# Curious and Interesting Things

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## Complex Networks | @networksvox
## CSYS/MATH 303, Spring, 2019

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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# Outline



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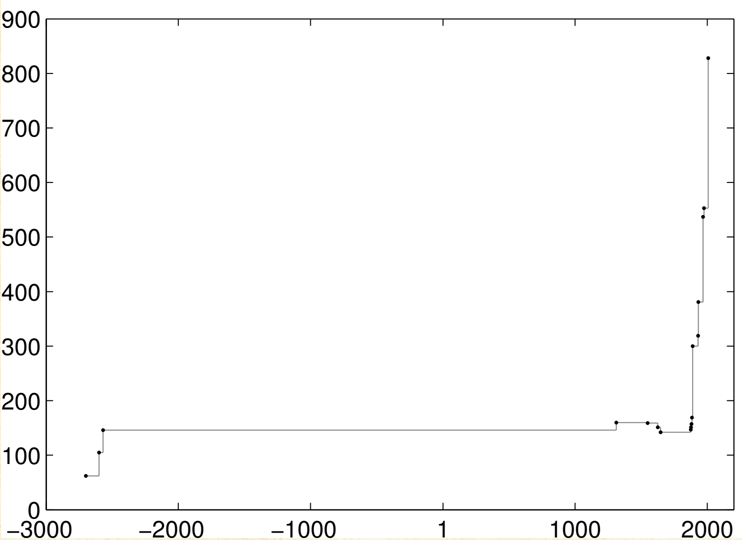
# What's this?

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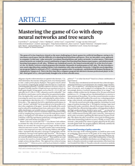
# Advances in sociotechnical algorithms:


ARTICLE

Mastering the game of Go with deep neural networks and tree search

"Mastering the game of Go with deep neural networks and tree search"
Silver and Silver,
Nature, **529**, 484–489, 2016. [6]

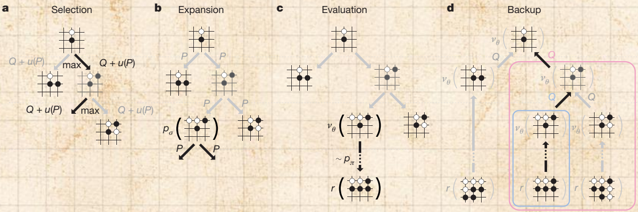
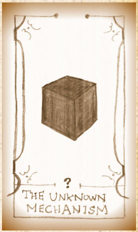



**Figure 3 | Monte Carlo tree search in AlphaGo. a**, Each simulation traverses the tree by selecting the edge with maximum action value $Q$, plus a bonus $u(P)$ that depends on a stored prior probability $P$ for that edge. **b**, The leaf node may be expanded; the new node is processed once by the policy network $p_\sigma$ and the output probabilities are stored as prior probabilities $P$ for each action. **c**, At the end of a simulation, the leaf node is evaluated in two ways: using the value network $v_\theta$; and by running a rollout to the end of the game with the fast rollout policy $p_\pi$, then computing the winner with function $r$. **d**, Action values $Q$ are updated to track the mean value of all evaluations $r(\cdot)$ and $v_\theta(\cdot)$ in the subtree below that action.

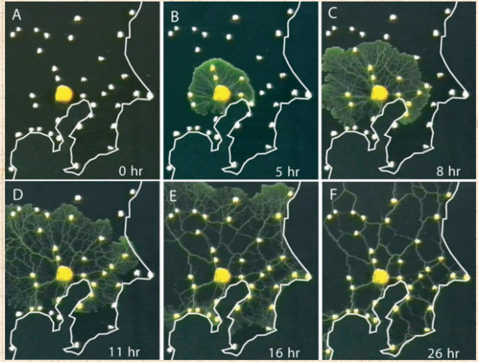


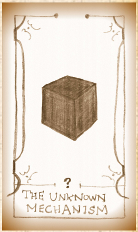

- Nature News (2016): Digital Intuition
- Wired (2012): Network Science of the game of Go


# "Rules for Biologically Inspired Adaptive Network Design"

Tero et al.,
Science, **327**, 439-442, 2010. [7]






Urban deslime in action:
https://www.youtube.com/watch?v=GwKuFREOgmo

"Citations to articles citing Benford's law: A Benford analysis"
Tariq Ahmad Mir,
Preprint available at http://arxiv.org/abs/1602.01205, 2016. [4]

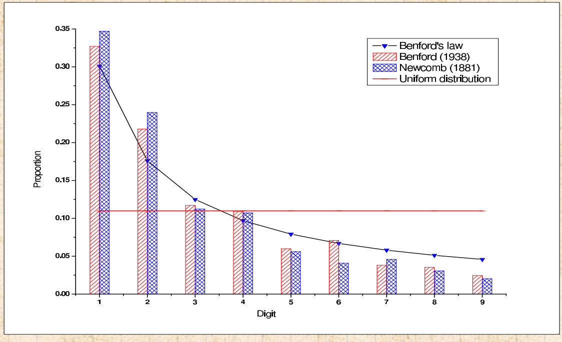
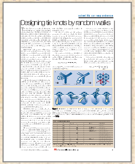



Fig. 1: The observed proportions of first digits of citations received by the articles citing FB and SN on September 30, 2012. For comparison the proportions expected from BL and uniform distributions are also shown.

