

Overview of Complex Networks

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Principles of Complex Systems, Vol. 1 | @pocsvox
CSYS/MATH 300, Fall, 2020

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Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



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- Etymology
- Popularity
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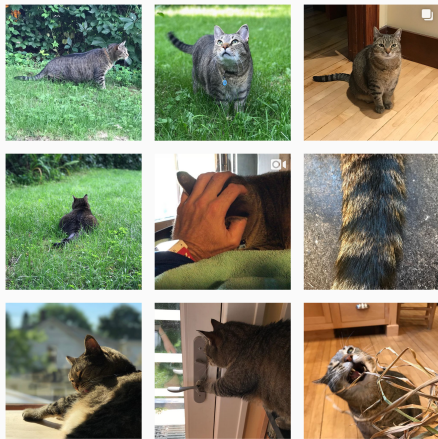
- Physical networks
- Interaction networks
- Relational networks



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net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*



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Thesaurus deliciousness:

network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.



From Keith Briggs's excellent etymological
investigation: 



Opus
reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

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-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

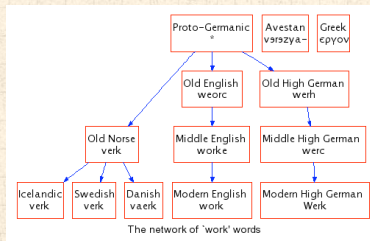
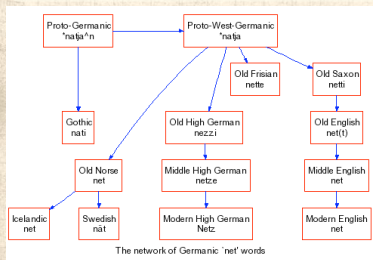


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Net and Work are venerable old words:


 **'Net'** first used to mean spider web (King Ælfréd, 888).


 **'Work'** appear to have long meant purposeful action.

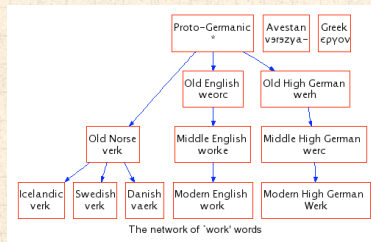
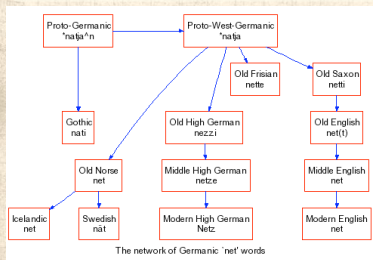



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 **'Network'** = something built based on the idea of natural, flexible lattice or web.

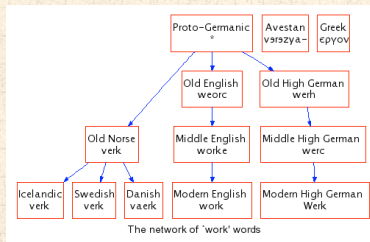
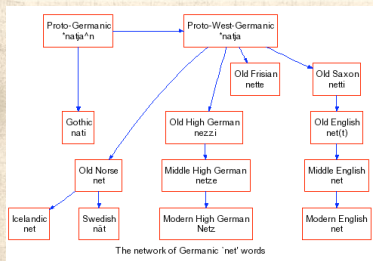



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 **'Network'** = something built based on the idea of natural, flexible lattice or web.

 c.f., ironwork, stonework, fretwork.



Key Observation:



Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.

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Opens door to mathematical and numerical analysis.

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 *Piranha physicus*




 Hunt in packs.



