Norwegian army.
- 🗞 24,000+ chains

We were lucky and contagious (more later):

"Using E-Mail to Count Connections" C, Sarah Milstein, New York Times, Circuits Section (December, 2001) The PoCSverse Small-world networks 17 of 68

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# All targets:

#### Table S1

Target	City	Country	Occupation	Gender	N	$N_c(\%)$	r (r <sub>0</sub> )	<l></l>
1	Novosibirsk	Russia	PhD student	F	8234	20(0.24)	64 (76)	4.05
2	New York	USA	Writer	F	6044	31 (0.51)	65 (73)	3.61
3	Bandung	Indonesia	Unemployed	М	8151	0	66 (76)	n/a
4	New York	USA	Journalist	F	5690	44 (0.77)	60 (72)	3.9
5	Ithaca	USA	Professor	М	5855	168 (2.87)	54 (71)	3.84
6	Melbourne	Australia	Travel Consultant	F	5597	20 (0.36)	60 (71)	5.2
7	Bardufoss	Norway	Army veterinarian	М	4343	16 (0.37)	63 (76)	4.25
8	Perth	Australia	Police Officer	М	4485	4 (0.09)	64 (75)	4.5
9	Omaha	USA	Life Insurance	F	4562	2 (0.04)	66 (79)	4.5
			Agent					
10	Welwyn Garden City	UK	Retired	М	6593	1 (0.02)	68 (74)	4
11	Paris	France	Librarian	F	4198	3 (0.07)	65 (75)	5
12	Tallinn	Estonia	Archival Inspector	М	4530	8 (0.18)	63(79)	4
13	Munich	Germany	Journalist	М	4350	32 (0.74)	62 (74)	4.66
14	Split	Croatia	Student	М	6629	0	63 (77)	n/a
15	Gurgaon	India	Technology	М	4510	12 (0.27)	67 (78)	3.67
			Consultant					
16	Managua	Nicaragua	Computer analyst	М	6547	2 (0.03)	68 (78)	5.5
17	Katikati	New Zealand	Potter	М	4091	12 (0.3)	62 (74)	4.33
18	Elderton	USA	Lutheran Pastor	М	4438	9 (0.21)	68 (76)	4.33
Totals			an and she had		98,847	384 (0.4)	63 (75)	4.05

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### Milgram's participation rate was roughly 75%

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Milgram's participation rate was roughly 75%
 Email version: Approximately 37% participation rate.

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- Milgram's participation rate was roughly 75%
   Email version: Approximately 37% participation rate.
- Probability of a chain of length 10 getting through:

 $.37^{10}\simeq 5\times 10^{-5}$ 



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References

- Milgram's participation rate was roughly 75%
   Email version: Approximately 37% participation rate.
- Probability of a chain of length 10 getting through:

 $.37^{10}\simeq 5\times 10^{-5}$ 

 $\Rightarrow$  384 completed chains (1.6% of all chains).

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Motivation/Incentives/Perception matter.
 If target *seems* reachable
 participation more likely.



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References

local Motivation/Incentives/Perception matter.

- ♣ If target seems reachable ⇒ participation more likely.
- Small changes in attrition rates ⇒ large changes in completion rates



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References

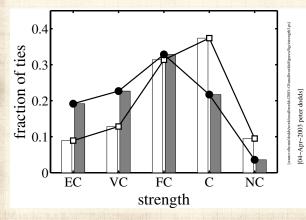
location/Incentives/Perception matter.

- ♣ If target seems reachable ⇒ participation more likely.
- Small changes in attrition rates
   ⇒ large changes in completion rates
- $\mathfrak{F}$  e.g.,  $\searrow$  15% in attrition rate
  - $\Rightarrow$   $\nearrow$  800% in completion rate



# Comparing successful to unsuccessful chains:

🚳 Successful chains used relatively weaker ties:



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## Successful chains disproportionately used:



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## Successful chains disproportionately used:

🚳 Weak ties, Granovetter [5]

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## Successful chains disproportionately used:

- 🚳 Weak ties, Granovetter [5]
- Professional ties (34% vs. 13%)

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## Successful chains disproportionately used:

- 🚳 Weak ties, Granovetter [5]
- Professional ties (34% vs. 13%)
- Ties originating at work/college



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## Successful chains disproportionately used:

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- 🗞 Target's work (65% vs. 40%)

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- 🗞 Target's work (65% vs. 40%)

...and disproportionately avoided



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## ...and disproportionately avoided

\lambda hubs (8% vs. 1%) (+ no evidence of funnels)

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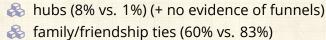
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- Ties originating at work/college
- 🗞 Target's work (65% vs. 40%)

### ...and disproportionately avoided

hubs (8% vs. 1%) (+ no evidence of funnels)
 family/friendship ties (60% vs. 83%)

 $\textbf{Geography} \rightarrow \textbf{Work}$ 

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Senders of successful messages showed little absolute dependency on



🚳 age, gender

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Senders of successful messages showed little absolute dependency on



\delta age, gender

🚳 country of residence

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Senders of successful messages showed little absolute dependency on

- 🗞 age, gender
- 🚳 country of residence
- 🚳 income

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Senders of successful messages showed little absolute dependency on

- 🚳 age, gender
- 🚳 country of residence
- 🗞 income
- 🚳 religion



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- 🚳 religion
- 🚳 relationship to recipient



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Senders of successful messages showed little absolute dependency on

- 🚳 age, gender
- 🚳 country of residence
- 🗞 income
- 🚳 religion
- 🚳 relationship to recipient

Range of completion rates for subpopulations:

30% to 40%



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### Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain." The PoCSverse Small-world networks 24 of 68

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## Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

### Why:

🚳 Specificity important

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## Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

### Why:

- 🚳 Specificity important
- Successful links used relevant information. (e.g. connecting to someone who shares same profession as target.)

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### Basic results:

 $\langle L \rangle = 4.05$  for all completed chains

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### Basic results:

- $\langle L \rangle = 4.05$  for all completed chains
- L<sub>\*</sub> = Estimated 'true' median chain length (zero attrition)

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## Basic results:

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lntra-country chains:  $L_* = 5$ 

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lnter-country chains:  $L_* = 7$ 

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- lntra-country chains:  $L_* = 5$
- lnter-country chains:  $L_* = 7$
- All chains:  $L_* = 7$
- $\clubsuit$  Milgram:  $L_* \simeq 9$

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### Harnessing social search:

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#### Harnessing social search:

Can distributed social search be used for something big/good? The PoCSverse Small-world networks 26 of 68

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#### Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)

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#### Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)

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- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
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- For real social search, we have an incentives problem.

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#### Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?

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#### Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?
- 🗞 Fun, money, prestige, ...?

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#### Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?
- 🗞 Fun, money, prestige, ...?
- 🚳 Must be 'non-gameable.'

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### A Grand Challenge:

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### A Grand Challenge:

#### 1969: The Internet is born (the ARPANET —four nodes!).

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#### A Grand Challenge:

- 1969: The Internet is born (the ARPANET —four nodes!).
- Originally funded by DARPA who created a grand Network Challenge for the 40th anniversary.

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References



\*DARPA = Defense Advanced Research Projects Agency .

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References

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- Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.

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- Challenge: Find the latitude and longitude of each balloon.
- 🗞 Prize: \$40,000.

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### Where the balloons were:



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🙈 MIT's Media Lab 🗹 won in less than 9 hours. [7]

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- Recursive incentive structure with exponentially decaying payout:
  - \$2000 for correctly reporting the coordinates of a balloon.

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  - ♥ \$1000 for recruiting a person who finds a balloon.

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  - \$500 for recruiting a person who recruits the balloon finder, ...

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  - 🝞 (Not a Ponzi scheme.)

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  - (Not a Ponzi scheme.)

True victory: Colbert interviews Riley Crane 🗹

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### Clever scheme:



🚳 Max payout = \$4000 per balloon.

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#### Clever scheme:

- 🚳 Max payout = \$4000 per balloon.
- lndividuals have clear incentives to both
  - 1. involve/source more people (spread), and
  - 2. find balloons (goal action).



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#### Extra notes:

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- Limit to how much money a set of bad actors can extract.