# Applied knot theory:




"Designing tie knots by random walks"
Fink and Mao,
Nature, **398**, 31–32, 1999. [1]

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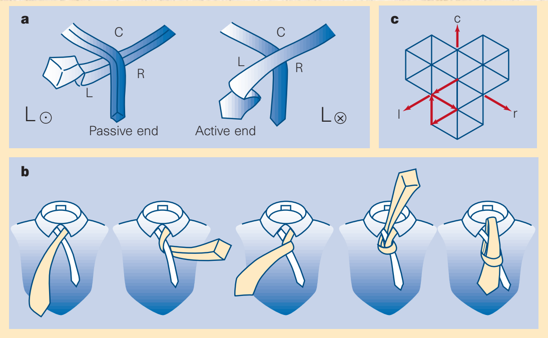


Figure 1 All diagrams are drawn in the frame of reference of the mirror image of the actual tie. **a,** The two ways of beginning a knot, $L_{\odot}$ and $L_{\otimes}$. For knots beginning with $L_{\odot}$, the tie must begin inside-out. **b,** The four-in-hand, denoted by the sequence $L_{\otimes} R_{\odot} L_{\otimes} C_{\odot} T$. **c,** A knot may be represented by a persistent random walk on a triangular lattice. The example shown is the four-in-hand, indicated by the walk $\hat{l}\,\hat{r}\,\hat{l}\,\hat{c}$.

# Applied knot theory:



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Table 1 **Aesthetic tie knots**

| $h$ | $\gamma$ | $\gamma/h$ | $K(h, \gamma)$ | $s$ | $b$ | Name | Sequence |
|---|---|---|---|---|---|---|---|
| 3 | 1 | 0.33 | 1 | 0 | 0 | | $L_\odot R_\otimes C_\odot T$ |
| 4 | 1 | 0.25 | 1 | −1 | 1 | Four-in-hand | $L_\otimes R_\odot L_\otimes C_\odot T$ |
| 5 | 2 | 0.40 | 2 | −1 | 0 | Pratt knot | $L_\odot C_\otimes R_\odot L_\otimes C_\odot T$ |
| 6 | 2 | 0.33 | 4 | 0 | 0 | Half-Windsor | $L_\otimes R_\odot C_\otimes L_\odot R_\otimes C_\odot T$ |
| 7 | 2 | 0.29 | 6 | −1 | 1 | | $L_\odot R_\otimes L_\odot C_\otimes R_\odot L_\otimes C_\odot T$ |
| 7 | 3 | 0.43 | 4 | 0 | 1 | | $L_\odot C_\otimes R_\odot C_\otimes L_\odot R_\otimes C_\odot T$ |
| 8 | 2 | 0.25 | 8 | 0 | 2 | | $L_\otimes R_\odot L_\otimes C_\odot R_\otimes L_\odot R_\otimes C_\odot T$ |
| 8 | 3 | 0.38 | 12 | −1 | 0 | Windsor | $L_\otimes C_\odot R_\otimes L_\odot C_\otimes R_\odot L_\otimes C_\odot T$ |
| 9 | 3 | 0.33 | 24 | 0 | 0 | | $L_\odot R_\otimes C_\odot L_\otimes R_\odot C_\otimes L_\odot R_\otimes C_\odot T$ |
| 9 | 4 | 0.44 | 8 | −1 | 2 | | $L_\odot C_\otimes R_\odot C_\otimes L_\odot C_\otimes R_\odot L_\otimes C_\odot T$ |

Knots are characterized by half-winding number $h$, centre number $\gamma$, centre fraction $\gamma/h$, knots per class $K(h, \gamma)$, symmetry $s$, balance $b$, name and sequence.

- $h$ = number of moves
- $\gamma$ = number of center moves
- $K(h,\gamma) = 2^{\gamma-1}\binom{h-\gamma-2}{\gamma-1}$

- $s = \sum_{i=1}^{h} x_i$ where $x$ = -1 for $L$ and +1 for $R$.
- $b = \frac{1}{2}\sum_{i=2}^{h-1} |\omega_i + \omega_{i-1}|$ where $\omega = \pm 1$ represents winding direction.