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-  *Piranha physicist*
-  Hunt in packs.
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- Piranha physicus*
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- See also: <https://xkcd.com/793/> 



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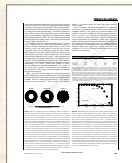
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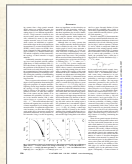



"Collective dynamics of 'small-world' networks" 

Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[16]


Times cited: 

(as of October 24, 2018)



"Emergence of scaling in random networks" 

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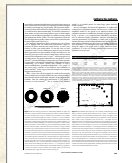
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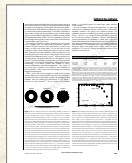
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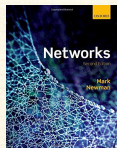
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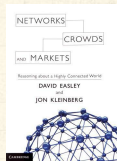
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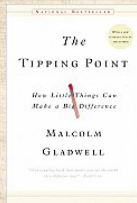


“Networks, crowds, and markets: Reasoning about a highly connected world” [a](#) [↗](#)
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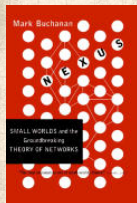
<http://cs.cornell.edu/home/kleinber/networks-book/> [↗](#)



Popularity according to books:



The Tipping Point: How Little Things can
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Gladwell [8]



Nexus: Small Worlds and the
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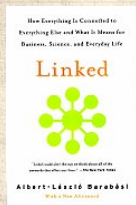
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Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts^[15]



Numerous others ...

-  **Complex Social Networks**—F. Vega-Redondo ^[14]
-  **Fractal River Basins: Chance and Self-Organization**—I. Rodríguez-Iturbe and A. Rinaldo ^[13]
-  **Random Graph Dynamics**—R. Durrette
-  **Scale-Free Networks**—Guido Caldarelli
-  **Evolution and Structure of the Internet: A Statistical Physics Approach**—Romu Pastor-Satorras and Alessandro Vespignani
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-  **Social Network Analysis**—Stanley Wasserman and Kathleen Faust
-  **Handbook of Graphs and Networks**—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
-  **Evolution of Networks**—S. N. Dorogovtsev and J. F. F. Mendes ^[6]



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But surely **networks aren't new** ...

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- We can now inform (alas) our theories with a much more measurable reality.*



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**If this is upsetting, maybe string theory is for you ...*



More observations



Web-scale data sets can be overly **exciting**.

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Witness:

 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

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 "The Unreasonable Effectiveness of Data,"
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 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" ^[17]





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 For scientists, description is only part of the battle.



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We still need to **understand**.



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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

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 e.g., people, forks in rivers, proteins, webpages, organisms, ...

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 **Links** may be directed or undirected.





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 e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes


 **Links** may be directed or undirected.

 **Links** may be binary or weighted.





Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

 **Links** may be directed or undirected.

 **Links** may be binary or weighted.

Other spiffing words: vertices and edges.



Super Basic definitions

Node degree = Number of links per node

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Node degree = Number of links per node

 Notation: Node i 's degree = k_i .

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 $k_i = 0, 1, 2, \dots$




Super Basic definitions

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 Notation: the average degree of a network = $\langle k \rangle$




Super Basic definitions

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 Notation: Node i 's degree = k_i .


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 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)





Super Basic definitions

Node degree = Number of links per node

 Notation: Node i 's degree = k_i .

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 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)


 Connection between number of edges m and
average degree:


$$\langle k \rangle = \frac{2m}{N}.$$





Super Basic definitions

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 Notation: Node i 's degree = k_i .

 $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)

 Connection between number of edges m and
average degree:

$$\langle k \rangle = \frac{2m}{N}.$$


 Defn: \mathcal{N}_i = the set of i 's k_i neighbors



Super Basic definitions

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Adjacency matrix:

 We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

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Super Basic definitions

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 We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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Super Basic definitions

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 (n.b., for numerical work, we always use sparse matrices.)

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So what passes for a complex network?

 Complex networks are **large** (in node number)



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So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)



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So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**



Examples

So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...



