System Robustness

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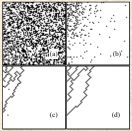
Principles of Complex Systems, Vol. 1 | @pocsvox
CSYS/MATH 300, Fall, 2020

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



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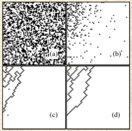

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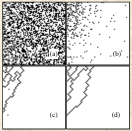
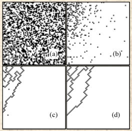
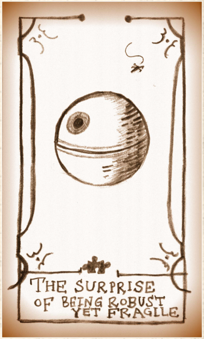


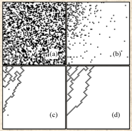
Outline




THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE DOORS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS
THE OVERCOMING OF CATASTROPHE
THE TROUBLES ILL-FORMED
THE NETWORKS OF MANY ILKS
THE CHAOS OF FETTERED CONTAGION
THE END

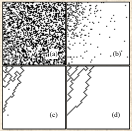

THE SURPRISE OF BEING ROBUST YET FRAGILE

Outline



Robustness

- Many complex systems are prone to cascading catastrophic failure:



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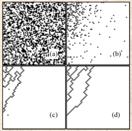

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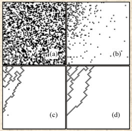

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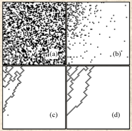

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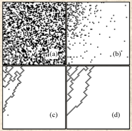

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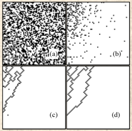

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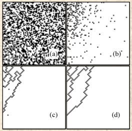

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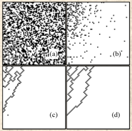

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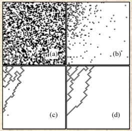

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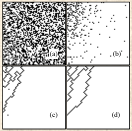

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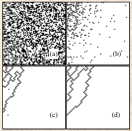

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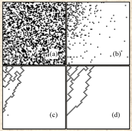

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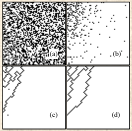
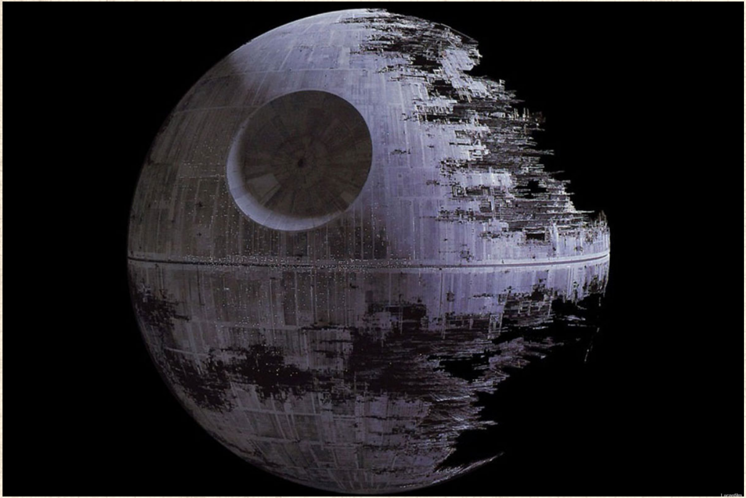
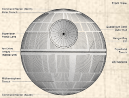
- But complex systems also show persistent robustness (not as exciting but important...)
- Robustness and Failure may be a power-law story...

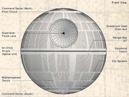


Our emblem of Robust-Yet-Fragile:



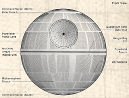


“Trouble ...”



Robustness

- System robustness may result from



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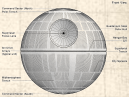

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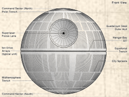

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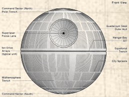
- Idea: Explore systems optimized to perform under uncertain conditions.



