

# The Amusing Law of Benford

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1 of 13



2 of 13



4 of 13

## Benford's Law—The Law of First Digits

### Observed for

- 📦 Fundamental constants (electron mass, charge, etc.)
- 📦 Utility bills
- 📦 Numbers on tax returns (ha!)
- 📦 Death rates
- 📦 Street addresses
- 📦 Numbers in newspapers

📦 Cited as [evidence of fraud](#) in the 2009 Iranian elections.



5 of 13



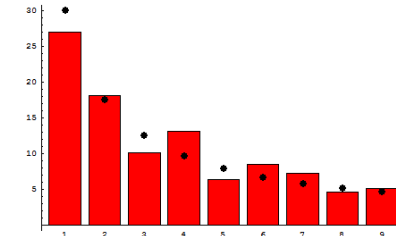
6 of 13



7 of 13

## Benford's Law—The Law of First Digits

### Population of countries:



Taken from [here](#).



8 of 13



9 of 13



10 of 13

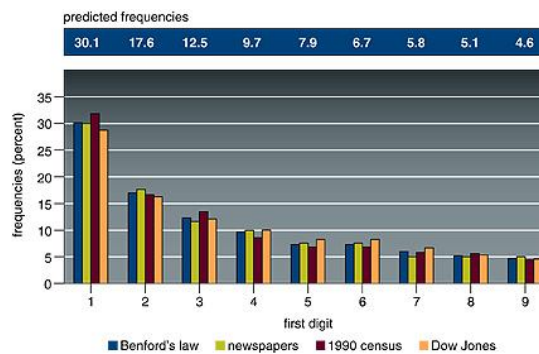
## Outline

### Benford's Law

### References

## Benford's Law—The Law of First Digits

### Real data:



From 'The First-Digit Phenomenon' by T. P. Hill (1998) [1]

## Essential story



$$P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right)$$

$$= \log_b \left( \frac{d+1}{d} \right)$$

$$= \log_b (d+1) - \log_b (d)$$

📦 Observe this distribution if numbers are distributed uniformly in log-space:

$$P(\log_e x) d(\log_e x) \propto 1 \cdot d(\log_e x) = x^{-1} dx = P(x) dx$$

- 📦 Power law distributions at work again...
- 📦 Extreme case of  $\gamma \approx 1$ .

## Benford's Law—The Law of First Digits



$$P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right)$$

for certain sets of 'naturally' occurring numbers in base  $b$

- 📦 Around 30.1% of first digits are '1', compared to only 4.6% for '9'.
- 📦 First observed by [Simon Newcomb](#) [3] in 1881 "Note on the Frequency of Use of the Different Digits in Natural Numbers"
- 📦 Independently discovered in 1938 by [Frank Benford](#).
- 📦 Newcomb almost always noted but Benford gets the stamp, according to [Stigler's Law of Eponymy](#).



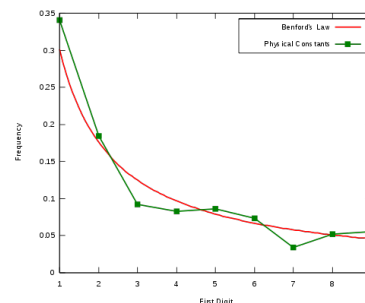
2 of 13



4 of 13

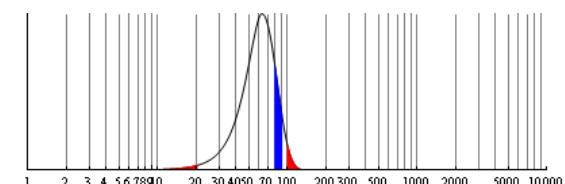
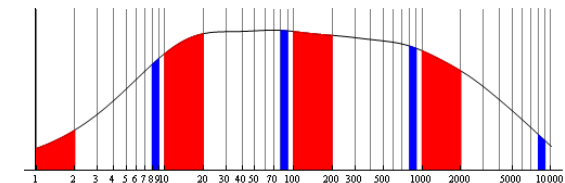
## Benford's Law—The Law of First Digits

### Physical constants of the universe:



Taken from [here](#).

## Benford's law



Taken from [here](#).



6 of 13



7 of 13



9 of 13



10 of 13

“Citations to articles citing Benford’s law: A Benford analysis”

Tariq Ahmad Mir,

Preprint available at

<http://arxiv.org/abs/1602.01205>, 2016. [2]

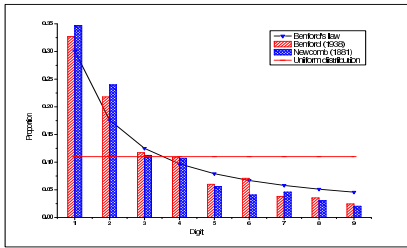


Fig. 1: The observed proportions of first digits of citations received by the articles citing FB and SN on September 30, 2012. For comparison the proportions expected from BL and uniform distributions are also shown.



On counting and logarithms:



Earlier: Listen to Radiolab's "Numbers."

Now: Benford's Law



References I

- [1] T. P. Hill. The first-digit phenomenon. *American Scientist*, 86:358–, 1998.
- [2] T. A. Mir. Citations to articles citing Benford's law: A Benford analysis, 2016. Preprint available at <http://arxiv.org/abs/1602.01205>. pdf
- [3] S. Newcomb. Note on the frequency of use of the different digits in natural numbers. *American Journal of Mathematics*, 4:39–40, 1881. pdf