#### Extra notes:

- 🚳 MIT's brand helped greatly.
- MIT group first heard about the competition a few days before. Ouch.
- A number of other teams did well C.
   Worthwhile looking at these competing
  - Worthwhile looking at these competing strategies.<sup>[7]</sup>

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#### **Collective Detective:**



#### 🚳 Finding an errant panda 🗹

Once again, social media proved to be a powerful dragnet. Around 1:15 p.m., a Washingtonian posted a picture on Twitter of Rusty in a patch of weeds in the Adams Morgan district, not far from the 163-acre zoo, which was created in 1889 by an act of Congress. "Red panda in our neighborhood," wrote Ashley Foughty, who identified herself as a singer, actress and traveler. "Please come save him!"

Another neighbor posted a photograph of two zoo workers, one in safari shorts standing on a rooftop, one holding a giant butterfly net. Soon the zoo announced: "Rusty the red panda has been recovered, crated & is headed safely back to the National Zoo!"

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#### Collective Detective:



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Nature News: "Crowdsourcing in manhunts can work: Despite mistakes over the Boston bombers, social media can help to find people quickly" C by Philip Ball (April 26, 2013)

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#### Collective Detective:



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🚳 Motherboard, Vice: One Degree of Separation in the Forever War 🖸 by Brian Castner (November 11, 2015)

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## Outline

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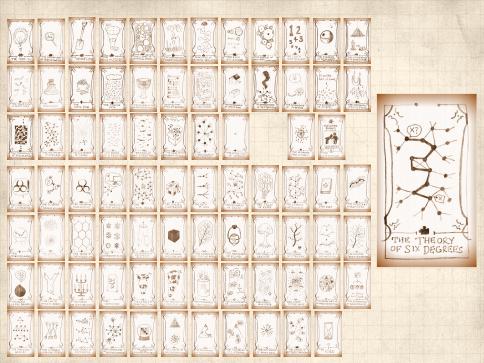
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The social world appears to be small ...why?

# Theory: how do we understand the small world property?

Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

N = population size,

 $d_{AB}$  = distance between nodes A and B.

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The social world appears to be small ...why?

# Theory: how do we understand the small world property?

Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

- N = population size,
- $d_{AB}$  = distance between nodes A and B.
- 🚳 But: social networks aren't random ...

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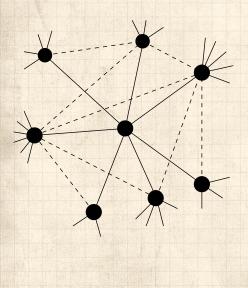
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## Simple socialness in a network:



Need "clustering" (your friends are likely to know each other): The PoCSverse Small-world networks 35 of 68

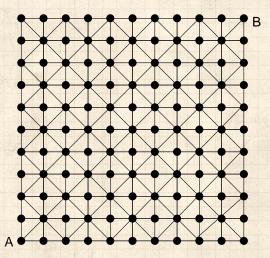
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## Non-randomness gives clustering:



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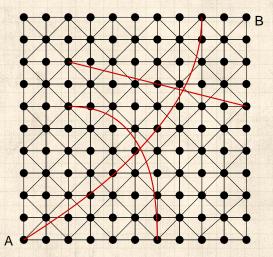
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 $d_{AB} = 10 \rightarrow$  too many long paths.

## Randomness + regularity



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Now have  $d_{AB} = 3$ 

 $\langle d \rangle$  decreases overall

Introduced by Watts and Strogatz (Nature, 1998)<sup>[11]</sup> "Collective dynamics of 'small-world' networks."



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Small-world networks were found everywhere:

🚳 neural network of C. elegans,

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 semantic networks of languages,
 actor collaboration graph,

🚳 food webs,

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Very weak requirements: local regularity The PoCSverse Small-world networks 38 of 68

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- 🚳 actor collaboration graph,
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- 🚳 social networks of comic book characters, ...