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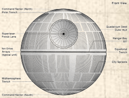
- The handle:
  'Highly Optimized Tolerance' (HOT) [4, 5, 6, 10]



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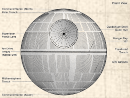

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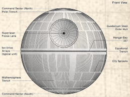

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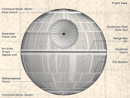
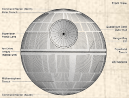
- Great abstracts of the world #73: "There aren't any." [7]



Robustness

Features of HOT systems: [5, 6]



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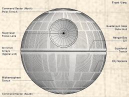
Features of HOT systems: [5, 6]

- High performance and robustness



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Features of HOT systems: [5, 6]

- High performance and robustness
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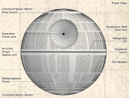

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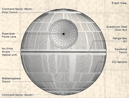

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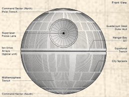

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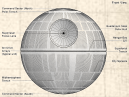
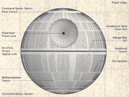
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)



Robustness

HOT combines things we've seen:

- Variable transformation



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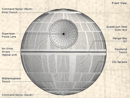

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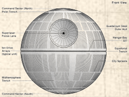

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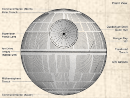

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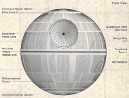

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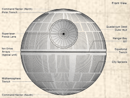

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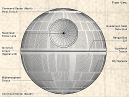
- MIWO is good: Mild In, Wild Out
- X has a characteristic size but Y does not



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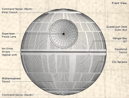
Forest fire example: [5]



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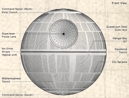

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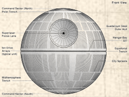

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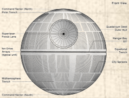

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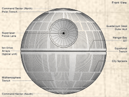
- Fires start at location (i, j) according to some distribution P_{ij}



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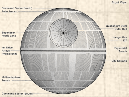
- Fires start at location (i, j) according to some distribution P_{ij}
- Fires spread from tree to tree (nearest neighbor only)



Robustness

Forest fire example: [5]

- Square $N \times N$ grid
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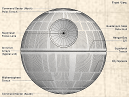

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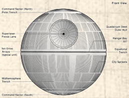

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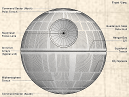
- Empty sites block fire
- Best case scenario:
  Build firebreaks to maximize average # trees left intact given one spark



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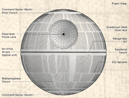

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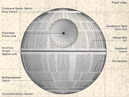
Forest fire example: [5]

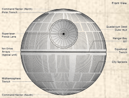
- Build a forest by adding one tree at a time



Robustness

Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree



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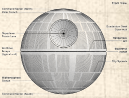

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Forest fire example: [5]

- Build a forest by adding one tree at a time
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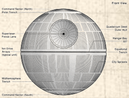
- Average over P_{ij} = spark probability



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Forest fire example: [5]

- Build a forest by adding one tree at a time
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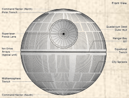
- D = design parameter
- Average over P_{ij} = spark probability
- $D = 1$: random addition



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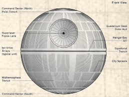
- $D = N^2$: test all possibilities



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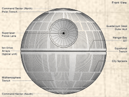

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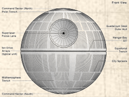

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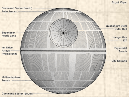
Measure average area of forest left untouched

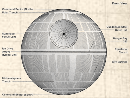
- $f(c)$ = distribution of fire sizes c (= cost)
- Yield = $Y = \rho - \langle c \rangle$



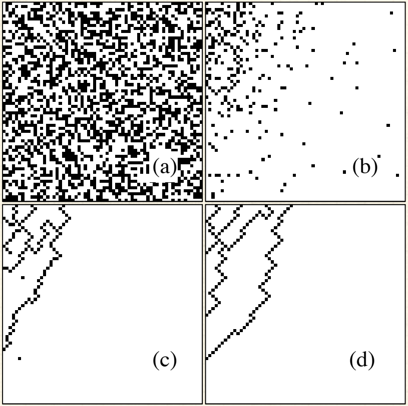
Robustness

Specifics:

- $$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$
  where
  $$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$
- In the original work, $b_y > b_x$
- Distribution has more width in y direction.