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River networks



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Neural networks



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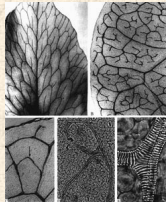
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Examples

Physical networks

-  River networks
-  Neural networks
-  Trees and leaves



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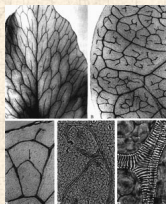
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-  River networks
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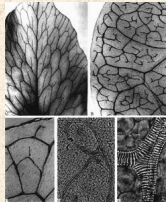
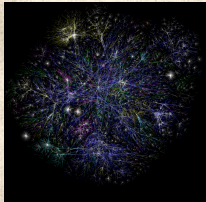


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-  River networks
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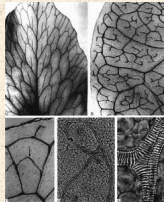
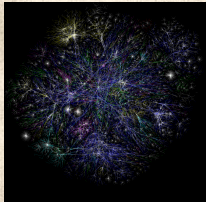


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-  River networks
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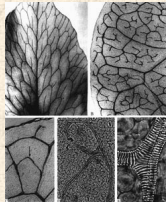
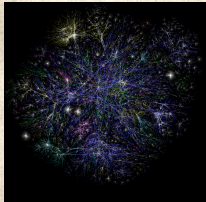


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-  River networks
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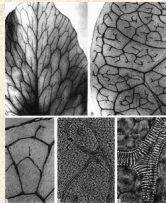
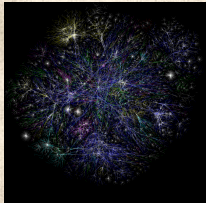



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-  River networks
-  Neural networks
-  Trees and leaves
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-  The Internet
-  Road networks
-  Power grids



-  **Distribution** (branching) versus **redistribution** (cyclical)



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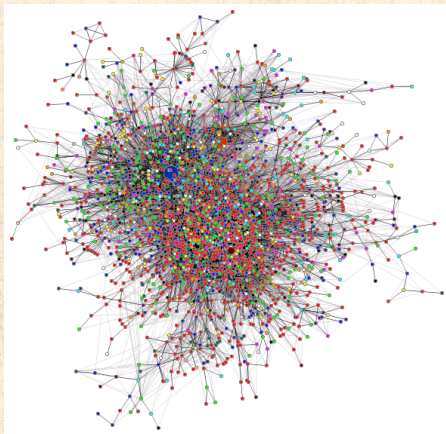



