Contagion

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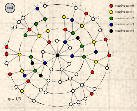
Principles of Complex Systems, Vols. 1 & 2
CSYS/MATH 300 and 303, 2021–2022 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



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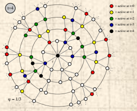

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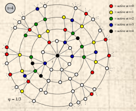
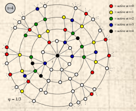


Outline

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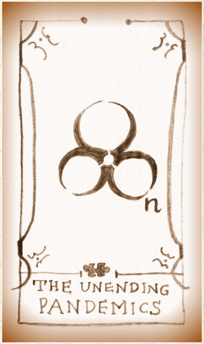
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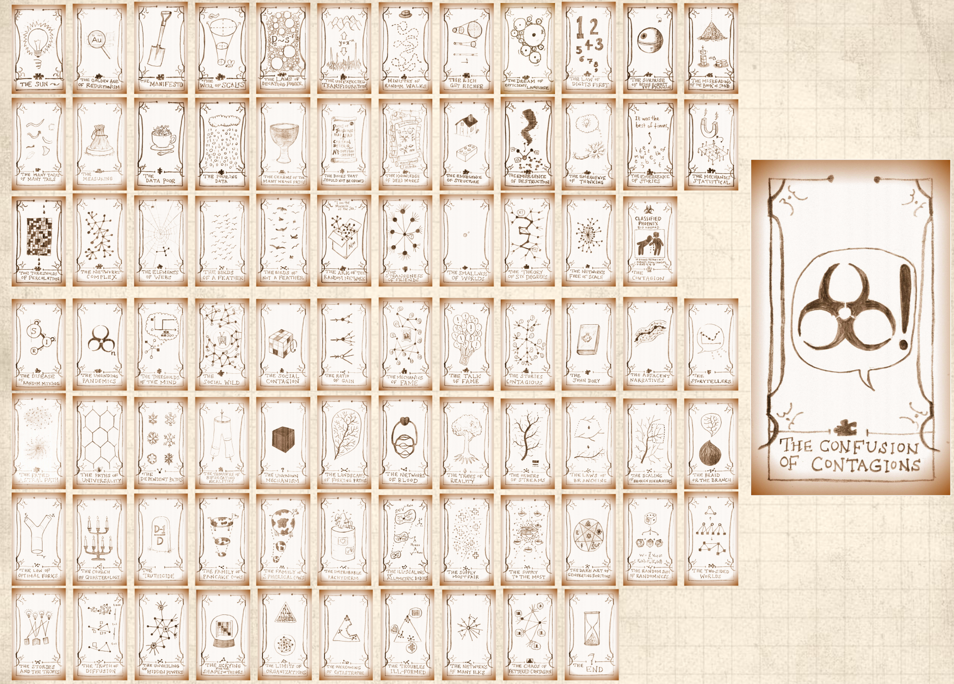
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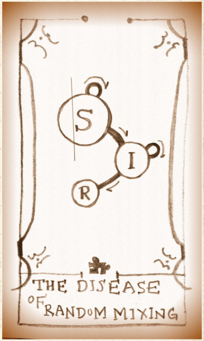
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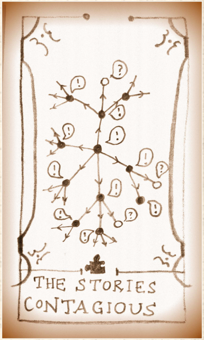
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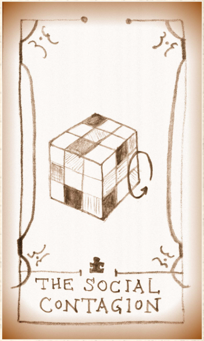
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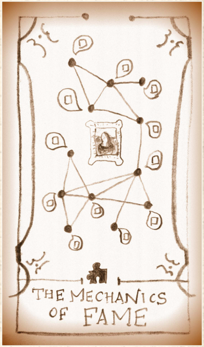
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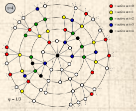
Contagion models

Some large questions concerning network contagion:



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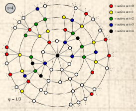

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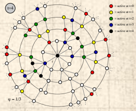

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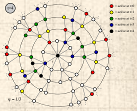

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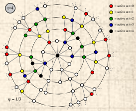

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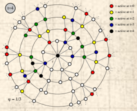

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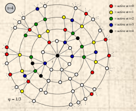
Next up: We'll look at some fundamental kinds of spreading on generalized random networks.



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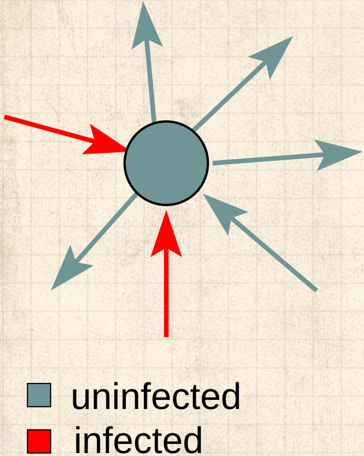
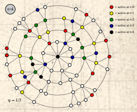


- General spreading mechanism: State of node i depends on history of i and i's neighbors' states.



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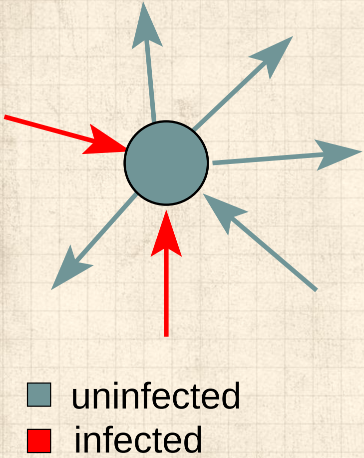

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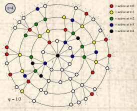
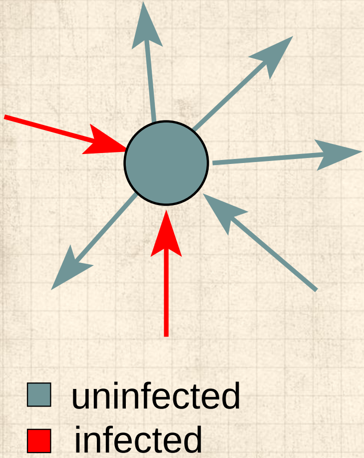
- Doses of entity may be stochastic and history-dependent.



Spreading mechanisms



- General spreading mechanism: State of node i depends on history of i and i's neighbors' states.
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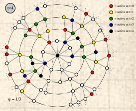

Spreading on Random Networks

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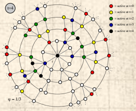

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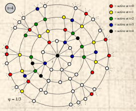

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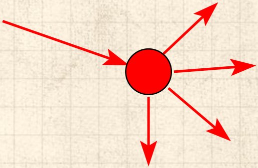
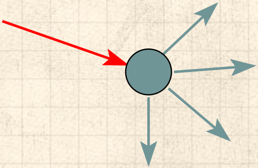
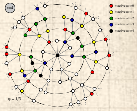


Failure:





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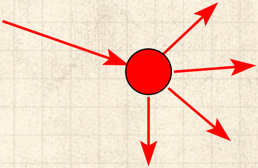

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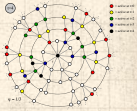

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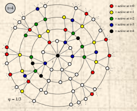

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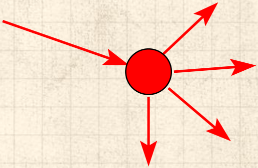

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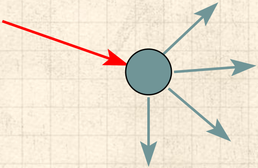
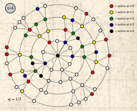


- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?

Global spreading condition

- We need to find: [5]
  R = the average # of infected edges that one random infected edge brings about.
- Call **R** the gain ratio.



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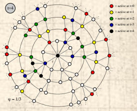

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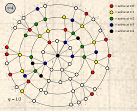

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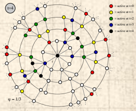

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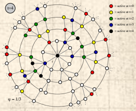

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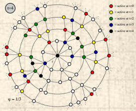

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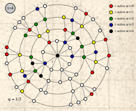

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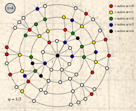

Global spreading condition

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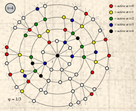

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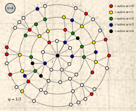

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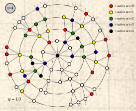

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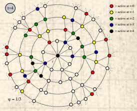

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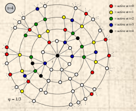
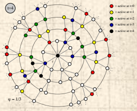
- Good: This is just our giant component condition again.



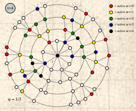
Global spreading condition

- Case 2:



Global spreading condition

- **Case 2:** If $B_{k1} = \beta < 1$



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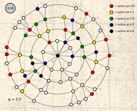

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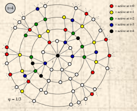

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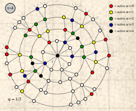

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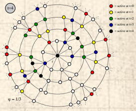

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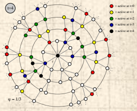
$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

Insert question from assignment 9



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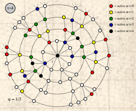
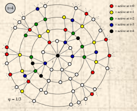
Insert question from assignment 9

- We can show $F_{\tilde{P}}(x) = F_P(\beta x + 1 - \beta)$.



Global spreading condition

- Cases 3, 4, 5, ...:



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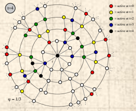

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- Cases 3, 4, 5, ...: Now allow B_{k1} to depend on k
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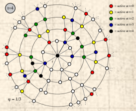

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- Cases 3, 4, 5, ...: Now allow B_{k1} to depend on k
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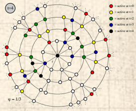
- Possibility: B_{k1} increases with k... .