Very weak requirements:

local regularity + random short cuts

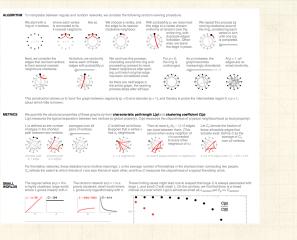
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#### Papers should be apps:



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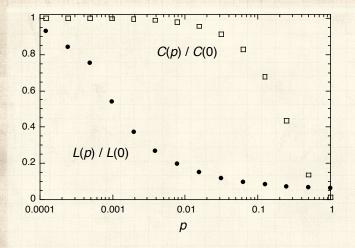
References

The Theory of Str. Decases



Interactive figures and tables = windows into large data sets (empirical or simulated).

#### The structural small-world property:



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L(p) = average shortest path length as a function of p
 C(p) = average clustring as a function of p

But are these short cuts findable?

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But are these short cuts findable?

Nope.<sup>[6]</sup>

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But are these short cuts findable?

Nope.<sup>[6]</sup>

Nodes cannot find each other quickly with any local search method.

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But are these short cuts findable?

Nope.<sup>[6]</sup>

Nodes cannot find each other quickly with any local search method.

Need a more sophisticated model ...

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#### 🚳 What can a local search method reasonably use?



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What can a local search method reasonably use?How to find things without a map?

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What can a local search method reasonably use?
How to find things without a map?
Need some measure of distance between friends and the target.

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- What can a local search method reasonably use?
  How to find things without a map?
- Need some measure of distance between friends and the target.

#### Some possible knowledge:

- Target's identity
   Friends' popularity
- 🚳 Friends' identities
- 🚳 Where message has been



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Jon Kleinberg (Nature, 2000)<sup>[6]</sup> "Navigation in a small world." The PoCSverse Small-world networks 43 of 68

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Jon Kleinberg (Nature, 2000)<sup>[6]</sup> "Navigation in a small world."

#### Allowed to vary:

1. local search algorithm

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Jon Kleinberg (Nature, 2000)<sup>[6]</sup> "Navigation in a small world."

#### Allowed to vary:

- 1. local search algorithm and
- 2. network structure.

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#### Kleinberg's Network:

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#### Kleinberg's Network:

1. Start with regular d-dimensional cubic lattice.



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#### Kleinberg's Network:

- 1. Start with regular d-dimensional cubic lattice.
- 2. Add local links so nodes know all nodes within a distance *q*.

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#### Kleinberg's Network:

- 1. Start with regular d-dimensional cubic lattice.
- 2. Add local links so nodes know all nodes within a distance *q*.
- 3. Add m short cuts per node.

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#### Kleinberg's Network:

- 1. Start with regular d-dimensional cubic lattice.
- 2. Add local links so nodes know all nodes within a distance *q*.
- 3. Add m short cuts per node.
- 4. Connect i to j with probability

$$p_{ij} \propto {x_{ij}}^{-\alpha}$$

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#### Kleinberg's Network:

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$$p_{ij} \propto {x_{ij}}^{-\alpha}$$

 $\begin{array}{l} \displaystyle \textcircled{\begin{subarray}{ll} \hline \& & \alpha = 0 \\ \displaystyle \textcircled{\begin{subarray}{ll} \hline \& & \alpha \end{array}} & \alpha \mbox{ large: reinforce local connections.} \\ \displaystyle \textcircled{\begin{subarray}{ll} \hline \& & \alpha = d \\ \displaystyle \end{matrix}} & \mbox{ connections grow logarithmically in space.} \end{array}$ 

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#### Theoretical optimal search:

🚳 "Greedy" algorithm.

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#### Theoretical optimal search:

- 🚳 "Greedy" algorithm.
- Solution Number of connections grow logarithmically (slowly) in space:  $\alpha = d$ .

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#### Theoretical optimal search:

- 🚳 "Greedy" algorithm.
- Solution Number of connections grow logarithmically (slowly) in space:  $\alpha = d$ .
- 🚳 Social golf.

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#### Theoretical optimal search:

- 🚳 "Greedy" algorithm.
- Solution Number of connections grow logarithmically (slowly) in space:  $\alpha = d$ .
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Search time grows slowly with system size (like  $\log^2 N$ ).