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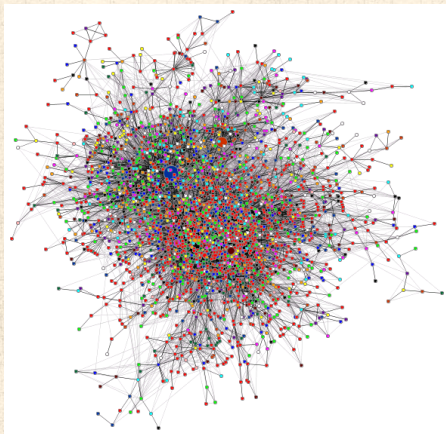
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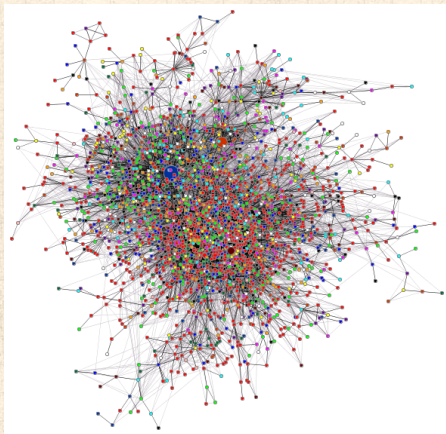
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
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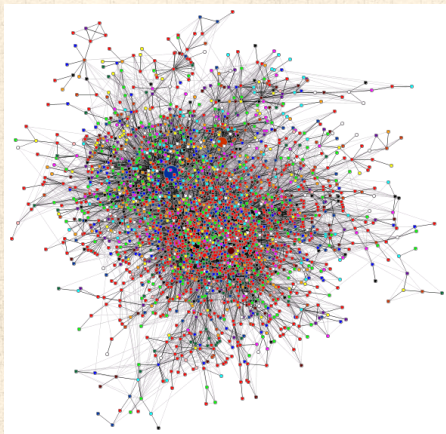
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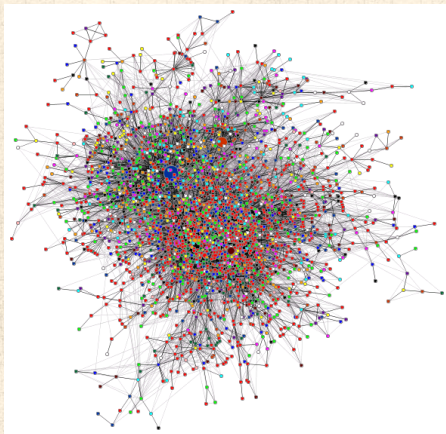
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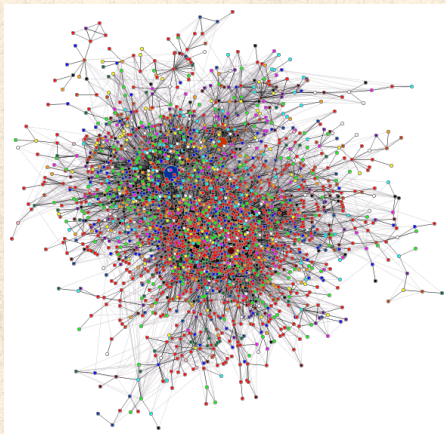
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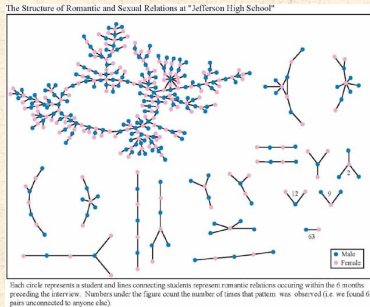
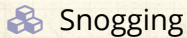
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Interaction networks: social networks



(Bearman *et al.*, 2004)

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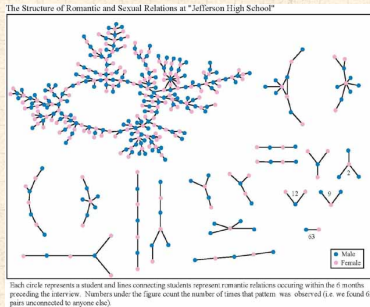


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Interaction networks: social networks

 Snogging

 Friendships



(Bearman *et al.*, 2004)

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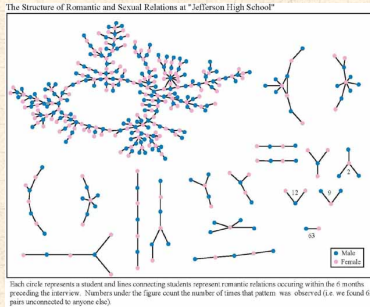
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Examples

Interaction networks: social networks

-  Snogging
-  Friendships
-  Acquaintances



(Bearman *et al.*, 2004)

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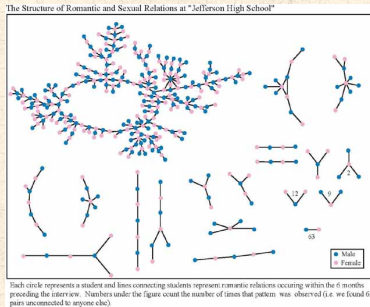
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Interaction networks: social networks

-  Snogging
-  Friendships
-  Acquaintances
-  Boards and directors



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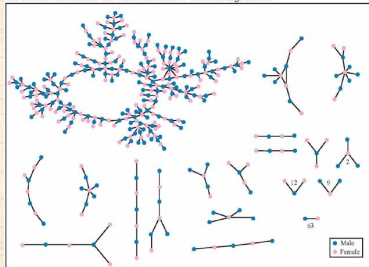


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Interaction networks: social networks

