P o C S What's The Story?

Principles of Complex Systems, Vol. 1, CSYS/MATH 300
University of Vermont, Fall 2020
Assignment 12 • code name: Basic Rocket Science

Due: Friday, December 4, by 4:59 pm, 2020.
Relevant clips, episodes, and slides are listed on the assignment's page:
http://www.uvm.edu/pdodds/teaching/courses/2020-08UVM-300/assignments/12/
Some useful reminders:
Deliverator: Prof. Peter Sheridan Dodds (contact through Teams)
Assistant Deliverator: Michael Arnold (contact through Teams)
Office: The Ether
Office hours: Tuesdays, 12 to 12:50 pm; Wednesdays, 1:15 pm to 2:05 pm; Thursdays, 12 to 12:50 pm; all scheduled on Teams
Course website: http://www.uvm.edu/pdodds/teaching/courses/2020-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

For coding, we recommend you improve your skills with Python, R, and/or Julia. The Deliverator uses Matlab.

Graduate students are requested to use LaTeX (or related TeX variant). If you are new to LaTeX, please endeavor to submit at least n questions per assignment in LaTeX, where n is the assignment number.

Assignment submission: Via Blackboard.

Please submit your project's current draft in pdf format via Blackboard by the same time specified for this assignment. For teams, please list all team member names clearly at the start.

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   We take a look at the 80/20 rule, 1 per centers, and similar concepts.

   Take x to be the wealth held by an individual in a population of n people, and the number of individuals with wealth between x and $x + \mathrm{d}x$ to be approximately $N(x)\mathrm{d}x$.

   Given a power-law size frequency distribution $N(x) = cx^{-\gamma}$ where $x_{\mathrm{min}} \ll x \ll \infty$, determine the value of γ for which the so-called 80/20 rule holds.

   In other words, find γ for which the bottom 4/5 of the population holds 1/5 of the overall wealth, and the top 1/5 holds the remaining 4/5.

   Assume the mean is finite, i.e., $\gamma > 2$.

(a) Determine the total wealth W in the system given $\int_{x_{\min}}^{\infty} \mathrm{d}x N(x) = n$.

(b) Imagine that $100q$ percent of the population holds $100(1-r)$ percent of the wealth.

Show γ depends on q and r as

$$\gamma = 1 + \frac{\ln \frac{1}{(1-q)}}{\ln \frac{1}{(1-q)} - \ln \frac{1}{r}}.$$

(c) Given the above, is every pairing of q and r possible?

(d) Find γ for the 80/20 requirement ($q = r = 4/5$).

(e) For the "80/20" γ you find, determine how much wealth $100q$ percent of the population possesses as a function of q and plot the result.