Overview of Complex Networks

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Principles of Complex Systems, Vols. 1 & 2 CSYS/MATH 300 and 303, 2021-2022 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont























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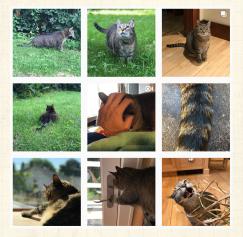
Examples of Complex Networks

Physical networks Interaction networks Relational networks



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net•work | 'net,wərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : a network of railroads.
- 2 a group or system of interconnected people or things: a trade network.
 - a group of people who exchange information, contacts, and experience for professional or social purposes : a support network.
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : the introduction of a second TV network | [as adj.] network television.
 - a number of interconnected computers, machines, or operations: specialized computers that manage multiple outside connections to a network | a local cellular phone network.
 - · a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network: the stock exchanges have proven to be resourceful in networking these deals.

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) networked workstations.
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career: the skills of networking, bargaining, and negotiation.

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Thesaurus deliciousness:

network

noun

- 1 a network of arteries WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2 a network of lanes MAZE, labyrinth, warren, tangle.
- **3** a network of friends SYSTEM, complex, nexus, web, webwork.

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From Keith Briggs's excellent etymological investigation:

Opus reticulatum:

A Latin origin?



[http://serialconsign.com/2007/11/we-put-netnetwork]

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First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

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1883-: distribution network of electrical cables

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🙈 1869-: railways

1883-: distribution network of electrical cables

1914-: wireless broadcasting networks

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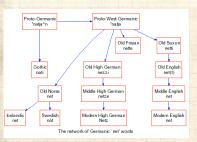
Net and Work are venerable old words:

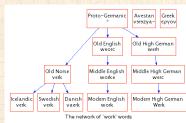


'Net' first used to mean spider web (King Ælfréd, 888).



'Work' appear to have long meant purposeful action.





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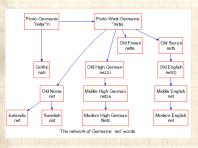
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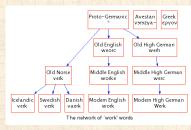


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'Network' = something built based on the idea of natural, flexible lattice or web.

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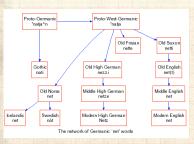
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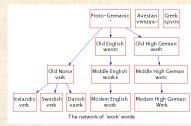


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c.f., ironwork, stonework, fretwork.

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Many complex systems can be viewed as complex networks of physical or abstract interactions.

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Many complex systems can be viewed as complex networks of physical or abstract interactions.

Opens door to mathematical and numerical analysis.

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- Many complex systems can be viewed as complex networks of physical or abstract interactions.
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Piranha physicus



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Hunt in packs.

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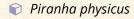
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- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

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Complex Networks Basics

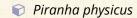
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- Hunt in packs.
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- See also: https://xkcd.com/793/

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Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"

Watts and Strogatz, Nature, **393**, 440–442, 1998. ^[16]

Times cited: ☑

(as of October 24, 2018)



"Emergence of scaling in random networks" Barabási and Albert, Science, **286**, 509–511, 1999. [2]

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Review articles:



"Complex Networks: Structure and Dynamics"
Boccaletti et al., Physics Reports, **424**, 175–308, 2006. [3]



"The structure and function of complex networks"
M. E. I. Newman,

SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: \checkmark ~ 17,782 (as of October 24, 2018)



"Statistical mechanics of complex networks"

Albert and Barabási, Rev. Mod. Phys., **74**, 47–97, 2002. [1]

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Popularity according to textbooks:

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Popularity according to textbooks:



"Networks" a 2 by Mark Newman (2018). [11]



"Networks, crowds, and markets: Reasoning about a highly connected world" a 2 by Easley and Kleinberg (2010). [7]

http://cs.cornell.edu/home/kleinber/networks-book/

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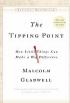
Graph theory?

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Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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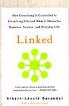
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Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [15]

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Numerous others ...

