

Overview of Complex Networks

Last updated: 2020/09/12, 13:39:25 EDT

Principles of Complex Systems, Vol. 1 | @pocsvox
CSYS/MATH 300, Fall, 2020

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



Licensed under the [Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/).

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



1 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



2 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



5 of 41

Thesaurus deliciousness:

network
noun

- 1 a *network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plus.
- 2 a *network of lanes* MAZE, labyrinth, warren, tangle.
- 3 a *network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent [etymological investigation](#):

- Opus reticulatum:
- A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network/>]

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

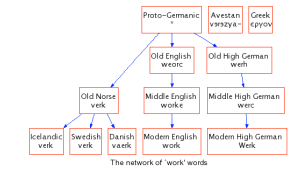
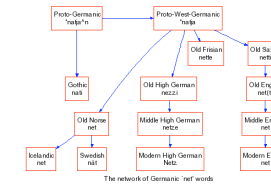
From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mech** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>

Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"
Watts and Strogatz, Nature, **393**, 440–442, 1998. [16]

Times cited: ~ 37,460 (as of October 24, 2018)



"Emergence of scaling in random networks"
Barabási and Albert, Science, **286**, 509–511, 1999. [2]

Times cited: ~ 32,093 (as of October 24, 2018)

Outline

Complex Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

Examples of Complex Networks

- Physical networks
- Interaction networks
- Relational networks

References

net•work |netwɜːk|
noun

- 1 an arrangement of intersecting horizontal and vertical lines.
 - a complex system of roads, railroads, or other transportation routes : a *network of railroads*.
- 2 a group or system of interconnected people or things : a *trade network*.
 - a group of people who exchange information, contacts, and experience for professional or social purposes : a *support network*.
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television*.
 - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | a *local cellular phone network*.
 - a system of connected electrical conductors.

verb [trans.]

- connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals*.
- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations*.
 - [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation*.



5 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



6 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



7 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



8 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



9 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



10 of 41

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks
References



12 of 41

Review articles:



"Complex Networks: Structure and Dynamics" Boccaletti et al., Physics Reports, **424**, 175–308, 2006. [3]

Times cited: ~ 8,533 (as of October 24, 2018)



"The structure and function of complex networks" M. E. J. Newman, SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: ~ 17,782 (as of October 24, 2018)



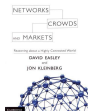
"Statistical mechanics of complex networks" Albert and Barabási, Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited: ~ 20,531 (as of October 24, 2018)

Popularity according to textbooks:



"Networks" by Mark Newman (2018). [11]



"Networks, crowds, and markets: Reasoning about a highly connected world" by Easley and Kleinberg (2010). [7]

<http://cs.cornell.edu/home/kleinber/networks-book/>

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

13 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

14 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

15 of 41

Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [15]

Numerous others ...

- Complex Social Networks—F. Vega-Redondo [14]
- Fractal River Basins: Chance and Self-Organization—I. Rodriguez-Iturbe and A. Rinaldo [13]
- Random Graph Dynamics—R. Durrett
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish **mechanistic** explanations.

*If this is upsetting, maybe string theory is for you ...

More observations

Web-scale data sets can be overly **exciting**.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- "The Unreasonable Effectiveness of Data," Halevy et al. [9].
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [17]

But:

- For scientists, description is only part of the battle.
- We still need to **understand**.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

Super Basic definitions

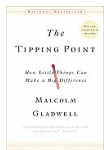
Node degree = Number of links per node

- Notation: Node i 's degree = k_i .
- $k_i = 0, 1, 2, \dots$
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- Connection between number of edges m and average degree:

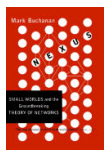
$$\langle k \rangle = \frac{2m}{N}$$

Defn: \mathcal{N}_i = the set of i 's k_i neighbors

Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

16 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

17 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

19 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

20 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

22 of 41

PoCS, Vol. 1
@pocsvox
Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

23 of 41

Super Basic definitions

Adjacency matrix:

We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(n.b., for numerical work, we always use sparse matrices.)

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

24 of 41

Examples

So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

25 of 41

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids



Distribution (branching) versus **redistribution** (cyclical)

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

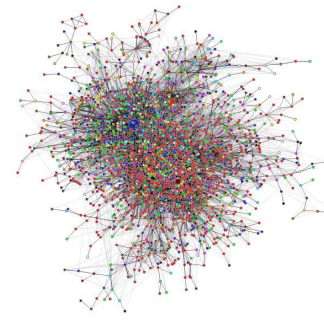
References

27 of 41

Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media



datamining.typepad.com

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

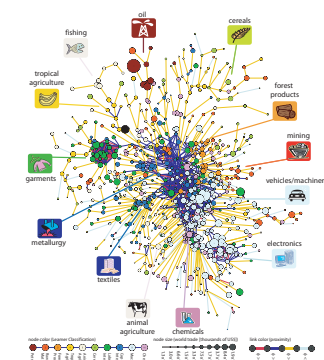
Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

29 of 41

topics:

- Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [10]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