# Irregular verbs

## Cleaning up the code that is English:


LETTERS
Quantifying the evolutionary dynamics of language

"Quantifying the evolutionary dynamics of language"
Lieberman et al.,
Nature, **449**, 713–716, 2007. [2]


- Exploration of how verbs with irregular conjugation gradually become regular over time.
- Comparison of verb behavior in Old, Middle, and Modern English.

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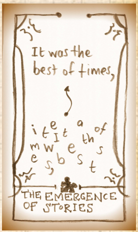
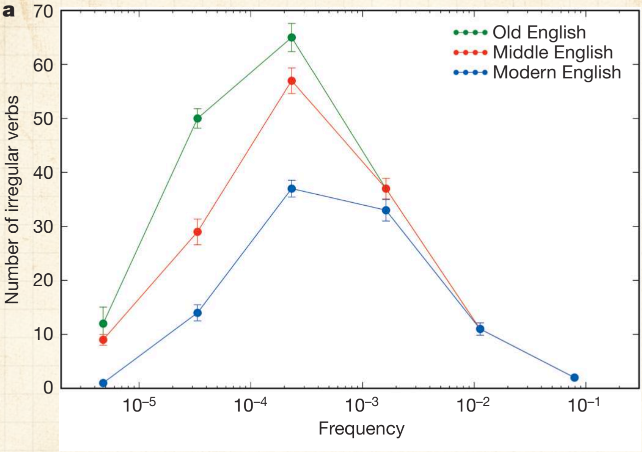
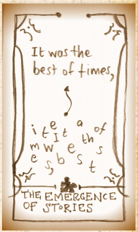


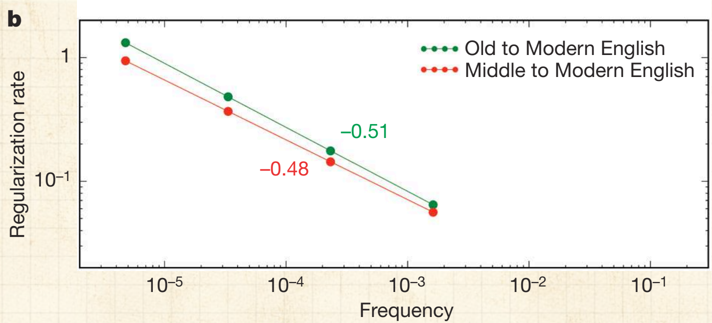
# Irregular verbs



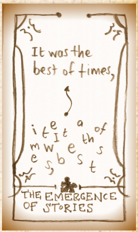

- Universal tendency towards regular conjugation
- Rare verbs tend to be regular in the first place

# Irregular verbs




- Rates are relative.





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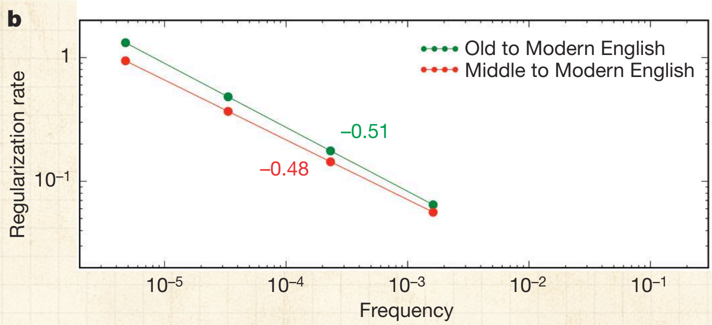


- Rates are relative.
- The more common a verb is, the more resilient it is to change.

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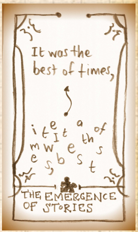