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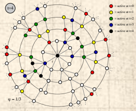

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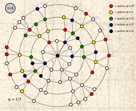

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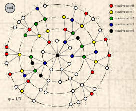

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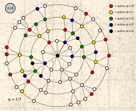

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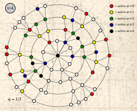

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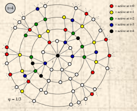
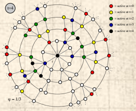
- The story:
  More well connected people are harder to influence.



Global spreading condition

- Example: $B_{k1} = 1/k$.



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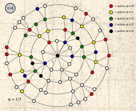

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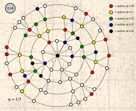

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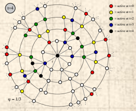

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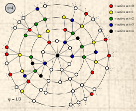

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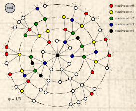

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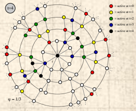

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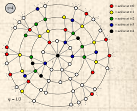

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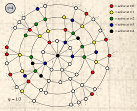
- Decay of B_{k1} is too fast.
- Result is independent of degree distribution.



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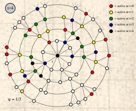
where $0 < \phi \leq 1$ is a threshold and H is the Heaviside function.



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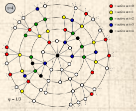

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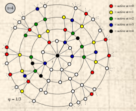

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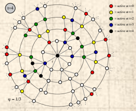

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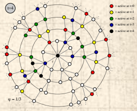

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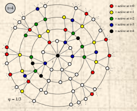
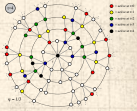


Global spreading condition

- The uniform threshold model global spreading condition:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} (k-1) \bullet \frac{kP_k}{\langle k \rangle} > 1.$$



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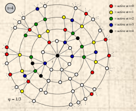

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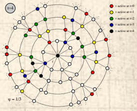

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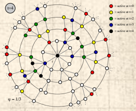
- Key: If we fix ϕ and then vary $\langle k \rangle$, we may see two phase transitions.



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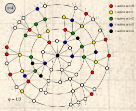
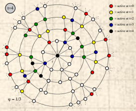
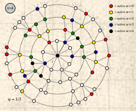
- Added to our standard giant component transition, we will see a cut off in spreading as nodes become more connected.



Virtual contagion: Corrupted Blood, a 2005 virtual plague in World of Warcraft:



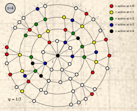
Outline



Social Contagion

Some important models (recap from CSYS 300)

- Tipping models—Schelling (1971) [11, 12, 13]



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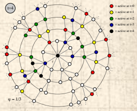

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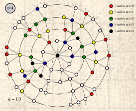

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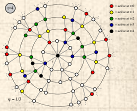

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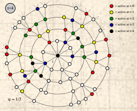

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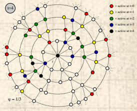
  - Social learning theory, Informational cascades,...



Threshold model on a network

Original work:


A simple model of global cascades on random networks
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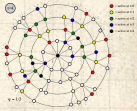

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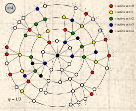
- Mean field Granovetter model → network model



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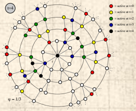
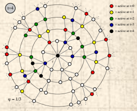
- Mean field Granovetter model $\rightarrow$ network model
- Individuals now have a limited view of the world



Threshold model on a network

- Interactions between individuals now represented by a network



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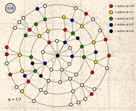

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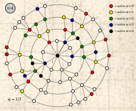
- Individual i has k_i contacts



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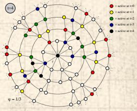

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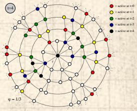

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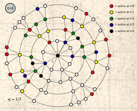

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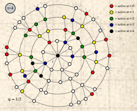

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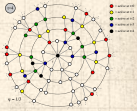

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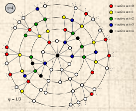
- Activation is permanent (SI)



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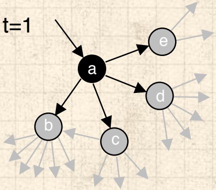
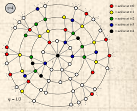


- All nodes have threshold $\phi = 0.2$.



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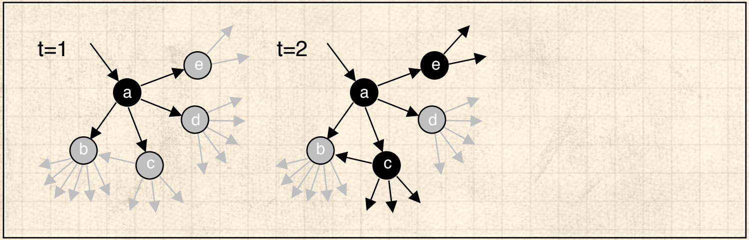
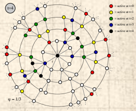


- All nodes have threshold $\phi = 0.2$.



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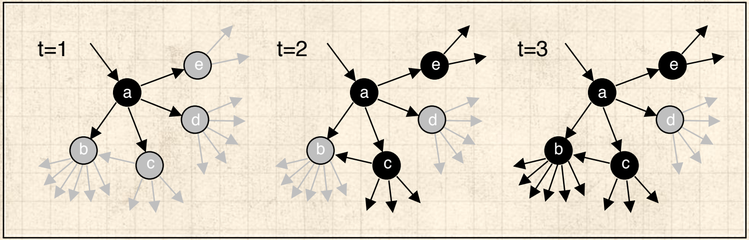

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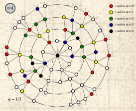
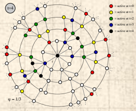


The most gullible

Vulnerables:



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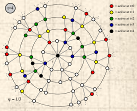

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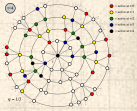
- The vulnerability condition for node i: $1/k_i \geq \phi_i$.



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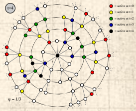

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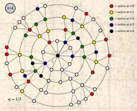

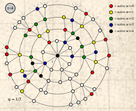

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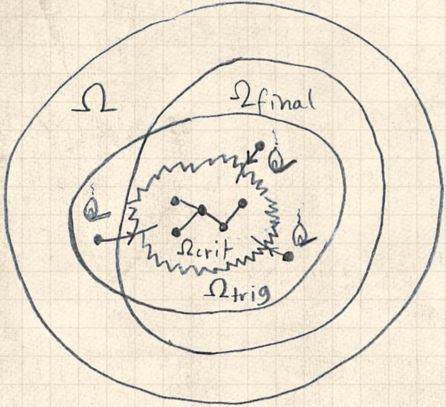
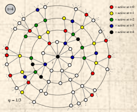
- Key: For global spreading events (cascades) on random networks, must have a *global component of vulnerables* [15]
- For a uniform threshold ϕ, our global spreading condition tells us when such a component exists:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} \frac{k P_k}{\langle k \rangle} \bullet (k-1) > 1.$$

Example random network structure:

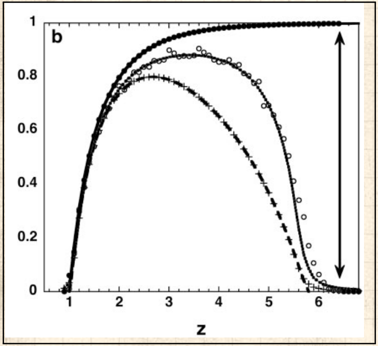


- Ω_{crit} = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network



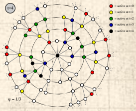
$\Omega_{\text{crit}} \subset \Omega_{\text{trig}}$; $\Omega_{\text{crit}} \subset \Omega_{\text{final}}$; and $\Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega$.

Global spreading events on random networks [15]



$z = \langle k \rangle$

- Top curve: final fraction infected if successful.



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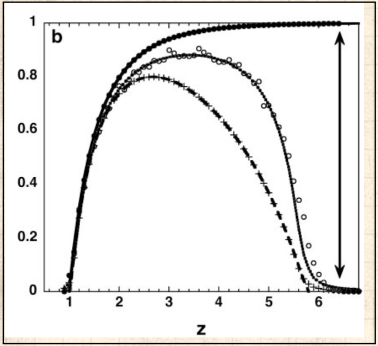

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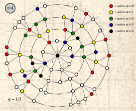

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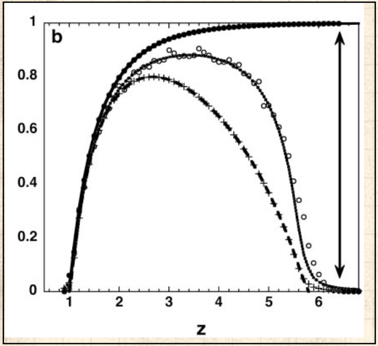

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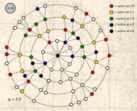

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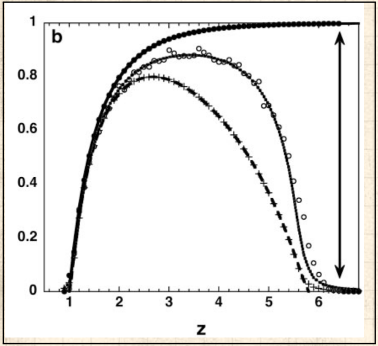

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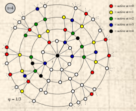
- Global spreading events occur only if size of vulnerable subcomponent > 0.