- & Complex Social Networks—F. Vega-Redondo [14]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [13]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- & Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

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But surely networks aren't new ...

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Graph theory is well established ...

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But surely networks aren't new ...



Graph theory is well established ...



Study of social networks started in the 1930's ...

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So why all this 'new' research on networks?

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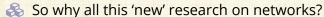
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Answer: Oodles of Easily Accessible Data.

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 *If this is upsetting, maybe string theory is for you ...

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Web-scale data sets can be overly exciting.

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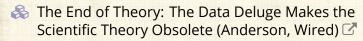
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Witness:



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Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- "The Unreasonable Effectiveness of Data," Halevy et al. [9].
- 🚓 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [17]

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But:

- For scientists, description is only part of the battle.
- We still need to understand.

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Nodes = A collection of entities which have properties that are somehow related to each other

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Nodes = A collection of entities which have properties that are somehow related to each other



🙈 e.g., people, forks in rivers, proteins, webpages, organisms, ...

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Links = Connections between nodes

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Links may be directed or undirected.

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Other spiffing words: vertices and edges.

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Node degree = Number of links per node

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Node degree = Number of links per node

 \mathbb{A} Notation: Node *i*'s degree = k_i .

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 $k_i = 0,1,2,...$

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Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)

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 $\& k_i = 0,1,2,...$

Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)

& Connection between number of edges m and average degree:

 $\langle k \rangle = \frac{2m}{N}.$

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Node degree = Number of links per node

 \aleph Notation: Node *i*'s degree = k_i .

 $\& k_i = 0,1,2,...$

Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)

& Connection between number of edges m and average degree:

 $\langle k \rangle = \frac{2m}{N}.$

 \bowtie Defn: \mathcal{N}_i = the set of i's k_i neighbors

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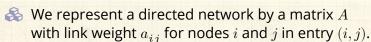
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Adjacency matrix:



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Adjacency matrix:

We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i,j).

ቆ e.g.,

$$A = \left[\begin{array}{ccccc} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \right]$$

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Adjacency matrix:

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(n.b., for numerical work, we always use sparse matrices.)

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So what passes for a complex network?

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So what passes for a complex network?

Complex networks are large (in node number)

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So what passes for a complex network?



Complex networks are large (in node number)



Complex networks are sparse (low edge to node) ratio)

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So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving

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So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

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Physical networks



River networks



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Physical networks



River networks



Neural networks



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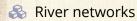
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Physical networks



Neural networks

Trees and leaves





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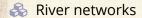
Examples of Complex Networks

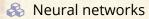
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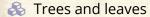
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Physical networks







Blood networks





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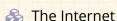
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River networks

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Trees and leaves.

Blood networks



The Internet



Road networks







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River networks

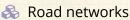
Neural networks

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Blood networks



The Internet



Power grids







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Distribution (branching) versus redistribution (cyclical)

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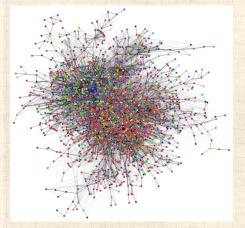
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Interaction networks



The Blogosphere



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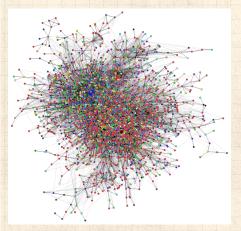
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The Blogosphere



Biochemical networks



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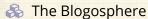
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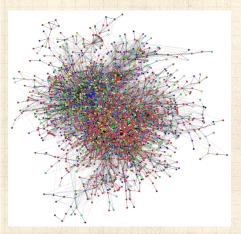


Interaction networks



Biochemical networks

Gene-protein networks



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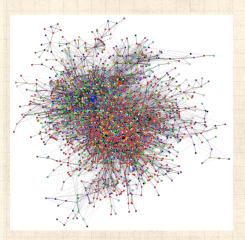
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Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom



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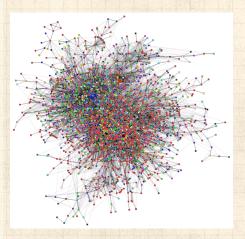
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Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)