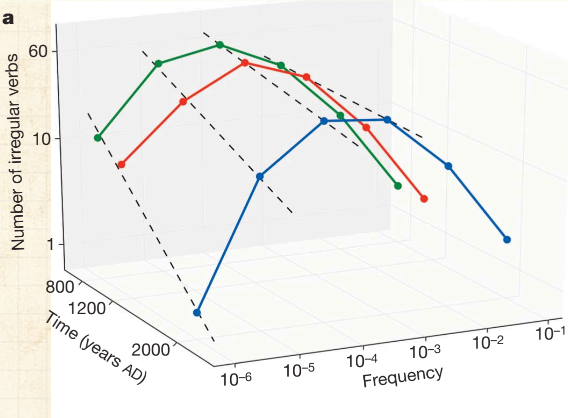
# Irregular verbs

**Table 1 | The 177 irregular verbs studied**

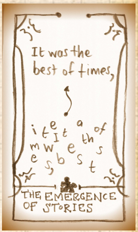
| Frequency | Verbs | Regularization (%) | Half-life (yr) |
|---|---|---|---|
| $10^{-1}$–1 | be, have | 0 | 38,800 |
| $10^{-2}$–$10^{-1}$ | come, do, find, get, give, go, know, say, see, take, think | 0 | 14,400 |
| $10^{-3}$–$10^{-2}$ | begin, break, bring, buy, choose, draw, drink, drive, eat, fall, fight, forget, grow, hang, help, hold, leave, let, lie, lose, reach, rise, run, seek, set, shake, sit, sleep, speak, stand, teach, throw, understand, walk, win, work, write | 10 | 5,400 |
| $10^{-4}$–$10^{-3}$ | arise, bake, bear, beat, bind, bite, blow, bow, burn, burst, carve, chew, climb, cling, creep, dare, dig, drag, flee, float, flow, fly, fold, freeze, grind, leap, lend, lock, melt, reckon, ride, rush, shape, shine, shoot, shrink, sigh, sing, sink, slide, slip, smoke, spin, spring, starve, steal, step, stretch, strike, stroke, suck, swallow, swear, sweep, swim, swing, tear, wake, wash, weave, weep, weigh, wind, yell, yield | 43 | 2,000 |
| $10^{-5}$–$10^{-4}$ | bark, bellow, bid, blend, braid, brew, cleave, cringe, crow, dive, drip, fare, fret, glide, gnaw, grip, heave, knead, low, milk, mourn, mow, prescribe, redden, reek, row, scrape, seethe, shear, shed, shove, slay, slit, smite, sow, span, spurn, sting, stink, strew, stride, swell, tread, uproot, wade, warp, wax, wield, wring, writhe | 72 | 700 |
| $10^{-6}$–$10^{-5}$ | bide, chide, delve, flay, hew, rue, shrive, slink, snip, spew, sup, wreak | 91 | 300 |

177 Old English irregular verbs were compiled for this study. These are arranged according to frequency bin, and in alphabetical order within each bin. Also shown is the percentage of verbs in each bin that have regularized. The half-life is shown in years. Verbs that have regularized are indicated in red. As we move down the list, an increasingly large fraction of the verbs are red; the frequency-dependent regularization of irregular verbs becomes immediately apparent.

- Red = regularized
- Estimates of half-life for regularization ($\propto f^{1/2}$)




- 'Wed' is next to go.
- -ed is the winning rule...
- But 'snuck' is sneaking up on sneaked. [3]



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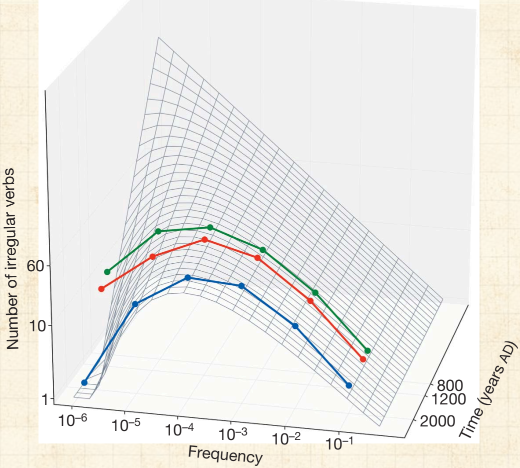
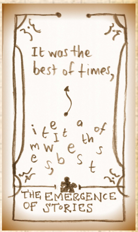
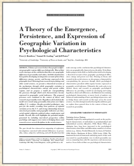



- Projecting back in time to proto-Zipf story of many tools.

# Personality distributions:

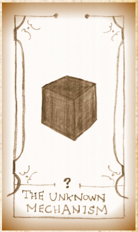

A Theory of the Emergence, Persistence, and Expression of Geographic Variation in Psychological Characteristics

"A Theory of the Emergence, Persistence, and Expression of Geographic Variation in Psychological Characteristics"
Rentfrow, Gosling, and Potter,
Perspectives on Psychological Science, **3**, 339–369, 2008. [5]

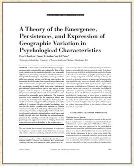
Five Factor Model (FFM):

- Extraversion [E]
- Agreeableness [A]
- Conscientiousness [C]
- Neuroticism [N]
- Openness [O]

"…a robust and widely accepted framework for conceptualizing the structure of personality… Although the FFM is not universally accepted in the field…" [5]