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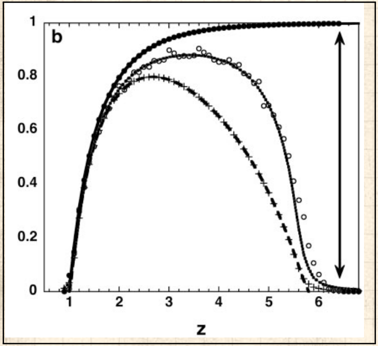

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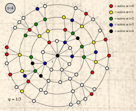
- System is robust-yet-fragile just below upper boundary [3, 4, 14]



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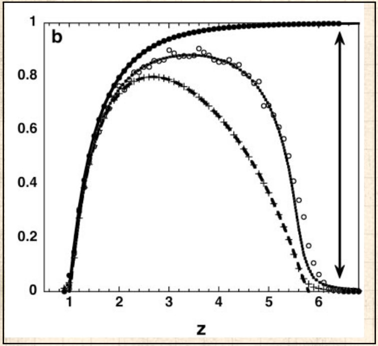

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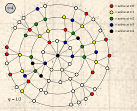

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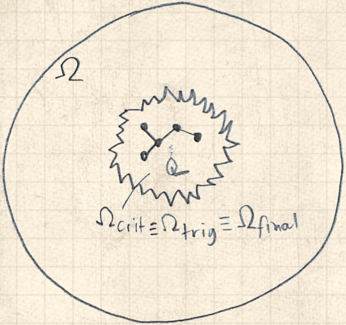
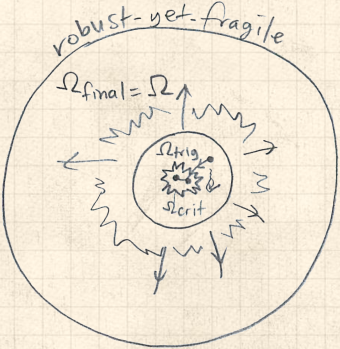
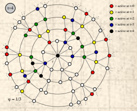
- System is robust-yet-fragile just below upper boundary [3, 4, 14]
- ‘Ignorance’ facilitates spreading.

Cascades on random networks





- Above lower phase transition
- Just below upper phase transition



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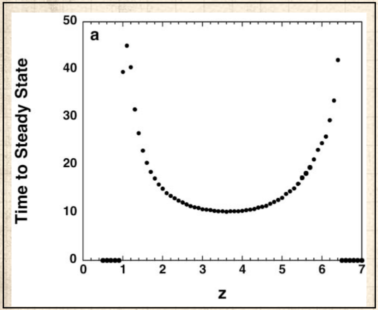
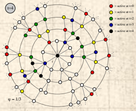


(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]



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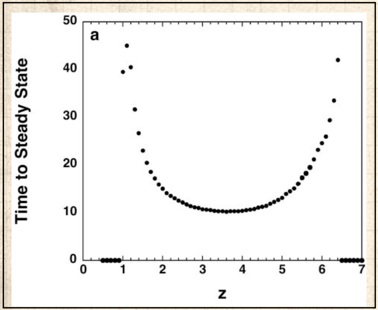

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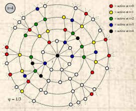
- Time taken for cascade to spread through network. [15]
- Two phase transitions.



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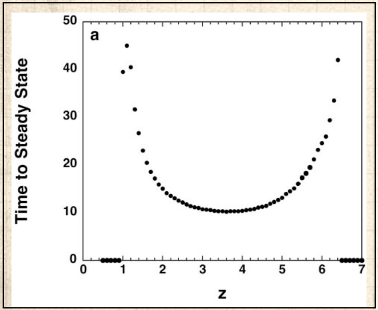

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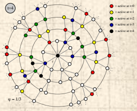
- Largest vulnerable component = critical mass.



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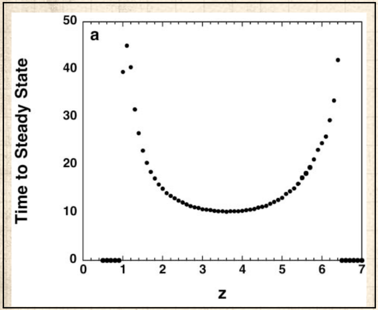
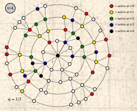


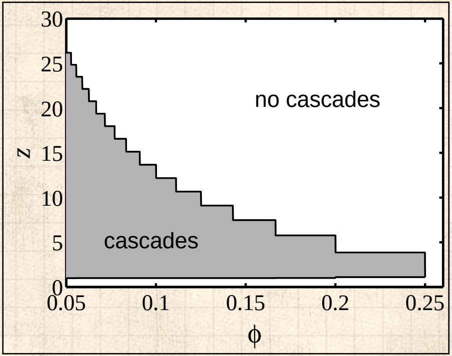
(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]
- Two phase transitions.

- Largest vulnerable component = critical mass.
- Now have endogenous mechanism for spreading from an individual to the critical mass and then beyond.

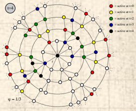


Cascade window for random networks



(n.b., $z = \langle k \rangle$)

- Outline of cascade window for random networks.



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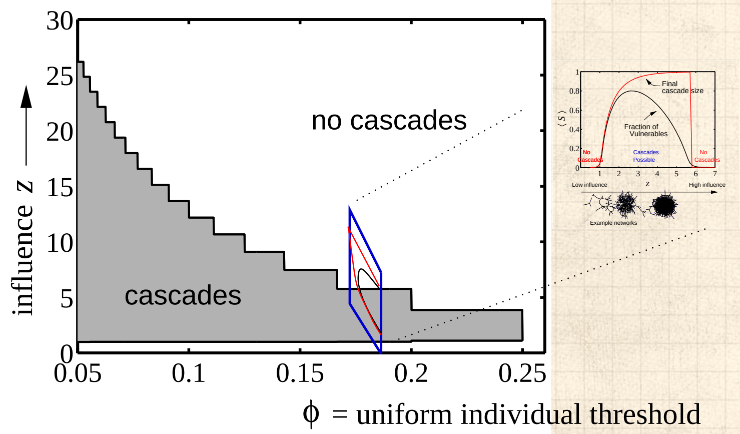
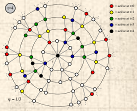
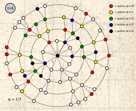
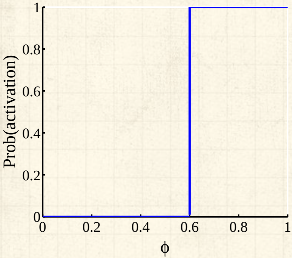




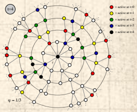
Outline



Granovetter's Threshold model—recap



- Assumes deterministic response functions



Social Contagion

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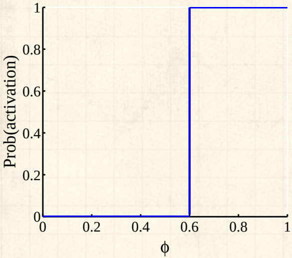
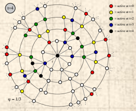
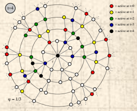


- Assumes deterministic response functions
- ϕ_* = threshold of an individual.





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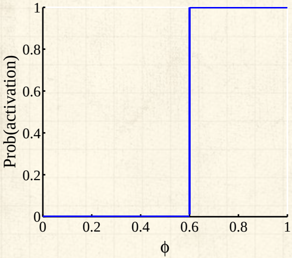

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- $f(\phi_*)$ = distribution of thresholds in a population.

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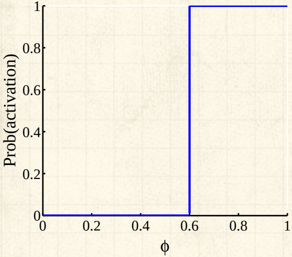

- Assumes deterministic response functions
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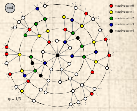
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$



Social Contagion

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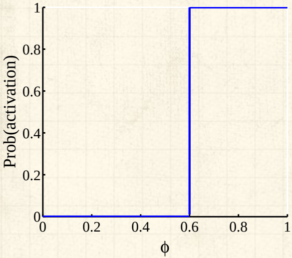

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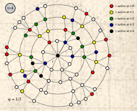
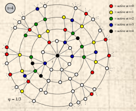
- ϕ_t = fraction of people 'rioting' at time step t.



Social Sciences—Threshold models

- At time $t + 1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.



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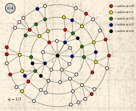

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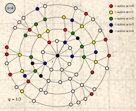
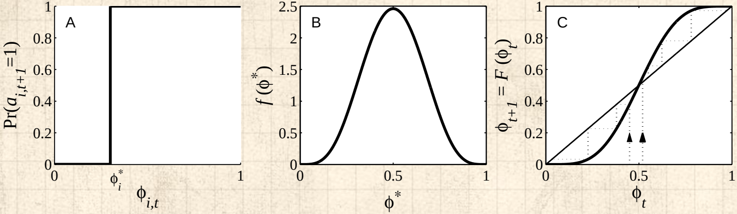
- $\Rightarrow$ Iterative maps of the unit interval $[0,1]$.

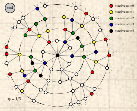


Social Sciences—Threshold models

Action based on perceived behavior of others.



