

# Overview of Complex Networks

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Principles of Complex Systems, Vol. 1 | @pocsvox  
CSYS/MATH 300, Fall, 2020

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Computational Story Lab | Vermont Complex Systems Center  
Vermont Advanced Computing Core | University of Vermont



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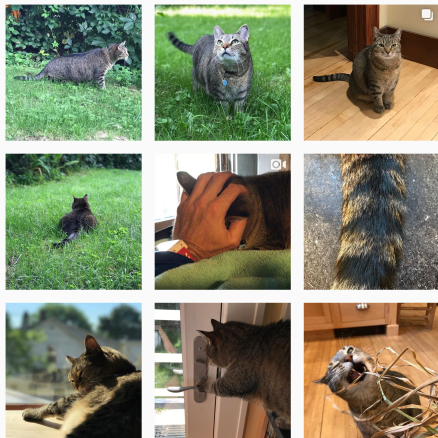
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# net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj. ] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [ trans. ]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj. ] ( **networked** ) *networked workstations.*
- [ intrans. ] [often as n. ] ( **networking** ) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*





## Thesaurus deliciousness:

# network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.

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From Keith Briggs's excellent etymological  
investigation: 



Opus  
reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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# Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

-  1658–: reticulate structures in animals
-  1839–: rivers and canals
-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

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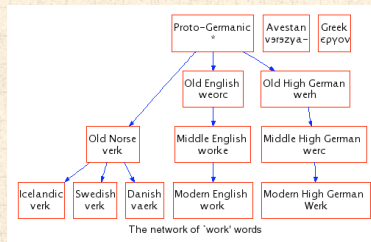
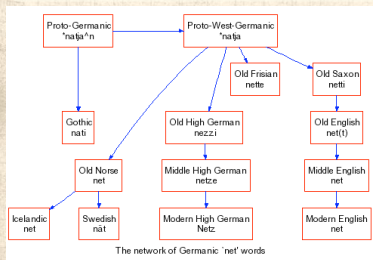



# Ancestry:


Net and Work are venerable old words:

 **'Net'** first used to mean spider web (King Ælfréd, 888).

 **'Work'** appear to have long meant purposeful action.



 **'Network'** = something built based on the idea of natural, flexible lattice or web.

 c.f., ironwork, stonework, fretwork.





## Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



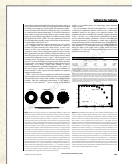
- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>




# Popularity (according to Google Scholar)


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"Collective dynamics of 'small-world' networks" 


Watts and Strogatz,  
Nature, **393**, 440–442, 1998. [16]

Times cited:  ~ 37,460 (as of October 24, 2018)



"Emergence of scaling in random networks" 

Barabási and Albert,  
Science, **286**, 509–511, 1999. [2]

Times cited:  ~ 32,093 (as of October 24, 2018)

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## Review articles:



### "Complex Networks: Structure and Dynamics"


Boccaletti et al.,  
Physics Reports, **424**, 175–308, 2006. [3]

Times cited:  ~ **8,533** (as of October 24, 2018)



### "The structure and function of complex networks"


M. E. J. Newman,  
SIAM Rev., **45**, 167–256, 2003. [12]

Times cited:  ~ **17,782** (as of October 24, 2018)



### "Statistical mechanics of complex networks"

Albert and Barabási,  
Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited:  ~ **20,531** (as of October 24, 2018)

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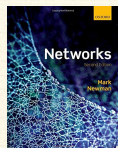
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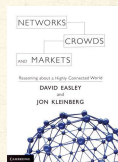


# Popularity according to textbooks:

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“Networks” [a](#) [↗](#)  
by Mark Newman (2018). <sup>[11]</sup>



“Networks, crowds, and markets: Reasoning  
about a highly connected world” [a](#) [↗](#)  
by Easley and Kleinberg (2010). <sup>[7]</sup>

<http://cs.cornell.edu/home/kleinber/networks-book/> [↗](#)

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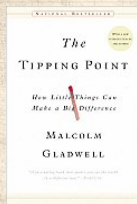




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The Tipping Point: How Little Things can  
make a Big Difference—Malcolm  
Gladwell [8]

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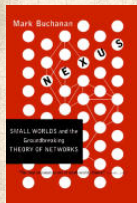
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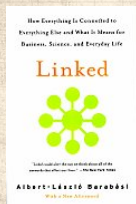


Nexus: Small Worlds and the  
Groundbreaking Science of  
Networks—Mark Buchanan



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Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts <sup>[15]</sup>

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## Numerous others ...

-  Complex Social Networks—F. Vega-Redondo <sup>[14]</sup>
-  Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo <sup>[13]</sup>
-  Random Graph Dynamics—R. Durrette
-  Scale-Free Networks—Guido Caldarelli
-  Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
-  Complex Graphs and Networks—Fan Chung
-  Social Network Analysis—Stanley Wasserman and Kathleen Faust
-  Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster <sup>[5]</sup>
-  Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes <sup>[6]</sup>



# More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.\*
- A worthy goal: establish **mechanistic explanations**.

*\*If this is upsetting, maybe string theory is for you ...*

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# More observations

 Web-scale data sets can be overly **exciting**.


Witness:


 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 "The Unreasonable Effectiveness of Data,"  
Halevy et al. [9].

 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [17]

But:

 For scientists, description is only part of the battle.

 We still need to **understand**.

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# Super Basic definitions

**Nodes** = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

**Links** = Connections between nodes

 **Links** may be directed or undirected.

 **Links** may be binary or weighted.

Other spiffing words: vertices and edges.

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## Node degree = Number of links per node


 Notation: Node  $i$ 's degree =  $k_i$ .