# Personality distributions: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A concern: self-reported data.

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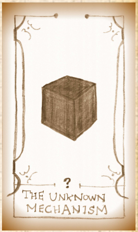
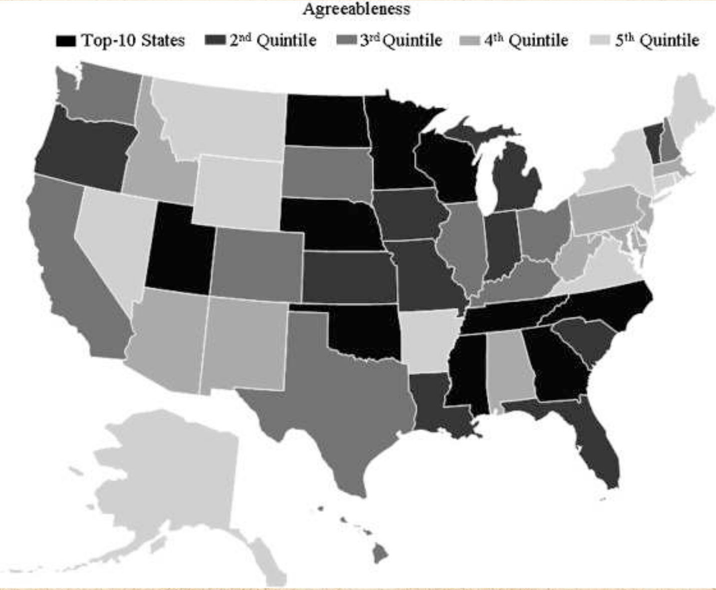
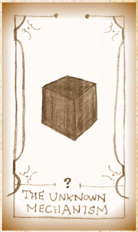


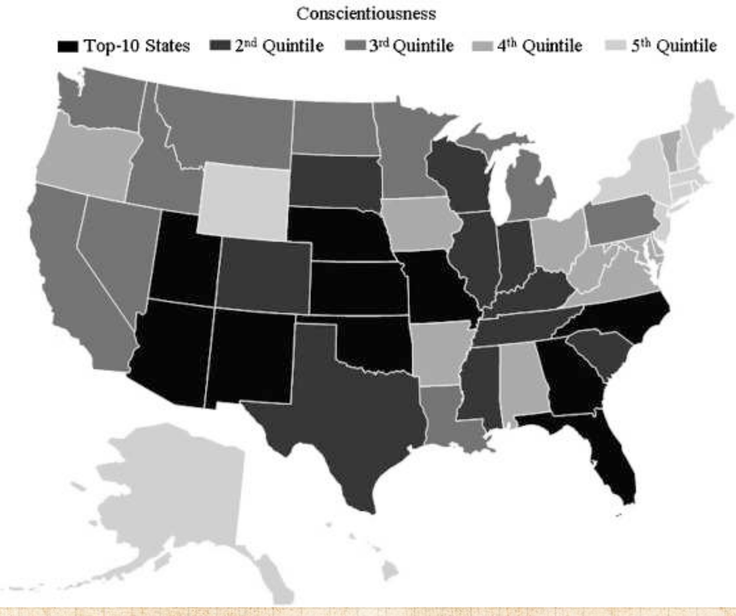
# Agreeableness:

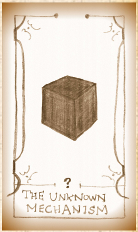




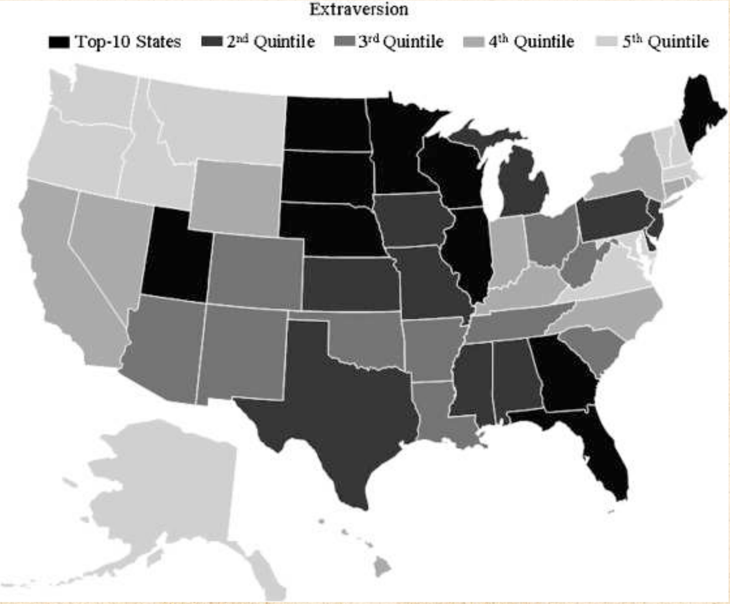
# Conscientiousness:

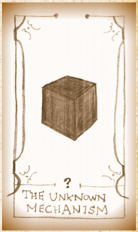


# Extraversion:

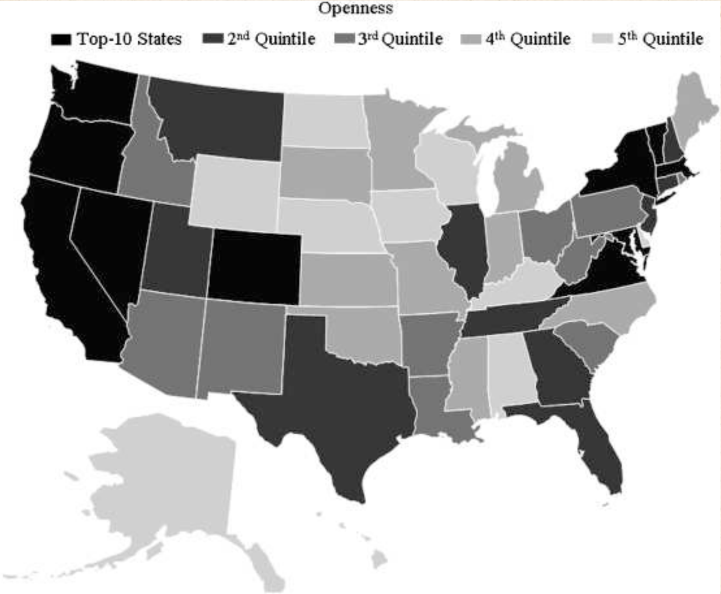


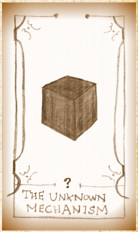


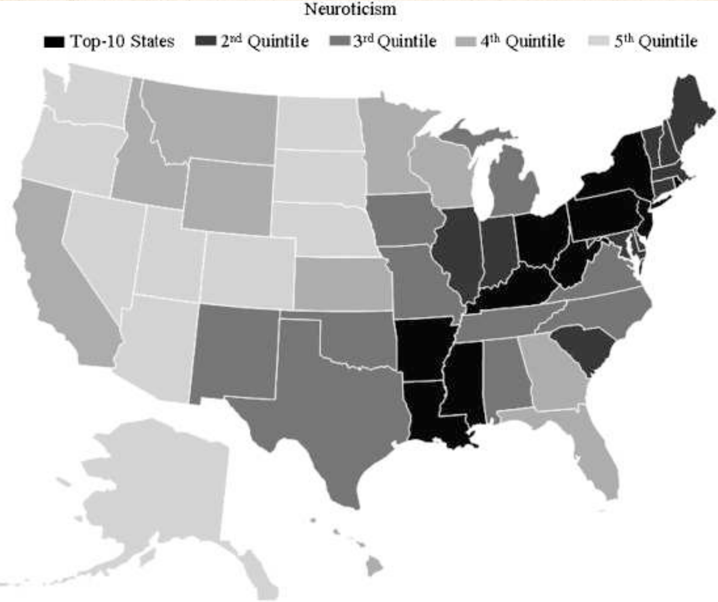
# Openness



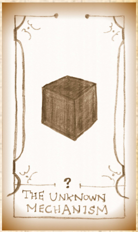

Openness
Top-10 States
2nd Quintile
3rd Quintile
4th Quintile
5th Quintile


THE UNKNOWN MECHANISM

# Neuroticism:

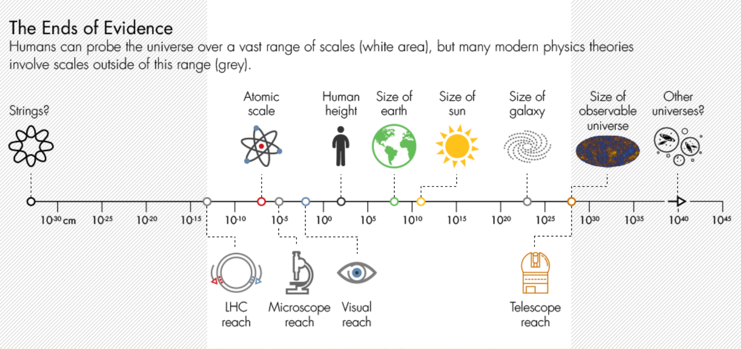




# Limits of testability and happiness in Science:

From A Fight for the soul of Science in Quanta Magazine (2016/02):

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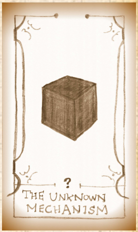

# Europe:

- Many errors called out in comments. Why hasn't this been done well?