- Two states: S and I
- Recover now possible (SIS)
- ϕ = fraction of contacts ‘on’ (e.g., rioting)



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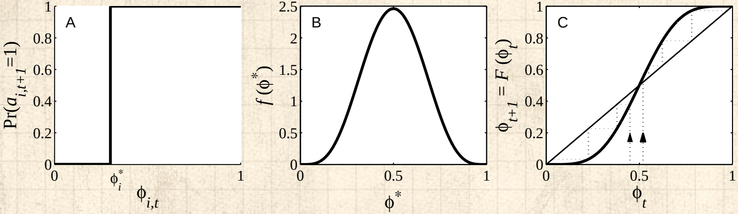

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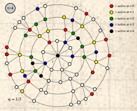

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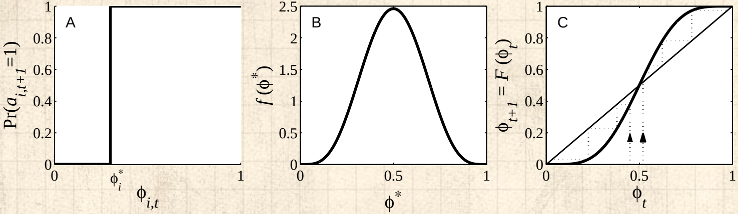

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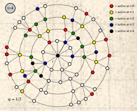
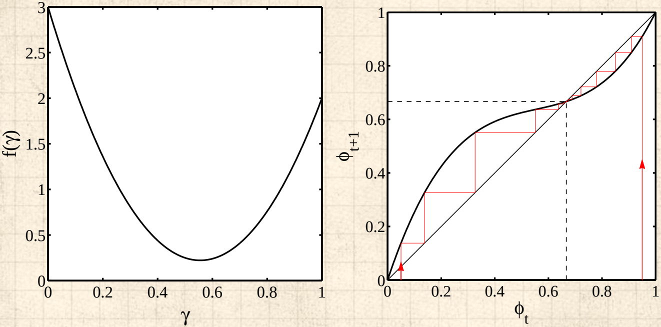
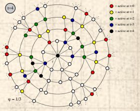
- ϕ = fraction of contacts ‘on’ (e.g., rioting)
- Discrete time, synchronous update (strong assumption!)
- This is a Critical mass model



Social Sciences—Threshold models

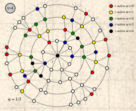


- Example of single stable state model



Social Sciences—Threshold models

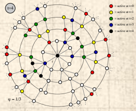
Implications for collective action theory:



Social Sciences—Threshold models

Implications for collective action theory:

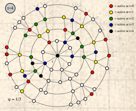
1. Collective uniformity $\not\Rightarrow$ individual uniformity



Social Sciences—Threshold models

Implications for collective action theory:

1. Collective uniformity $\nRightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes



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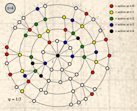

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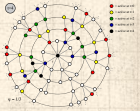
- Connect mean-field model to network model.



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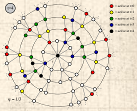

Social Sciences—Threshold models

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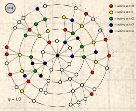
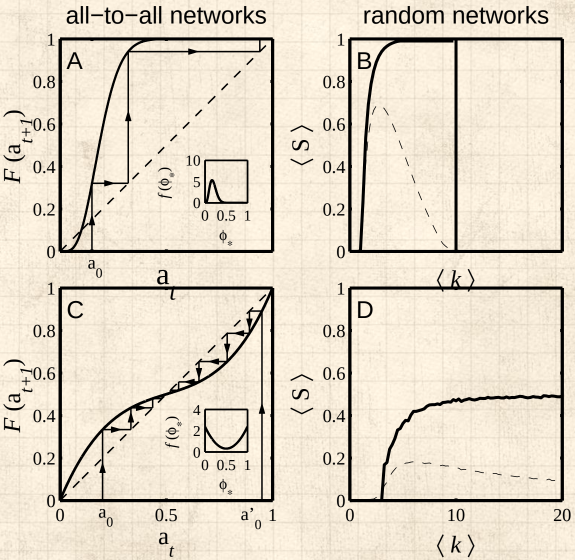
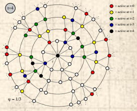
- Single seed for network model: $1/N \to 0$.
- Comparison between network and mean-field model sensible for vanishing seed size for the latter.



All-to-all versus random networks



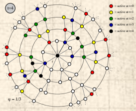


Spreadworthiness: Cat videos

Bowling with Ragdolls:

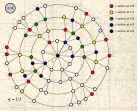
https://www.youtube.com/watch?v=XX-g2nmqL9Q?rel=0

- Organic extreme outlier?
- Success did not spread to other videos.



Threshold contagion on random networks

Three key pieces to describe analytically:



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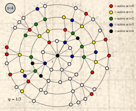
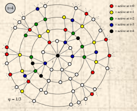


Three key pieces to describe analytically:

1. The fractional size of the largest subcomponent of vulnerable nodes, S_{vuln}.

Threshold contagion on random networks



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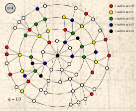
Threshold contagion on random networks



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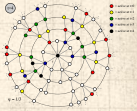
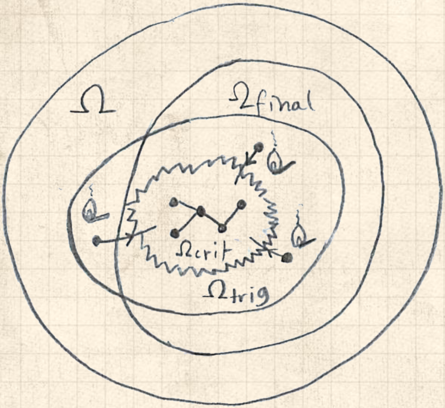
Threshold contagion on random networks

Three key pieces to describe analytically:

1. The fractional size of the largest subcomponent of vulnerable nodes, S_{vuln}.
2. The chance of starting a global spreading event, $P_{\text{trig}} = S_{\text{trig}}$.
3. The expected final size of any successful spread, S.
   - n.b., the distribution of S is almost always bimodal.

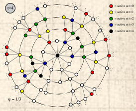


Example random network structure:



- $\Omega_{crit} = \Omega_{vuln}$ = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

$\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and $\Omega_{trig}, \Omega_{final} \subset \Omega$.



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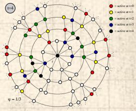

Threshold contagion on random networks

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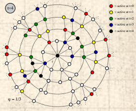
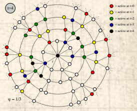


Threshold contagion on random networks

- First goal: Find the largest component of vulnerable nodes.
- Recall that for finding the giant component's size, we had to solve:

$$F_\pi(x) = xF_P\left(F_\rho(x)\right) \text{ and } F_\rho(x) = xF_R\left(F_\rho(x)\right)$$



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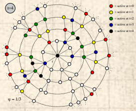

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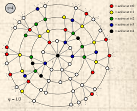

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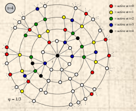
- This is a node-based percolation problem.
- For a general monotonic threshold distribution $f(\phi)$, a degree k node is vulnerable with probability

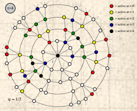
$$B_{k1} = \int_0^{1/k} f(\phi)\mathrm{d}\phi\,.$$



Threshold contagion on random networks

- We now have a generating function for the probability that a randomly chosen node is vulnerable and has degree k:

$$F_P^{(\text{vuln})}(x) = \sum_{k=0}^{\infty} P_k B_{k1} x^k.$$



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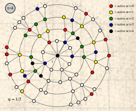

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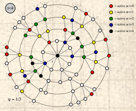

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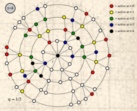

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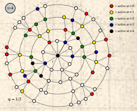
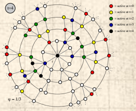
- Detail: We still have the underlying degree distribution involved in the denominator.



Threshold contagion on random networks

- Functional relations for component size g.f.'s are almost the same ...



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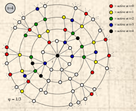

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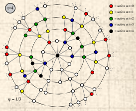
$$F_\pi^{(\text{vuln})}(x) = \underbrace{1 - F_P^{(\text{vuln})}(1)}_{\substack{\text{central node}\\ \text{is not}\\ \text{vulnerable}}} + x F_P^{(\text{vuln})}\left(F_\rho^{(\text{vuln})}(x)\right)$$



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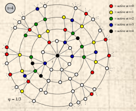

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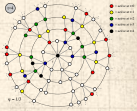

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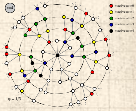
- Can now solve as before to find

$$S_{\text{vuln}} = 1 - F_\pi^{(\text{vuln})}(1).$$



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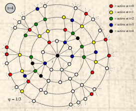
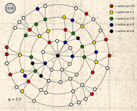


Threshold contagion on random networks

- **Second goal:** Find probability of triggering largest vulnerable component.



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- Second goal: Find probability of triggering largest vulnerable component.
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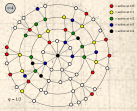

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- Same set up as for vulnerable component except now we don't care if the initial node is vulnerable or not:

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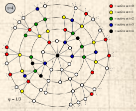

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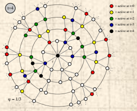
- Solve as before to find $P_{\text{trig}} = S_{\text{trig}} = 1 - F_{\pi}^{(\text{trig})}(1)$.



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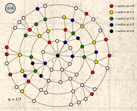
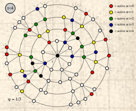


Physical derivation of possibility and probability of global spreading:

- Possibility: binary indicator of phase. Global spreading events are either possible or can never happen.



Physical derivation of possibility and probability of global spreading:

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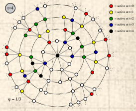
- For random networks, global spreading possibility is understood as meaning a giant component of vulnerable nodes exists.



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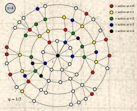

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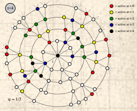

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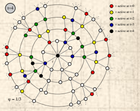

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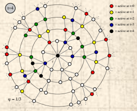

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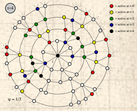
- Later: Generalize to more complex networks involving assortativity of all kinds.