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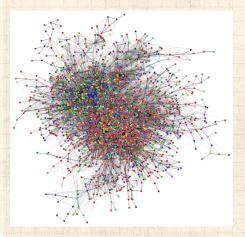
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Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks



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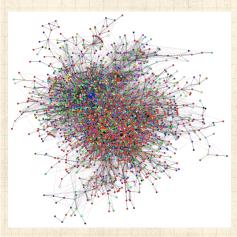
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Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)



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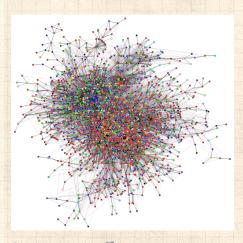
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- The Blogosphere
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- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- 🚓 The Media



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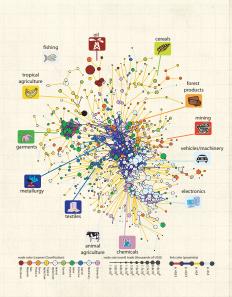


topics:

- Hidalgo et al.'s

 "The Product

 Space Conditions
 the Development
 of Nations" [10]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



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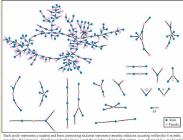


Interaction networks: social networks



Snogging

The Structure of Romantic and Sexual Relations at "Jefferson High School"



preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman et al., 2004)

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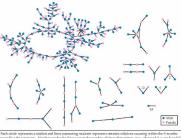


Snogging



Friendships

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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(Bearman et al., 2004)

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Interaction networks: social networks



Snogging

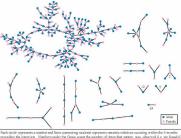


Friendships



Acquaintances

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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(Bearman et al., 2004)

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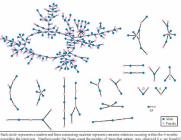




Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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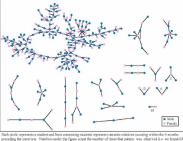
Physical networks Interaction networks Relational networks



Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations

The Structure of Romantic and Sexual Relations at "Jefferson High School"



proceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found to pairs unconnected to anyone elso).

(Bearman et al., 2004)



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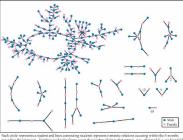
Physical networks Interaction networks Relational networks



Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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(Bearman et al., 2004)

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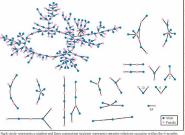
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Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- A facebook (twitter ().

The Structure of Romantic and Sexual Relations at "Jefferson High School"



preceding the interview. Numbers under the figure count the number of times that puttern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman et al., 2004)

'Remotely sensed' by: email activity, instant messaging, phone logs

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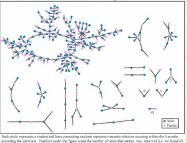
Interaction networks



Interaction networks: social networks

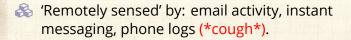
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- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook
 twitter
 twitter

The Structure of Romantic and Sexual Relations at "Jefferson High School"



proceding the interview. Numbers unset the figure ocum use number of times user pastern was observed (i.e. we techn pairs unconnected to anyone clse)

(Bearman et al., 2004)



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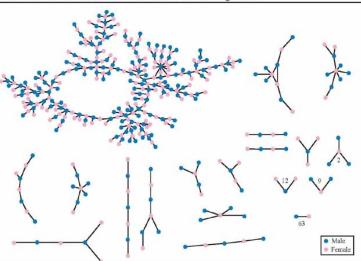
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The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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Consumer purchases

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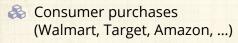
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Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings

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Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas

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Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- & Knowledge/Databases/Ideas

common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

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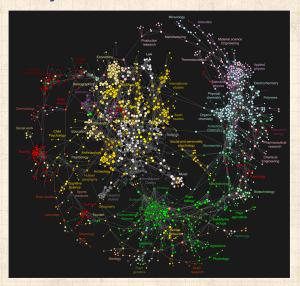
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Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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Neural reboot (NR):

Dog has fun.

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