# John Conway's Doomsday rule for determining a date's day of the week:

Doomsdays for the Gregorian calendar

| Mon. | Tue. | Wed. | Thu. | Fri. | Sat. | Sun. | Mon. | Tue. | Wed. | Thu. | Fri. | Sat. | Sun. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1898 | 1899 | 1900 | 1901 | 1902 | 1903 | → | 1904 | 1905 | 1906 | 1907 | → | 1908 | 1909 |
| 1910 | 1911 | → | 1912 | 1913 | 1914 | 1915 | → | 1916 | 1917 | 1918 | 1919 | → | 1920 |
| 1921 | 1922 | 1923 | → | 1924 | 1925 | 1926 | 1927 | → | 1928 | 1929 | 1930 | 1931 | → |
| 1932 | 1933 | 1934 | 1935 | → | 1936 | 1937 | 1938 | 1939 | → | 1940 | 1941 | 1942 | 1943 |
| → | 1944 | 1945 | 1946 | 1947 | → | 1948 | 1949 | 1950 | 1951 | → | 1952 | 1953 | 1954 |
| 1955 | → | 1956 | 1957 | 1958 | 1959 | → | 1960 | 1961 | 1962 | 1963 | → | 1964 | 1965 |
| 1966 | 1967 | → | 1968 | 1969 | 1970 | 1971 | → | 1972 | 1973 | 1974 | 1975 | → | 1976 |
| 1977 | 1978 | 1979 | → | 1980 | 1981 | 1982 | 1983 | → | 1984 | 1985 | 1986 | 1987 | → |
| 1988 | 1989 | 1990 | 1991 | → | 1992 | 1993 | 1994 | 1995 | → | 1996 | 1997 | 1998 | 1999 |
| → | 2000 | 2001 | 2002 | 2003 | → | 2004 | 2005 | 2006 | 2007 | → | 2008 | 2009 | 2010 |
| 2011 | → | 2012 | 2013 | 2014 | 2015 | → | 2016 | 2017 | 2018 | 2019 | → | 2020 | 2021 |
| 2022 | 2023 | → | 2024 | 2025 | 2026 | 2027 | → | 2028 | 2029 | 2030 | 2031 | → | 2032 |
| 2033 | 2034 | 2035 | → | 2036 | 2037 | 2038 | 2039 | → | 2040 | 2041 | 2042 | 2043 | → |
| 2044 | 2045 | 2046 | 2047 | → | 2048 | 2049 | 2050 | 2051 | → | 2052 | 2053 | 2054 | 2055 |
| → | 2056 | 2057 | 2058 | 2059 | → | 2060 | 2061 | 2062 | 2063 | → | 2064 | 2065 | 2066 |
| 2067 | → | 2068 | 2069 | 2070 | 2071 | → | 2072 | 2073 | 2074 | 2075 | → | 2076 | 2077 |
| 2078 | 2079 | → | 2080 | 2081 | 2082 | 2083 | → | 2084 | 2085 | 2086 | 2087 | → | 2088 |
| 2089 | 2090 | 2091 | → | 2092 | 2093 | 2094 | 2095 | → | 2096 | 2097 | 2098 | 2099 | 2100 |

- Works for Gregorian (1582–, haphazardly) and the increasingly inaccurate Julian calendars (400 and 28 years cycles).
- Apparently inspired by Lewis Carroll's work on a perpetual calendar.

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References

## Outline:

- Determine "anchor day" for a given century, then find Doomsday for a given year in that century.
- Remember special Doomsday dates and work from there.
- Naturally: Load this year's Doomsday into brain.

## Century's anchor day (Gregorian, Sunday $\equiv$ 0):

$$5 \times \left( \left\lfloor \frac{YYYY}{100} \right\rfloor \mod 4 \right) \mod 7 + \text{Tuesday}$$

## Offset:

$$\left( 365YY + \left\lfloor \frac{YY}{4} \right\rfloor \right) \mod 7 = \left( YY + \left\lfloor \frac{YY}{4} \right\rfloor \right) \mod 7$$

# Memorable Doomsdays:

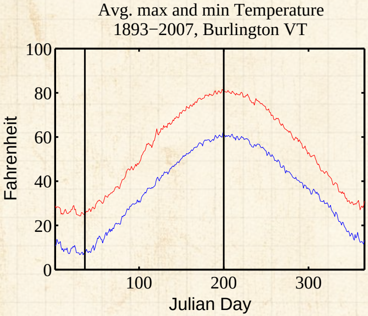
| Month | Memorable date | Month/Day | Mnemonic[6] |
|---|---|---|---|
| January | January 3 (common years), January 4 (leap years) | 1/3 or 1/4 | the 3rd **3** years in 4 and the 4th in the **4**th |
| February | February 28 (common years), February 29 (leap years) | 2/28 or 2/29 | last day of February |
| March | "March 0" | 3/0 | last day of February |
| April | April 4 | 4/4 | **4/4**, 6/6, 8/8, 10/10, 12/12 |
| May | May 9 | 5/9 | **9-to-5** at 7-11 |
| June | June 6 | 6/6 | 4/4, **6/6**, 8/8, 10/10, 12/12 |
| July | July 11 | 7/11 | 9-to-5 at **7-11** |
| August | August 8 | 8/8 | 4/4, 6/6, **8/8**, 10/10, 12/12 |
| September | September 5 | 9/5 | **9-to-5** at 7-11 |
| October | October 10 | 10/10 | 4/4, 6/6, 8/8, **10/10**, 12/12 |
| November | November 7 | 11/7 | 9-to-5 at **7-11** |
| December | December 12 | 12/12 | 4/4, 6/6, 8/8, 10/10, **12/12** |