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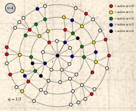

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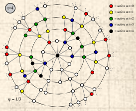

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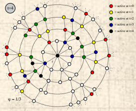

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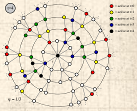

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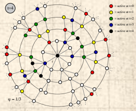

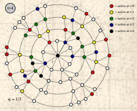

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- Put everything together and solve for Q_{trig}:

$$Q_{\text{trig}} = \sum_k \frac{kP_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - (1 - Q_{\text{trig}})^{k-1}\right].$$

Good things about our equation for Q_{trig}:

$$Q_{\text{trig}} = \sum_k \frac{kP_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - (1 - Q_{\text{trig}})^{k-1}\right] = f(Q_{\text{trig}}; P_k, B_{k1})$$

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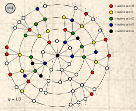

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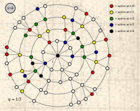

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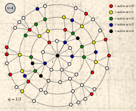

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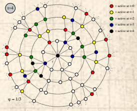

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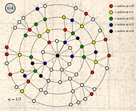

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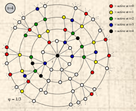
- Start with a suitably small seed $Q_{\text{trig}}^{(1)} > 0$ and iterate while rubbing hands together.



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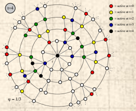

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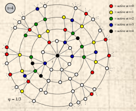

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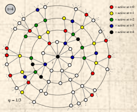

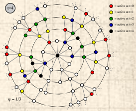

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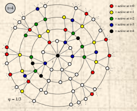
- As for S_{vuln}, P_{trig} is non-zero when $Q_{\text{trig}} > 0$.



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- We found that $F_\rho^{(\text{vuln})}(1)$—the probability that a random edge leads to a finite vulnerable component—satisfies

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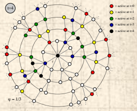

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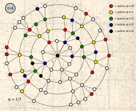

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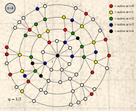

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- Some breathless algebra it all matches:

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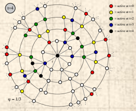

Fractional size of the largest vulnerable component:

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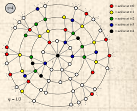

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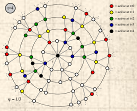

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- Excited scrabbling about gives us, as before:

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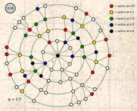

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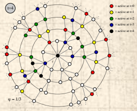
- Slight adjustment to the vulnerable component calculation.



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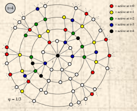

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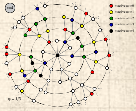
- We play these cards: $F_\rho^{(\text{vuln})}(1) = 1 - Q_{\text{trig}}$ and $F_P(x) = \sum_{k=0}^{\infty} P_k x^k$ to arrive at

$$1 - S_{\text{trig}} = 1 + \sum_{k=0}^{\infty} P_k \left(1 - Q_{\text{trig}}\right)^k.$$



Triggering probability for single-seed global spreading events:

- Slight adjustment to the vulnerable component calculation.
- $S_{\text{trig}} = 1 - F_{\pi}^{(\text{trig})}(1)$ where

$$F_{\pi}^{(\text{trig})}(1) = 1 \cdot F_P\left(F_{\rho}^{(\text{vuln})}(1)\right).$$

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- More scruffing around brings happiness:

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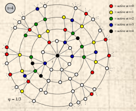

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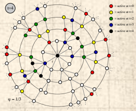

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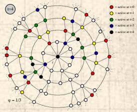

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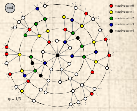

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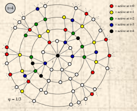

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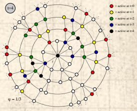

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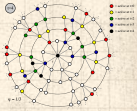
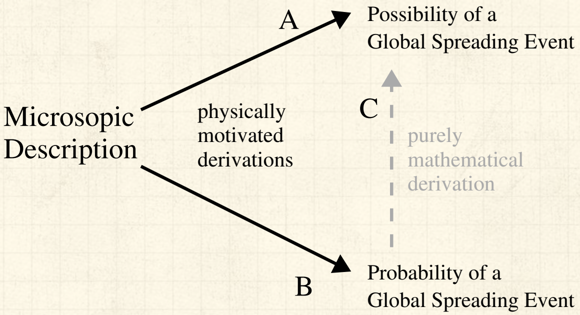
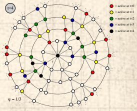
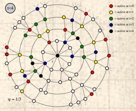
- We need to find out what happens as $Q_{\text{trig}} \to 0$. [9]



What we're doing:







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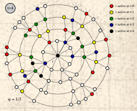

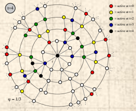

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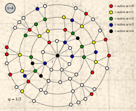

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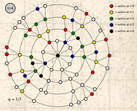

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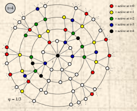
- Inequality?



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- From this connection, we don't know anything about a gain ratio **R** or how to arrange the pieces.

Outline

Basic Contagion Models

Global spreading condition

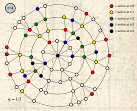
Social Contagion Models

- Network version
- All-to-all networks

Theory

- Spreading possibility
- Spreading probability
- Physical explanation
- Final size

References



Threshold contagion on random networks

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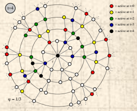

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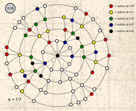

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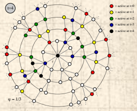

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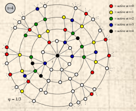

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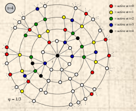
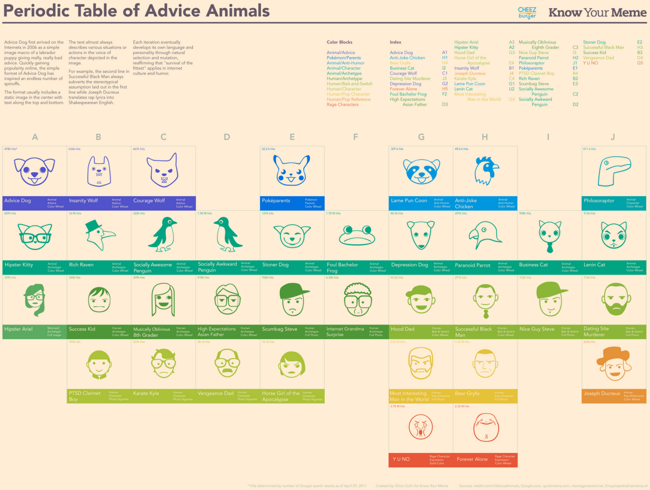
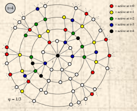
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [6]



Meme species:



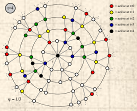


More here at http://knowyourmeme.com

Expected size of spread

Idea:

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$



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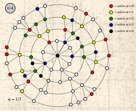

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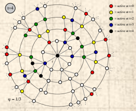

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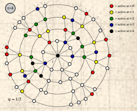

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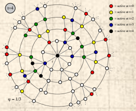

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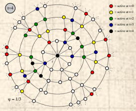

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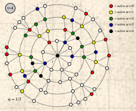
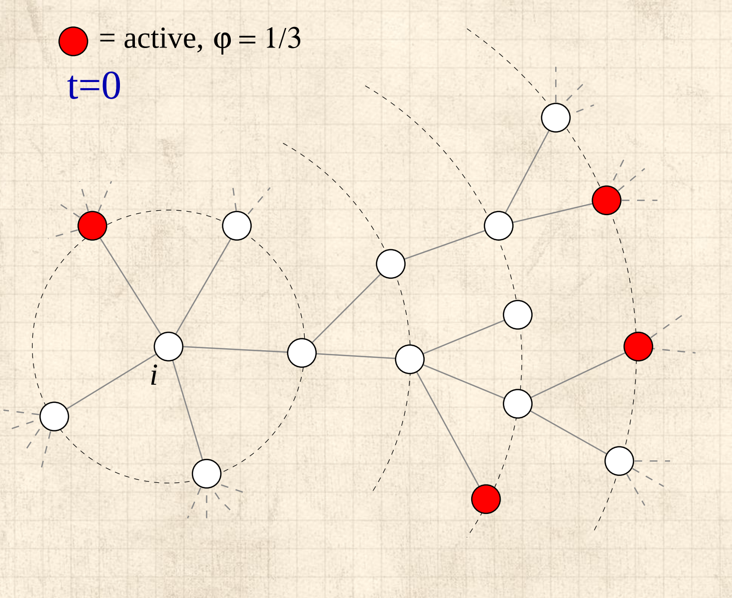
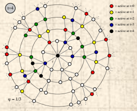
  - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i.



Expected size of spread





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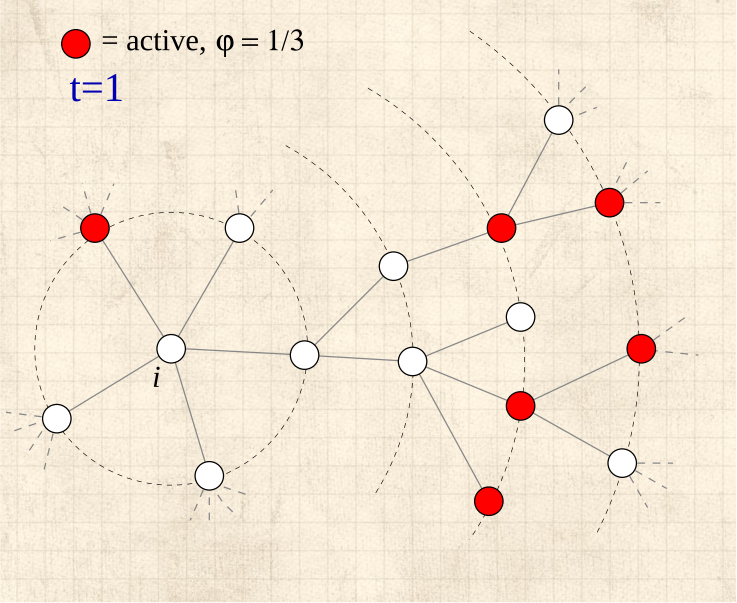