  $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network =  $\langle k \rangle$   
(and sometimes  $z$ )

 Connection between number of edges  $m$  and  
average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

 Defn:  $\mathcal{N}_i$  = the set of  $i$ 's  $k_i$  neighbors

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## Adjacency matrix:

 We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)

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## So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...

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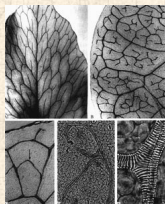
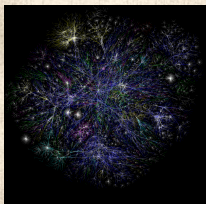



# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids



-  **Distribution** (branching) versus **redistribution** (cyclical)

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## Interaction networks


 The Blogosphere


 Biochemical networks


 Gene-protein networks

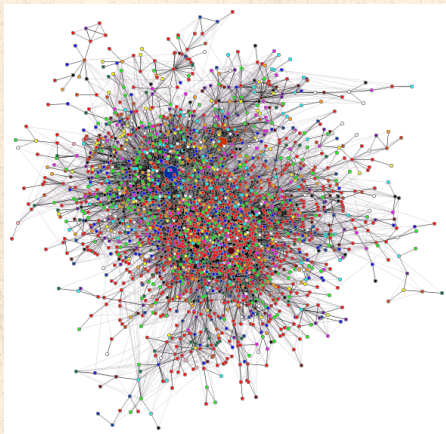
 Food webs: who eats whom


 The World Wide Web (?)

 Airline networks

 Call networks (AT&T)

 The Media



[datamining.typepad.com](http://datamining.typepad.com) 

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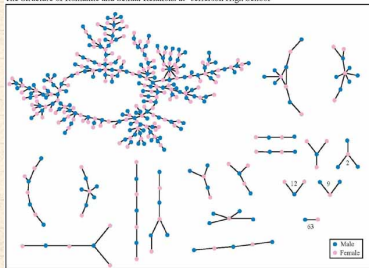


# Examples

## Interaction networks: social networks

- 🧱 Snogging
- 🧱 Friendships
- 🧱 Acquaintances
- 🧱 Boards and directors
- 🧱 Organizations
- 🧱 [facebook](#) ↗ [twitter](#) ↗,

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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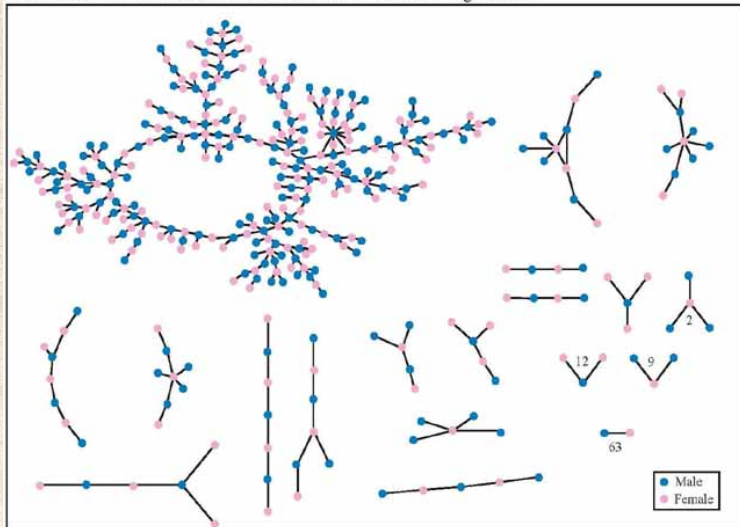
- 🧱 'Remotely sensed' by: email activity, instant messaging, phone logs (\*cough\*).



# Examples

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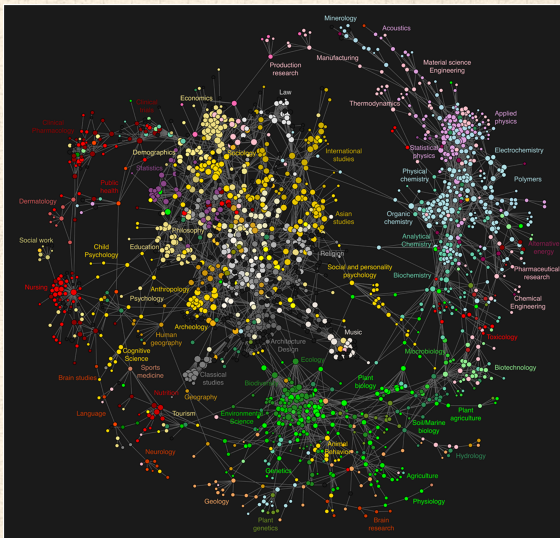
-  Consumer purchases (Walmart, Target, Amazon, ...)
-  Thesauri: Networks of words generated by meanings
-  Knowledge/Databases/Ideas
-  Metadata—Tagging: [bit.ly](http://bit.ly)  [flickr](https://www.flickr.com/photos/14914470@N00/) 

**common tags**    cloud | [list](#)

community daily dictionary education **encyclopedia**  
english free imported info information internet knowledge  
learning news **reference** research resource  
resources search tools useful web web2.0 **wiki**  
**wikipedia**



# Clickworthy Science:



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“Clickstream Data Yields High-Resolution Maps of Science”,  
Bollen et al. [4], 2009.





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[Physics Reports, 424:175–308, 2006. pdf](#)

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Clickstream data yields high-resolution maps of science.

[PLoS ONE, 4:e4803, 2009. pdf](#) 

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Science, 317:482–487, 2007. [pdf](#) 





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