HOT Forests



[5]

$N = 64$

(a) $D = 1$
(b) $D = 2$
(c) $D = N$
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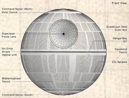
P_{ij} has a Gaussian decay



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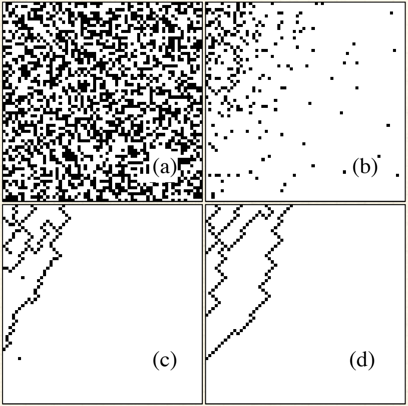

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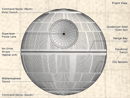

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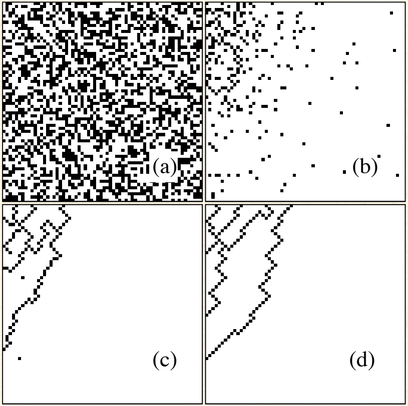

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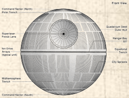
- But rare extreme events occur



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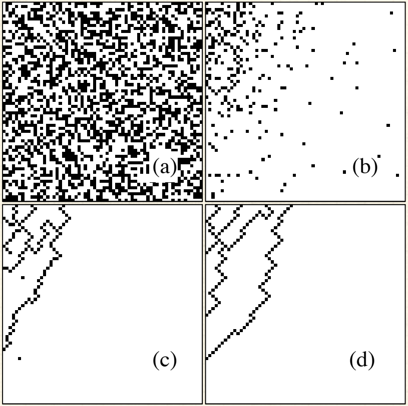

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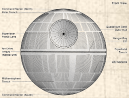
P_{ij} has a Gaussian decay

- Optimized forests do well on average (robustness)
- But rare extreme events occur



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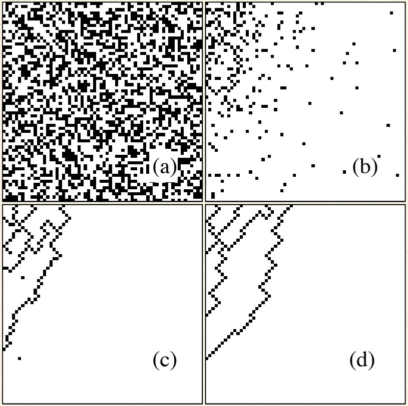

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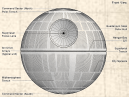
- But rare extreme events occur (fragility)



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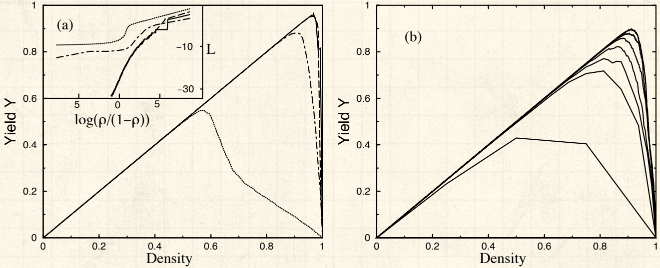
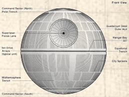


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters $D = 1$ (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with $N = 64$, and (b) for $D = 2$ and $N = 2, 2^2, \ldots, 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f\rangle/(1 - \langle f\rangle)]$, on a scale which more clearly differentiates between the curves.

[5]



HOT Forests:

- Y = ‘the average density of trees left unburned in a configuration after a single spark hits.’ [5]

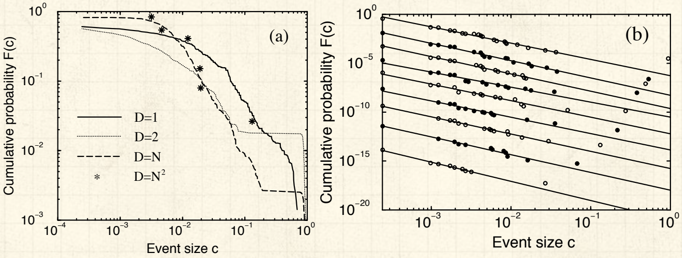
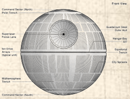
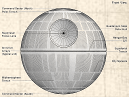


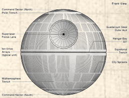
FIG. 3. Cumulative distributions of events $F(c)$: (a) at peak yield for $D = 1, 2, N$, and N^2 with $N = 64$, and (b) for $D = N^2$, and $N = 64$ at equal density increments of 0.1, ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).