-  Snogging
-  Friendships
-  Acquaintances
-  Boards and directors
-  Organizations

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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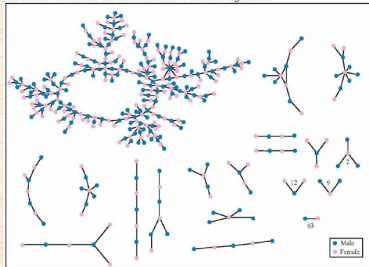


Examples

Interaction networks: social networks

-  Snogging
-  Friendships
-  Acquaintances
-  Boards and directors
-  Organizations
-  [facebook](#)  [twitter](#) 

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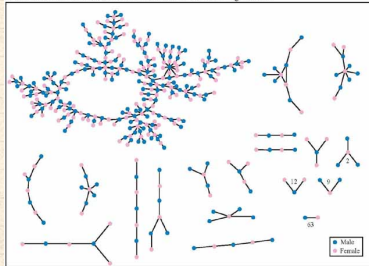
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Interaction networks: social networks

- 🧱 Snogging
- 🧱 Friendships
- 🧱 Acquaintances
- 🧱 Boards and directors
- 🧱 Organizations
- 🧱 [facebook](#) ↗ [twitter](#) ↗,

🧱 'Remotely sensed' by: email activity, instant messaging, phone logs

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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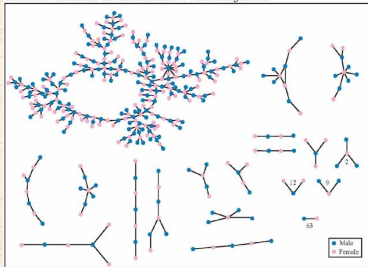


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Interaction networks: social networks

- 🧱 Snogging
- 🧱 Friendships
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- 🧱 Boards and directors
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- 🧱 [facebook](#) ↗ [twitter](#) ↗,

The Structure of Romantic and Sexual Relations at "Jefferson High School"



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- 🧱 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

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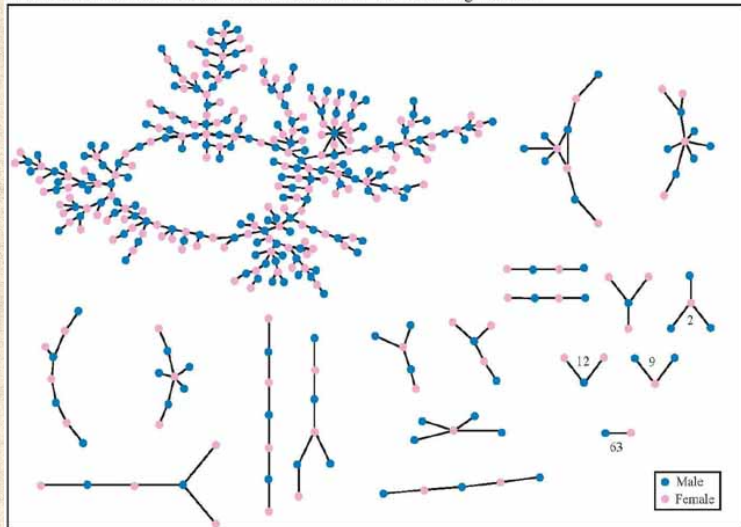
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The Structure of Romantic and Sexual Relations at "Jefferson High School"



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 Consumer purchases

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-  Consumer purchases
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Consumer purchases
(Walmart, Target, Amazon, ...)



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Consumer purchases
(Walmart, Target, Amazon, ...)



Thesauri: Networks of words generated by
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Consumer purchases
(Walmart, Target, Amazon, ...)





Thesauri: Networks of words generated by meanings



Knowledge/Databases/Ideas



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learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
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Neural reboot (NR):

Dog has fun.

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Examples of
Complex
Networks


Physical networks

Interaction networks

Relational networks

References



<https://www.youtube.com/watch?v=7xEX-48RHCY?rel=0> 

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