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#### Theoretical optimal search:

- 🚳 "Greedy" algorithm.
- Number of connections grow logarithmically (slowly) in space:  $\alpha = d$ .
- 🚳 Social golf.

Search time grows slowly with system size (like  $\log^2 N$ ).

But: social networks aren't lattices plus links.



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If networks have hubs can also search well: Adamic et al. (2001)<sup>[1]</sup>

 $P(k_i) \propto k_i^{-\gamma}$ 

#### where k = degree of node i (number of friends).

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Nutshell



If networks have hubs can also search well: Adamic et al. (2001)<sup>[1]</sup>

 $P(k_i) \propto k_i^{-\gamma}$ 

where k = degree of node i (number of friends).
Basic idea: get to hubs first (airline networks). The PoCSverse Small-world networks 46 of 68

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If networks have hubs can also search well: Adamic et al. (2001)<sup>[1]</sup>

 $P(k_i) \propto k_i^{-\gamma}$ 

where k = degree of node i (number of friends).
Basic idea: get to hubs first (airline networks).
But: hubs in social networks are limited.

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## Outline

## Small-world networks

## Generalized affiliation networks

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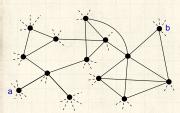
Theory

Generalized affiliation networks Nutshell



## The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of a is closest to the target b?

What does 'closest' mean?

What is 'social distance'?



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One approach: incorporate identity.

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One approach: incorporate identity.

Identity is formed from attributes such as:

- 🚳 Geographic location
- 🚳 Type of employment
- 🗞 Religious beliefs
- 🚳 Recreational activities.

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Generalized affiliation networks Nutshell



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Generalized affiliation networks Nutshell

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- 🚳 Geographic location
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Attributes  $\Leftrightarrow$  Contexts  $\Leftrightarrow$  Interactions  $\Leftrightarrow$  Networks.

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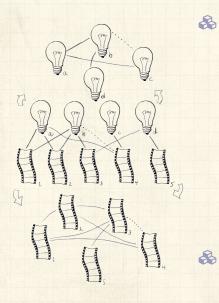
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Generalized affiliation networks Nutshell



#### Bipartite affiliation structures:



Many real-world networks have an underlying multi-partite structure.

- Stories-tropes.
- Boards and directors.
- Films-actorsdirectors.
- Classes-teachersstudents.
- Upstairsdownstairs.

Unipartite networks may be induced or co-exist. The PoCSverse Small-world networks 50 of 68

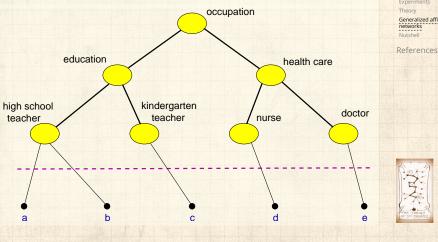
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## Social distance—Context distance



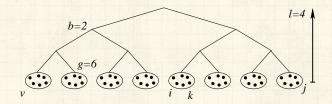
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# Distance between two individuals $x_{ij}$ is the height of lowest common ancestor.



$$x_{ij} = 3$$
,  $x_{ik} = 1$ ,  $x_{iv} = 4$ .

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Individuals are more likely to know each other the closer they are within a hierarchy.



Individuals are more likely to know each other the closer they are within a hierarchy.
 Construct *z* connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$

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Individuals are more likely to know each other the closer they are within a hierarchy.
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 $\alpha = 0$ : random connections.



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Individuals are more likely to know each other the closer they are within a hierarchy.
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 $\begin{array}{l} \displaystyle \textcircled{\&} \ \ \alpha = 0: \mbox{ random connections.} \\ \displaystyle \begin{array}{l} \displaystyle \textcircled{\&} \ \ \alpha \mbox{ large: local connections.} \end{array} \end{array}$ 



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#### geography occupation age 0 100 а e \lambda Blau & Schwartz<sup>[2]</sup>, Simmel<sup>[8]</sup>, Breiger<sup>[3]</sup>, Watts *et* al. <sup>[10]</sup>; see also Google+ Circles.