Outline


Front View
Command Sector (North)
Polar Trench
Quadanium Steel Outer Hull
Superlaser Focus Lens
Hangar Bay 327
Ion Drive Arrays (typical unit)
Equatorial Trench
City Sprawls
Midhemisphere Trench
Command Sector (South)

Narrative causality:



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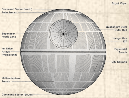
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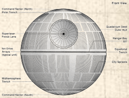
Random Forests

$D = 1$: Random forests = Percolation [11]

- Randomly add trees.



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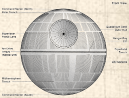

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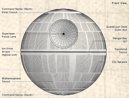

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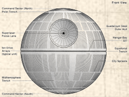

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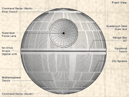
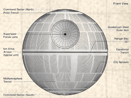
- Forest is random and featureless.



HOT forests nutshell:

- Highly structured
- Power law distribution of tree cluster sizes for a broad range of ρ, including below ρ_c.



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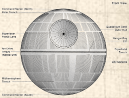

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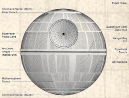

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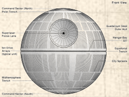

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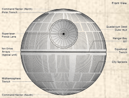

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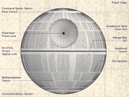

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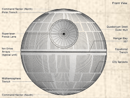
- Growth is key to toy model which is both algorithmic and physical.
- HOT theory is more general than just this toy model.



HOT forests—Real data:

“Complexity and Robustness,” Carlson & Dolye [6]

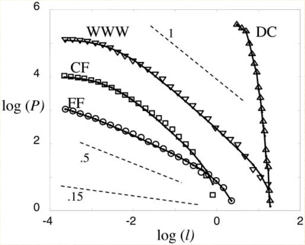
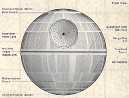


Fig. 1. Log–log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5$, 1 (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l_i)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km^2 (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.

- These are CCDFs (Eek: P, $\mathcal{P}(l \geq l_i)$)

- PLR = probability-loss-resource.
- Minimize cost subject to resource (barrier) constraints:
  $C = \sum_i p_i l_i$
  given
  $l_i = f(r_i)$ and $\sum r_i \leq R$.
- DC = Data Compression.



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"Complexity and Robustness," Carlson & Dolye [6]

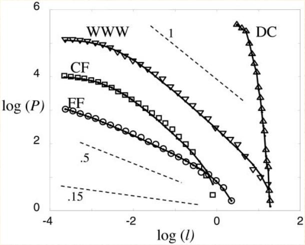
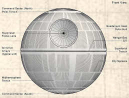


Fig. 1. Log–log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5$, 1 (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l_i)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km^2 (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.

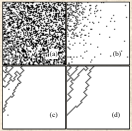
- These are CCDFs (Eek: P, $\mathcal{P}(l \geq l_i)$)

- PLR = probability-loss-resource.
- Minimize cost subject to resource (barrier) constraints: $C = \sum_i p_i l_i$ given $l_i = f(r_i)$ and $\sum r_i \leq R$.
- DC = Data Compression.
- Horror: log. Screaming: "The base! What is the base!? You monsters!"



HOT theory:

The abstract story, using figurative forest fires:



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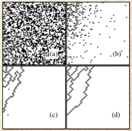
- Given some measure of failure size y_i and correlated resource size x_i with relationship $y_i = x_i^{-\alpha}$, $i = 1, \ldots, N_{\text{sites}}$.



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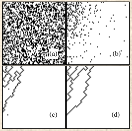

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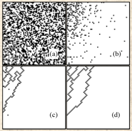

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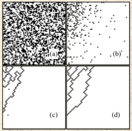
$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i.$$

a_i = area of ith site's region, and p_i = avg. prob. of fire at ith site over some time frame.