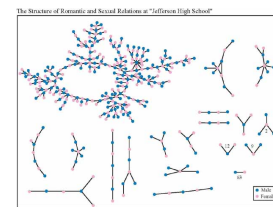
References

30 of 41

Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook twitter



(Bearman et al., 2004)

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

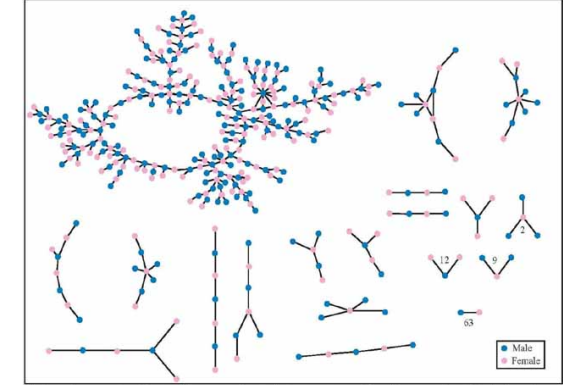
References

31 of 41

'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else)

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

32 of 41

Examples

Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: [bit.ly](#) [flickr](#)

common tags [cloud](#) | [list](#)

[community](#) [daily](#) [dictionary](#) [education](#) [encyclopedia](#) [english](#) [free](#) [imported](#) [info](#) [information](#) [internet](#) [knowledge](#) [learning](#) [news](#) [reference](#) [research](#) [resource](#) [resources](#) [search](#) [tools](#) [useful](#) [web](#) [web2.0](#) [wiki](#) [wikipedia](#)

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

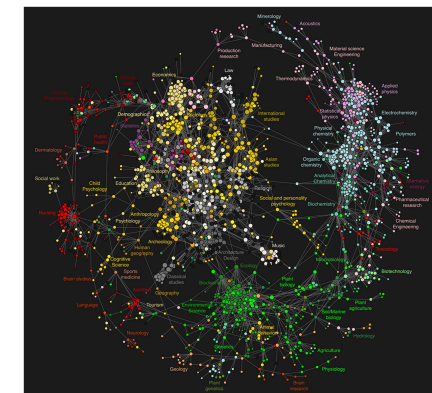
Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

34 of 41

Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

PoCS, Vol. 1
@pocsvox
Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks
Physical networks
Interaction networks
Relational networks

References

35 of 41

References I

- [1] R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. *Rev. Mod. Phys.*, 74:47–97, 2002. [pdf](#)
- [2] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–511, 1999. [pdf](#)
- [3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. *Physics Reports*, 424:175–308, 2006. [pdf](#)

PoCS, Vol. 1
@pocsvox
[Overview of Complex Networks](#)

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



37 of 41

References II

- [4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila. Clickstream data yields high-resolution maps of science. *PLoS ONE*, 4:e4803, 2009. [pdf](#)
- [5] S. Bornholdt and H. G. Schuster, editors. Handbook of Graphs and Networks. Wiley-VCH, Berlin, 2003.
- [6] S. N. Dorogovtsev and J. F. F. Mendes. Evolution of Networks. Oxford University Press, Oxford, UK, 2003.

PoCS, Vol. 1
@pocsvox
[Overview of Complex Networks](#)

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



38 of 41

References III

- [7] D. Easley and J. Kleinberg. Networks, crowds, and markets: Reasoning about a highly connected world. Cambridge University Press, 2010.
- [8] M. Gladwell. The Tipping Point. Little, Brown and Company, New York, 2000.
- [9] A. Halevy, P. Norvig, and F. Pereira. The unreasonable effectiveness of data. *IEEE Intelligent Systems*, 24:8–12, 2009. [pdf](#)
- [10] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman. The product space conditions the development of nations. *Science*, 317:482–487, 2007. [pdf](#)

PoCS, Vol. 1
@pocsvox
[Overview of Complex Networks](#)

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



39 of 41

References IV

- [11] M. Newman. Networks. Oxford university press, 2nd edition, 2018.
- [12] M. E. J. Newman. The structure and function of complex networks. *SIAM Rev.*, 45(2):167–256, 2003. [pdf](#)
- [13] I. Rodríguez-Iturbe and A. Rinaldo. Fractal River Basins: Chance and Self-Organization. Cambridge University Press, Cambridge, UK, 1997.
- [14] F. Vega-Redondo. Complex Social Networks. Cambridge University Press, 2007.

PoCS, Vol. 1
@pocsvox
[Overview of Complex Networks](#)

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



40 of 41

References V

- [15] D. J. Watts. Six Degrees. Norton, New York, 2003.
- [16] D. J. Watts and S. J. Strogatz. Collective dynamics of ‘small-world’ networks. *Nature*, 393:440–442, 1998. [pdf](#)
- [17] E. Wigner. The unreasonable effectiveness of mathematics in the natural sciences. *Communications on Pure and Applied Mathematics*, 13:1–14, 1960. [pdf](#)

PoCS, Vol. 1
@pocsvox
[Overview of Complex Networks](#)

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



41 of 41