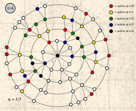

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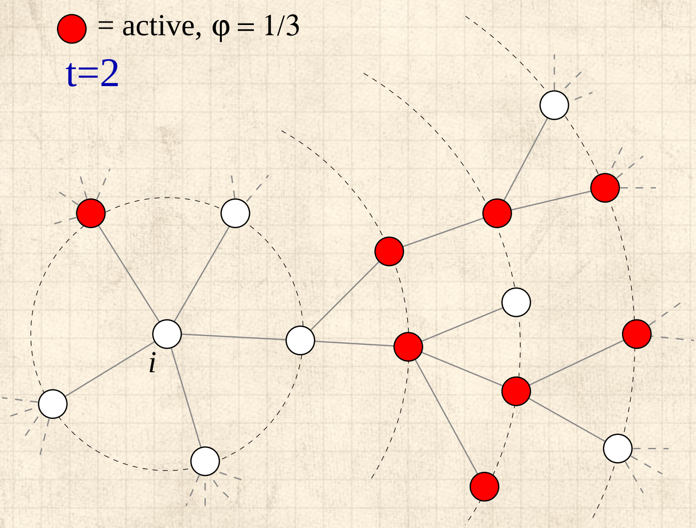

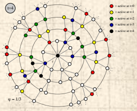

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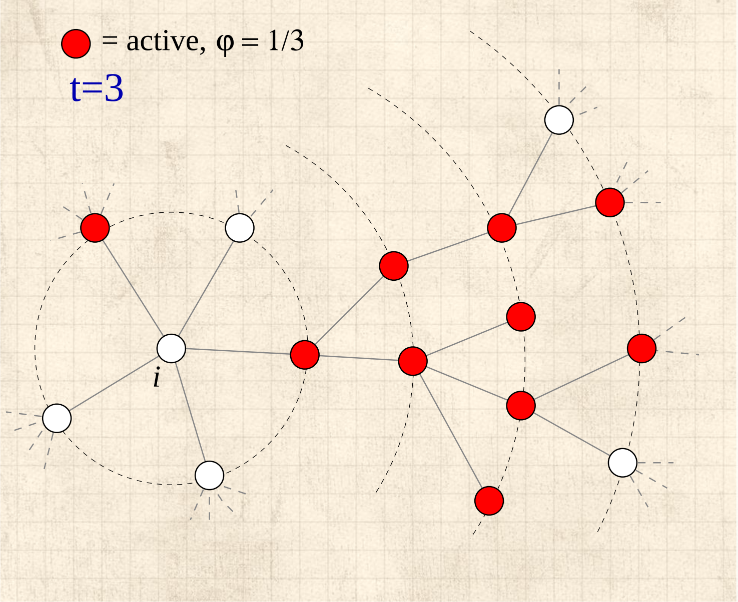

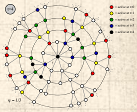

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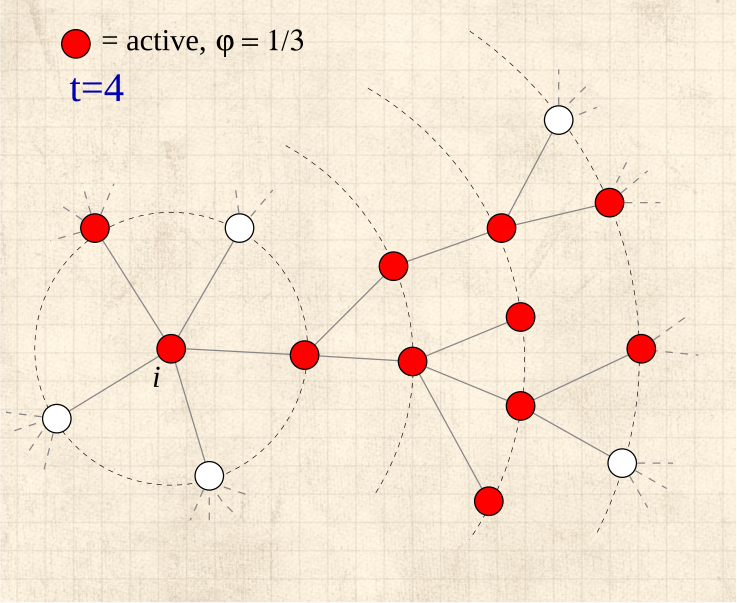

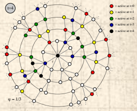

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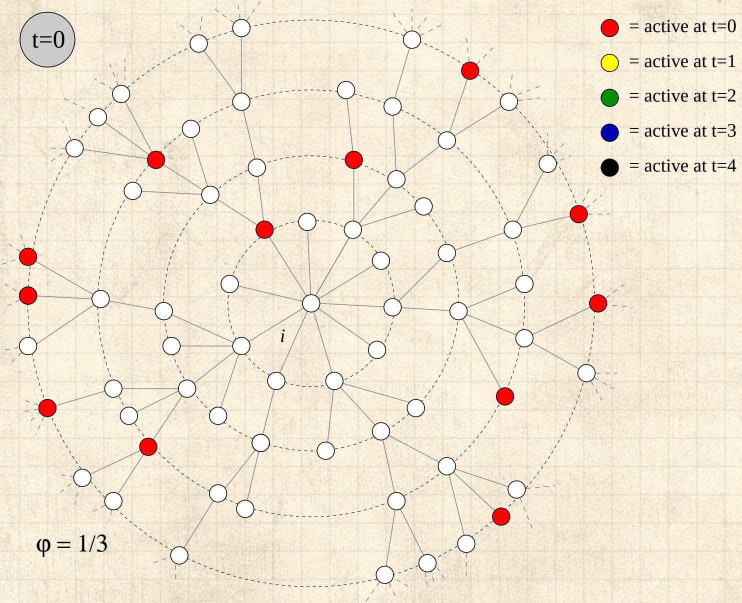

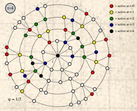

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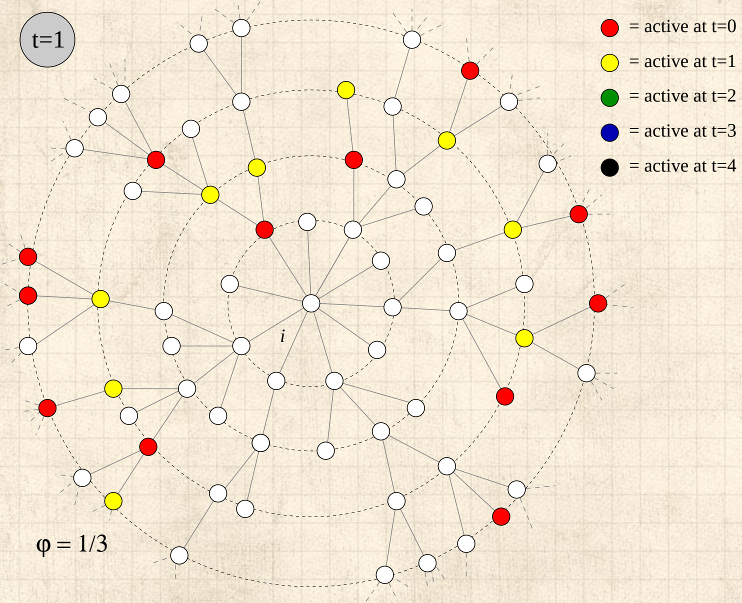
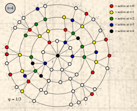
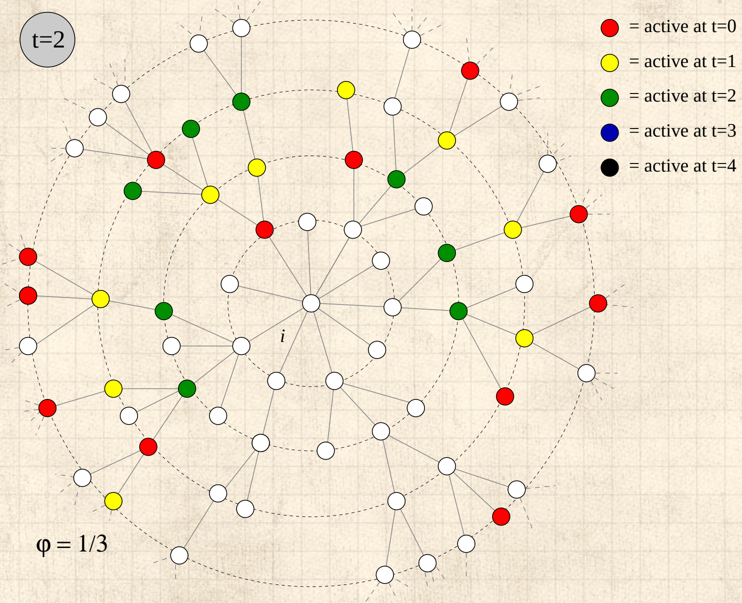
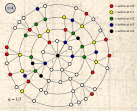




Expected size of spread





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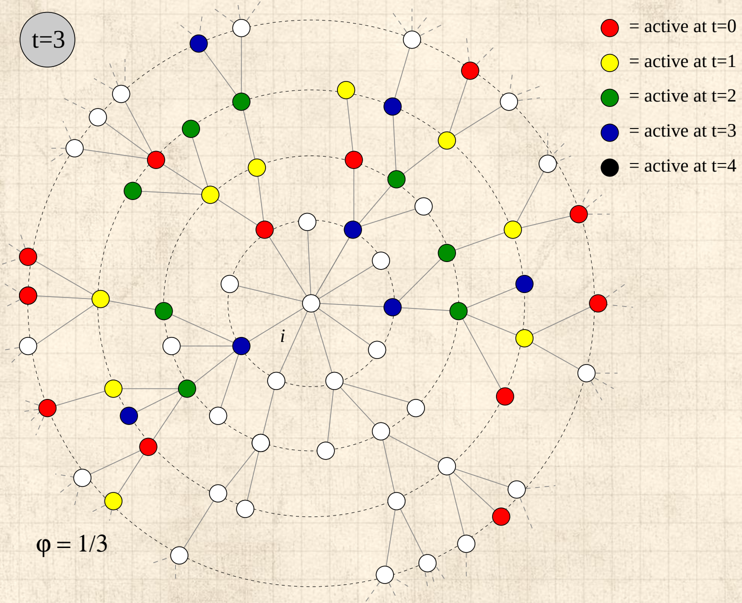