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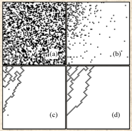

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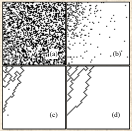

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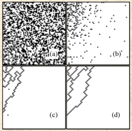

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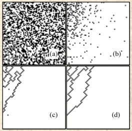

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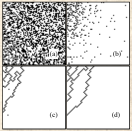
3. Insert question from assignment 7 to find:

$$\mathbf{Pr}(a_i) \propto a_i^{-\gamma} .$$

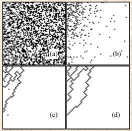


Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x}) p(\vec{x}) \mathrm{d}\vec{x}$$

   where C is some cost to be evaluated at each point in space $\vec{x}$ (e.g., $V(\vec{x})^{\alpha}$),



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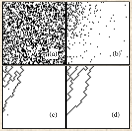

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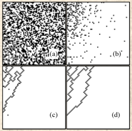

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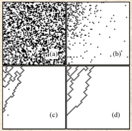

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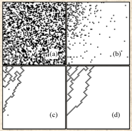
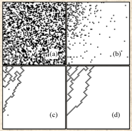
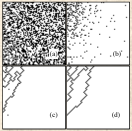
- For spatial systems with barriers: $\beta = d$.

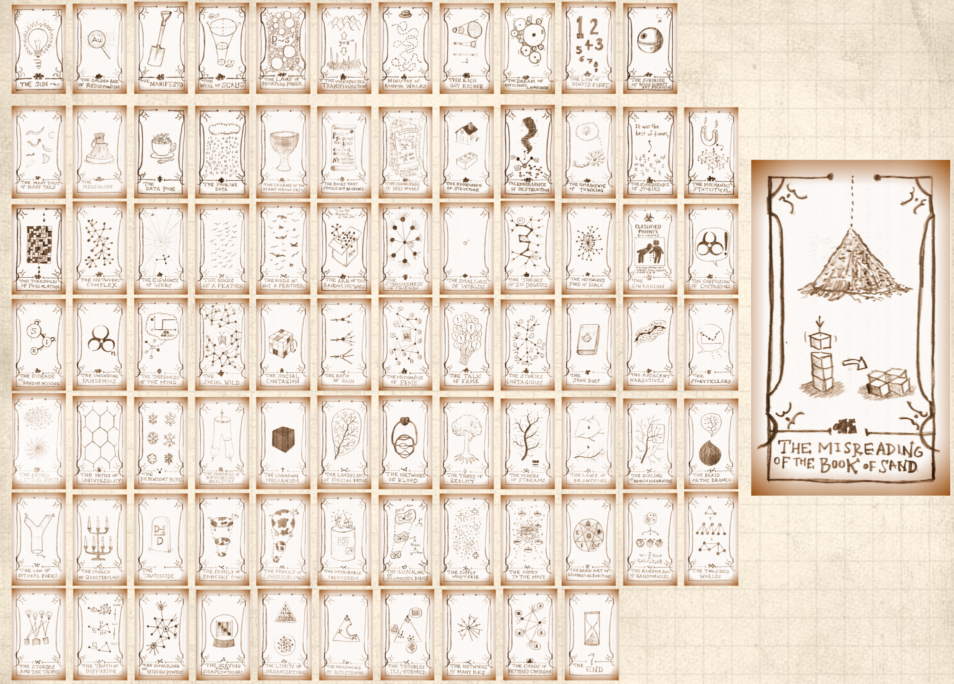


The Emperor's Robust-Yet-Fragileness:



Outline




THE SUN
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THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
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THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
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CLASSIFIED
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THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
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THE SOCIAL WILD
THE SOCIAL CONTAGION
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THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
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THE TROUSERS OF BIFURCATING REALITIES
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THE LANDSCAPES OF FORKING PATHS
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THE END
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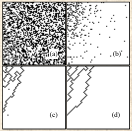
SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at 'critical states';



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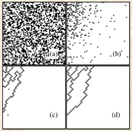

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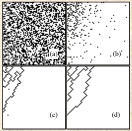

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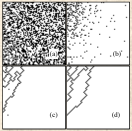

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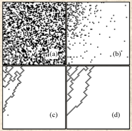

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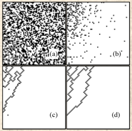

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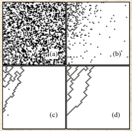
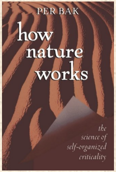
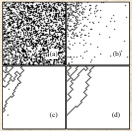
- Much criticism and arguing...