#### Generalized affiliation networks

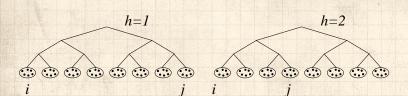


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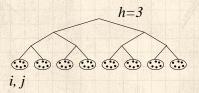
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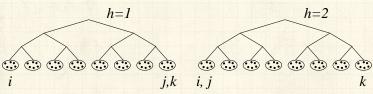


$$\begin{split} \vec{v}_i &= [1 \; 1 \; 1]^T, \vec{v}_j = [8 \; 4 \; 1]^T \\ x^1_{ij} &= 4, \; x^2_{ij} = 3, \; x^3_{ij} = 1. \end{split}$$

Social distance:  
$$y_{ij} = \min_h x_{ij}^h$$
.



#### Triangle inequality doesn't hold:



 $y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$ 



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#### lndividuals know the identity vectors of

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#### lndividuals know the identity vectors of 1. themselves,

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#### lndividuals know the identity vectors of

- 1. themselves,
- 2. their friends,

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#### lndividuals know the identity vectors of

- 1. themselves,
- 2. their friends, and
- 3. the target.

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#### lndividuals know the identity vectors of

- 1. themselves,
- 2. their friends,
  - and
- 3. the target.
- Individuals can estimate the social distance between their friends and the target.

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#### lndividuals know the identity vectors of

- 1. themselves,
- 2. their friends,
- and a the targe
- 3. the target.
- Individuals can estimate the social distance between their friends and the target.
- Use a greedy algorithm + allow searches to fail randomly.

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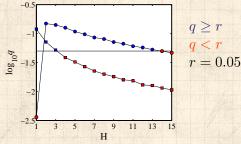
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#### The model-results—searchable networks

 $\alpha = 0$  versus  $\alpha = 2$  for  $N \simeq 10^5$ :



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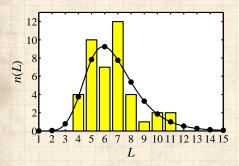
*q* = probability an arbitrary message chain reaches a target.

- 🚳 A few dimensions help.
- 🗞 Searchability decreases as population increases.
- Precise form of hierarchy largely doesn't matter.



#### The model-results

#### Milgram's Nebraska-Boston data:



Model parameters:  $N = 10^8$ , z = 300, g = 100, b = 10,  $\alpha = 1, H = 2$ ;

 $\left. \begin{array}{l} \bigotimes \ \left< L_{\rm model} \right> \simeq 6.7 \\ \bigotimes \ \ L_{\rm data} \simeq 6.5 \end{array} \right.$ 



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#### Social search—Data

#### Adamic and Adar (2003)

For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.



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#### Social search—Data

#### Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Solution Probability of connection as function of real distance  $\propto 1/r$ .

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### Social Search—Real world uses

Tags create identities for objects

- 🚳 Website tagging: bitly.com 🗹
- 🚳 (e.g., Wikipedia)
- 🚳 Photo tagging: flickr.com
- Dynamic creation of metadata plus links between information objects.
- 🗞 Folksonomy: collaborative creation of metadata



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#### Social Search—Real world uses

#### Recommender systems:

Amazon uses people's actions to build effective connections between books.

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# Social Search—Real world uses

#### Recommender systems:

- Amazon uses people's actions to build effective connections between books.
- Source of the hoi polloi.

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# Outline

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Small-world networks Experiments Theory Generalized affiliation networks

Nutshell



🚳 Bare networks are typically unsearchable.

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Nutshell



Bare networks are typically unsearchable.
 Paths are findable if nodes understand how network is formed.

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Nutshell



- 🚳 Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- lmportance of identity (interaction contexts).

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Nutshell



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Nutshell



- 🚳 Bare networks are typically unsearchable.
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- 🚳 Construction of peer-to-peer networks.

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Nutshell



- 🚳 Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- 🚳 Importance of identity (interaction contexts).
- 🚳 Improved social network models.
- 🗞 Construction of peer-to-peer networks.
- Construction of searchable information databases.

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Nutshell



# Neural reboot (NR):

Food-induced happiness

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Nutshell

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https://www.youtube.com/watch?v=vC8gJ0\_9o4M?rel=0

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