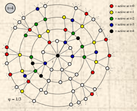

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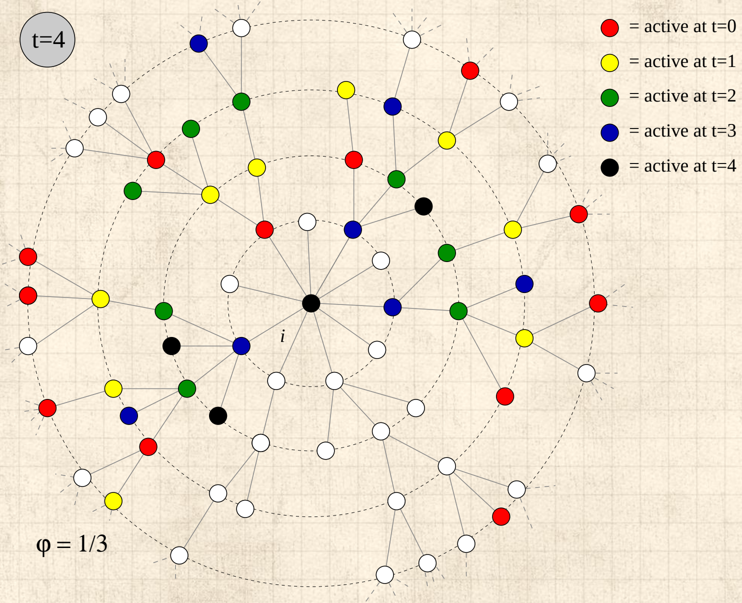
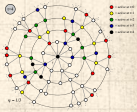
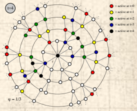




Expected size of spread

Notes:

- Calculations presume nodes do not become inactive (strong restriction, liftable)



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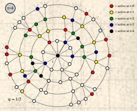

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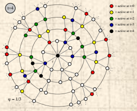

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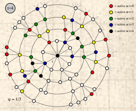

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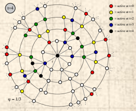

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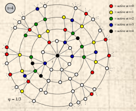

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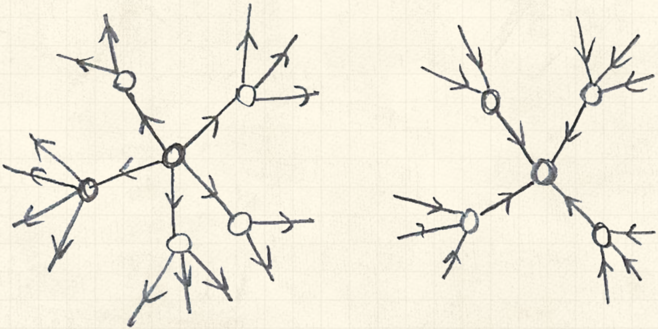
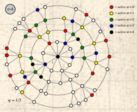
- We can in fact determine **Pr**(node of degree k switches on at time t).
- Even more, we can compute: **Pr**(specific node i switches on at time t).
- Asynchronous updating can be handled too.

Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.

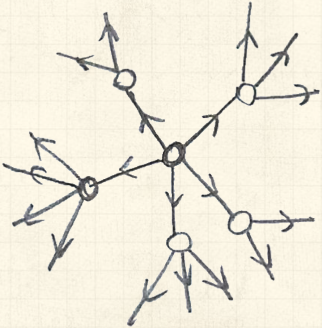


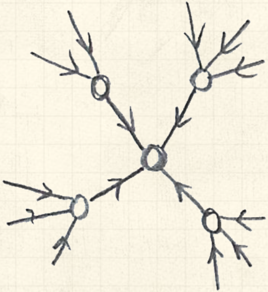


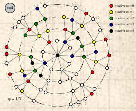
Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.

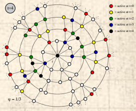






Expected size of spread

- **Notation:**
  $\phi_{k,t} = \mathbf{Pr}(\text{a degree } k \text{ node is active at time } t).$



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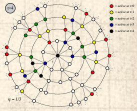

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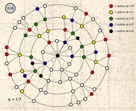

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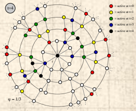

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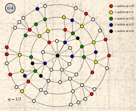

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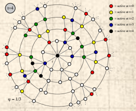

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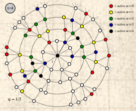
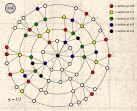
- Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1-\phi_0)\sum_{j=0}^{k}\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}B_{kj}.$$



Expected size of spread

- For general t, we need to know the probability an edge coming into a degree k node at time t is active.



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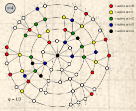

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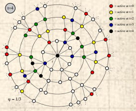

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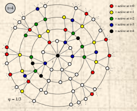
- Story analogous to $t = 1$ case. For specific node i:

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i - j} B_{k_i j}.$$



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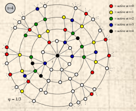
- Average over all nodes with degree k to obtain expression for ϕ_{t+1}:

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}.$$



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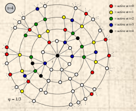
- So we need to compute θ_t...



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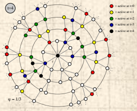
- So we need to compute θ_t... massive excitement...



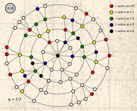
Expected size of spread

First connect θ_0 to θ_1:

- $\theta_1 = \phi_0 +$

$$(1-\phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^{\,j} (1-\theta_0)^{k-1-j} B_{kj}$$

- $\frac{k P_k}{\langle k \rangle} = Q_k$ = **Pr** (edge connects to a degree k node).
- $\sum_{j=0}^{k-1}$ piece gives **Pr** (degree node k activates if j of its $k-1$ incoming neighbors are active).
- ϕ_0 and $(1-\phi_0)$ terms account for state of node at time $t=0$.



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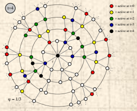
- ϕ_0 and $(1-\phi_0)$ terms account for state of node at time $t=0$.
- See this all generalizes to give θ_{t+1} in terms of θ_t...



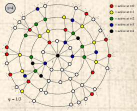
Expected size of spread

Two pieces: edges first, and then nodes

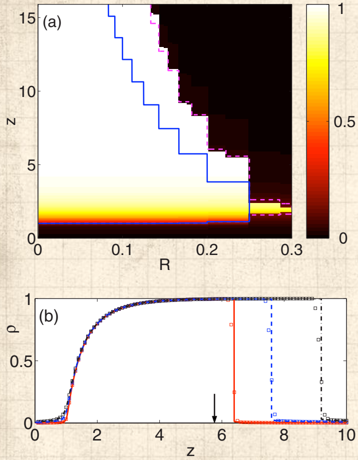
1. $$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + \underbrace{(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_t^{\,j}(1-\theta_t)^{k-1-j}B_{kj}}_{\text{social effects}}$$

   with $\theta_0 = \phi_0$.

2. $$\phi_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + \underbrace{(1-\phi_0)\sum_{k=0}^{\infty}P_k\sum_{j=0}^{k}\binom{k}{j}\theta_t^{\,j}(1-\theta_t)^{k-j}B_{kj}}_{\text{social effects}}.$$

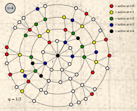


Comparison between theory and simulations



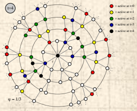
From Gleeson and Cahalane [7]

- Pure random networks with simple threshold responses
- R = uniform threshold (our ϕ_*); z = average degree; $\rho = \phi$; $q = \theta$; $N = 10^5$.
- $\phi_0 = 10^{-3}$, 0.5×10^{-2}, and 10^{-2}.
- Cascade window is for $\phi_0 = 10^{-2}$ case.
- Sensible expansion of cascade window as ϕ_0 increases.



Notes:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.



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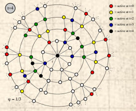

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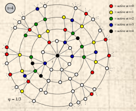
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.



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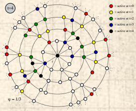
  meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.
- If $\theta = 0$ is a fixed point of G (i.e., $G(0;\phi_0) = 0$) then spreading occurs for a small seed if

$$G'(0;\phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

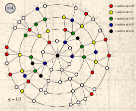


Insert question from assignment 10

Notes:

In words:

- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.



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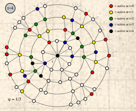
- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an unstable fixed point at $\theta = 0$, then cascades are also always possible.



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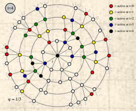
- Cascade condition is more complicated for $\phi_0 > 0$.



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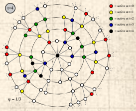

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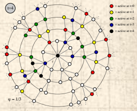
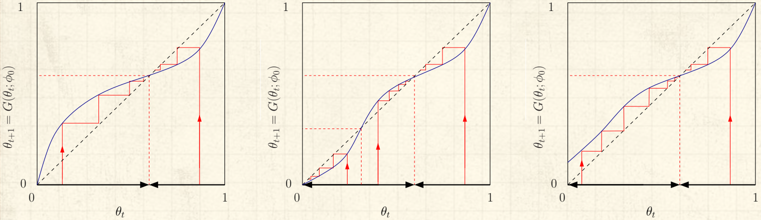
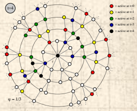
- If G has a stable fixed point at $\theta = 0$, and an unstable fixed point for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- Tricky point: G depends on ϕ_0, so as we change ϕ_0, we also change G.



General fixed point story:



- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.