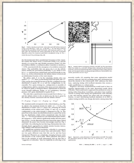




"How Nature Works: the Science of Self-Organized Criticality"
by Per Bak (1997). [2]

Avalanches of Sand and Rice ...





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HOT versus SOC

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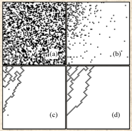

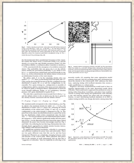

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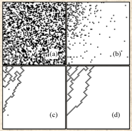

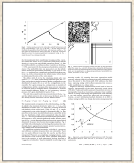

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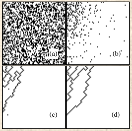

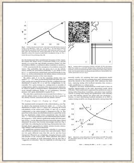

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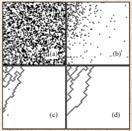
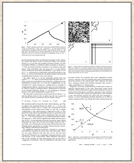
- SOC systems have one special density

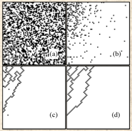


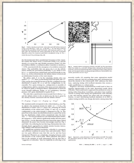


"Complexity and Robustness"
Carlson and Doyle,
Proc. Natl. Acad. Sci., **99**, 2538–2545, 2002. [6]

HOT versus SOC

- Both produce power laws
- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
- SOC systems have one special density
- HOT systems produce specialized structures





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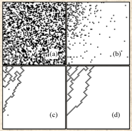
HOT versus SOC

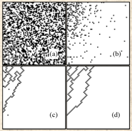
- Both produce power laws
- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
- SOC systems have one special density
- HOT systems produce specialized structures
- SOC systems produce generic structures



HOT theory—Summary of designed tolerance [6]

Table 1. Characteristics of SOC, HOT, and data

	Property	SOC	HOT and Data
1	Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2	Robustness	Generic	Robust, yet fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for power laws	Critical internal fluctuations	Robust performance
7	Exponent α	Small	Large
8	α vs. dimension d	$\alpha \approx (d-1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large (∞)
10	Increase model resolution	No change	New structures, new sensitivities
11	Response to forcing	Homogeneous	Variable



Outline

Robustness

HOT theory

Narrative causality

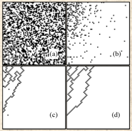
Random forests

Self-Organized Criticality

COLD theory

Network robustness

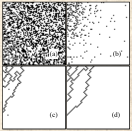
References



COLD forests

Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]



COLD forests

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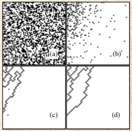
- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly



COLD forests

Avoidance of large-scale failures

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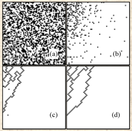
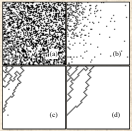
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...



COLD forests

Avoidance of large-scale failures

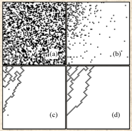
- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe



COLD forests

Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated



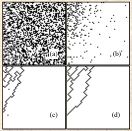
Cutoffs

Observed:

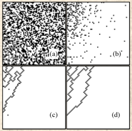
- Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.



Cutoffs



Observed:

- Power law distributions often have an exponential cutoff

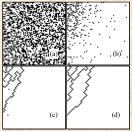
$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.

- May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-a x^{-\gamma+1}}$$

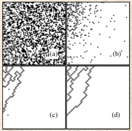
Outline



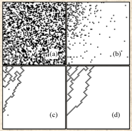
Robustness

We'll return to this later on:

- Network robustness.
- Albert et al., Nature, 2000:
  "Error and attack tolerance of complex networks" [1]
- General contagion processes acting on complex networks. [13, 12]
- Similar robust-yet-fragile stories ...



The Emperor’s Robust-Yet-Fragileness:



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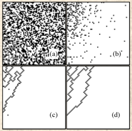

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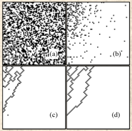

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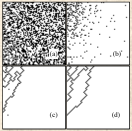

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