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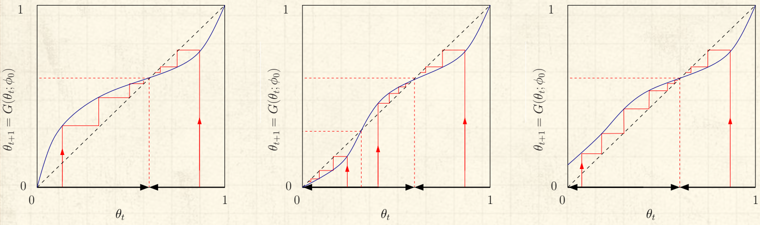
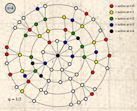


- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.



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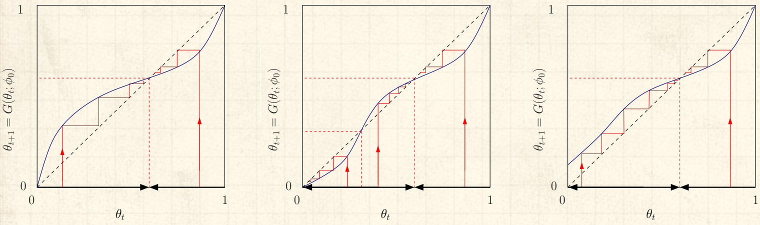

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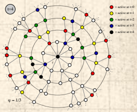

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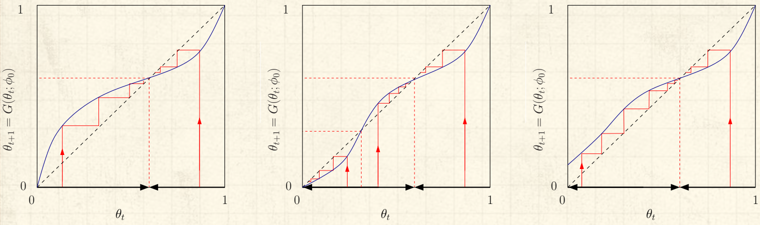

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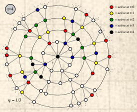

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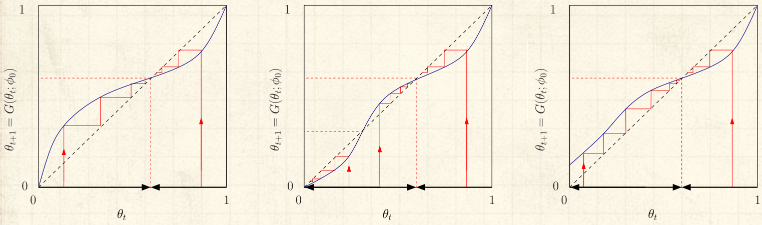

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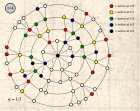

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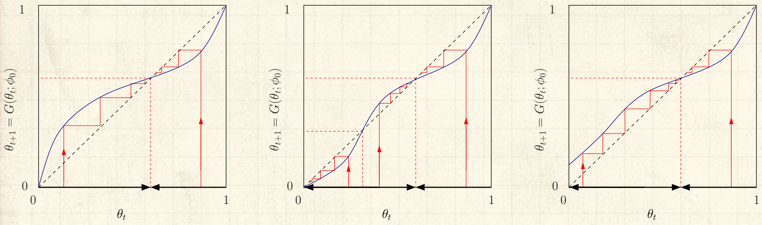

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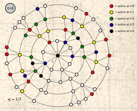
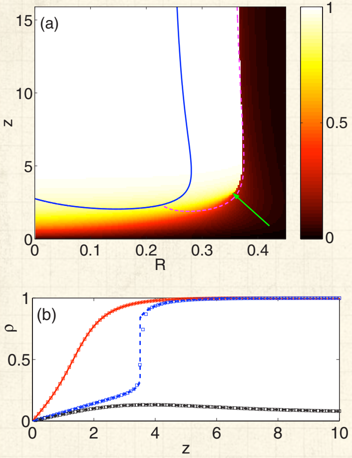
- Second: $G'(\theta; \phi_0) \geq 0$, $0 \leq \theta \leq 1$.

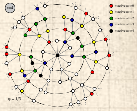


Interesting behavior:



- Now allow thresholds to be distributed according to a Gaussian with mean R.
- R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.

From Gleeson and Cahalane [7]



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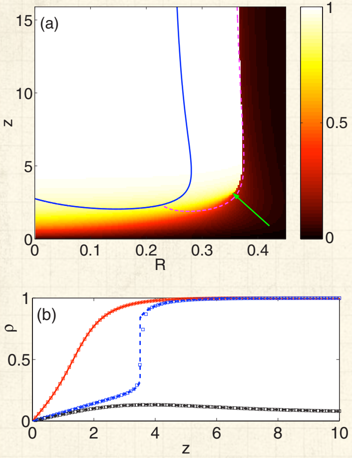

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- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.

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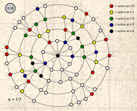

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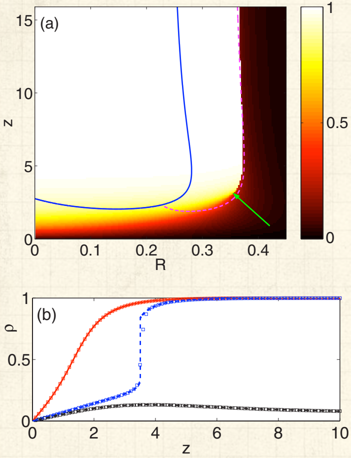

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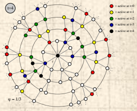
- R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.
- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.
- Now see a (nasty) discontinuous phase transition for low $\langle k \rangle$.



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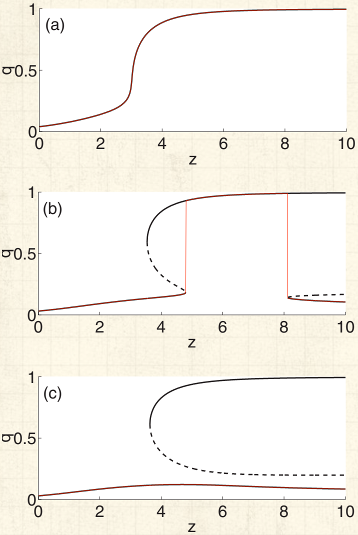
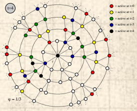
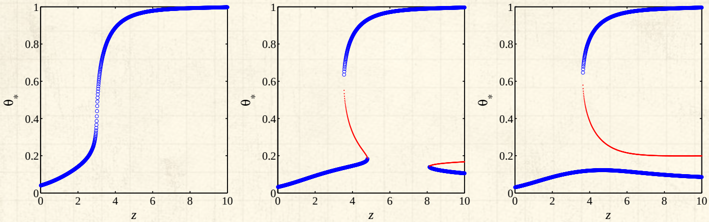


- Plots of stability points for $\theta_{t+1} = G(\theta_t; \phi_0)$.
- n.b.: 0 is not a fixed point here: $\theta_0 = 0$ always takes off.
- Top to bottom: R = 0.35, 0.371, and 0.375.
- Saddle node bifurcations appear and merge (b and c).

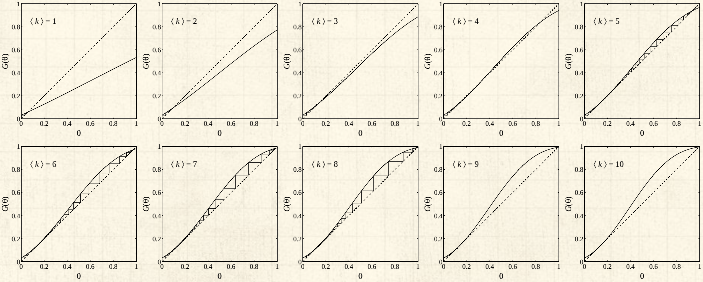
From Gleeson and Cahalane [7]

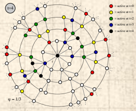


What's happening:



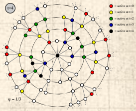
Fixed points slip above and below the $\theta_{t+1} = \theta_t$ line:





Time-dependent solutions

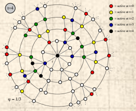
Synchronous update



Time-dependent solutions

Synchronous update

- Done: Evolution of ϕ_t and θ_t given exactly by the maps we have derived.



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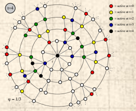
Asynchronous updates

- Update nodes with probability α.



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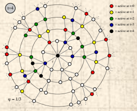

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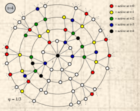
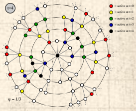
- Now can talk about $\phi(t)$ and $\theta(t)$.



Nutshell:

- Solid dive into understanding contagion on generalized random networks.



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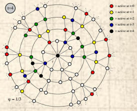
- Threshold model leads to idea of vulnerables and a critical mass. [16, 8]



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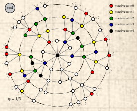

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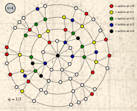

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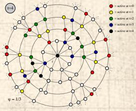

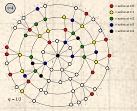

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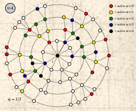

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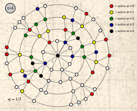
- The single seed contagion condition and triggering probability can be fully developed using a physical story. [5, 9]
- Many connections to other kinds of models: Voter models, Ising models, ...

Neural reboot (NR):

Pangolin happiness:

https://www.youtube.com/watch?v=LMiYjkG4onM?rel=0



References I

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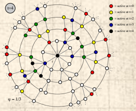

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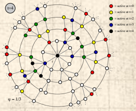

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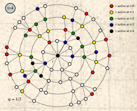

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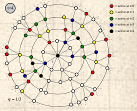

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