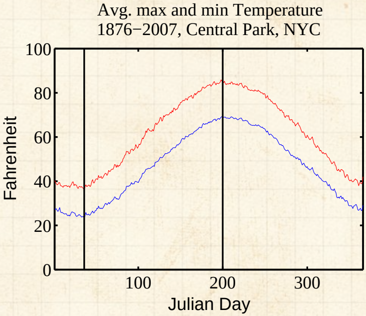
- Pi day (March 14), July 4, Halloween, and Boxing Day are always Doomsdays.

# The bissextile year

“The Julian calendar, which was developed in 46 BC by Julius Caesar, and became effective in 45 BC, distributed an extra ten days among the months of the Roman Republican calendar. Caesar also replaced the intercalary month by a single intercalary day, located where the intercalary month used to be. To create the intercalary day, the existing ante diem sextum Kalendas Martias (February 24) was doubled, producing ante diem bis sextum Kalendas Martias. Hence, the year containing the doubled day was a bissextile (bis sextum, ”twice sixth”) year. For legal purposes, the two days of the bis sextum were considered to be a single day, with the second half being intercalated; but in common practice by 238, when Censorinus wrote, the intercalary day was followed by the last five days of February, a. d. VI, V, IV, III and pridie Kal. Mart. (the days numbered 24, 25, 26, 27, and 28 from the beginning of February in a common year), so that the intercalated day was the first half of the doubled day. Thus the intercalated day was effectively inserted between the 23rd and 24th days of February.”

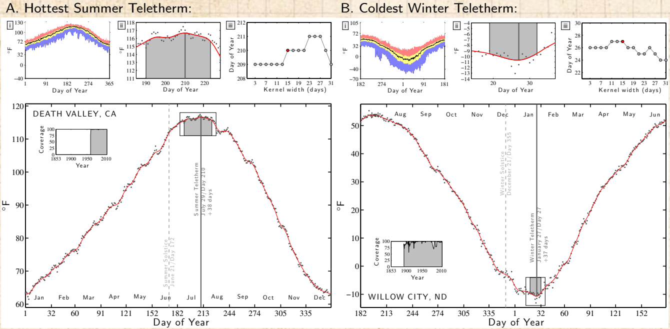
# The Teletherm, an early conception:




- Hibernal Teletherm ≈ February 4.
- Halfway between Winter Solstice and Spring Equinox
- Bonus: Groundhog Day, Imbolc, …
- Aesteval Teletherm ≈ July 19 (164 days later).

In review: “Tracking the Teletherms: The spatiotemporal dynamics of the hottest and coldest days of the year”, Dodds, Mitchell, Reagan, and Danforth.



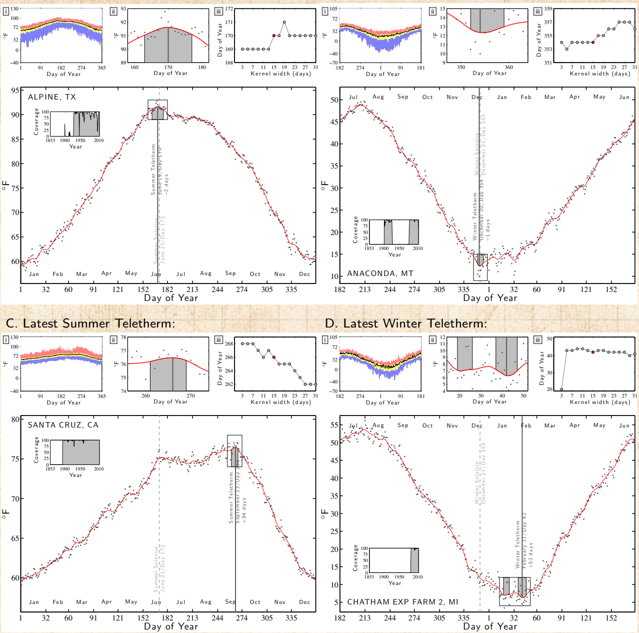

- 2 × 1218 similar figures for the US.
- 6000ish pages of Supplementary Information (all figures)
- Interactive website.

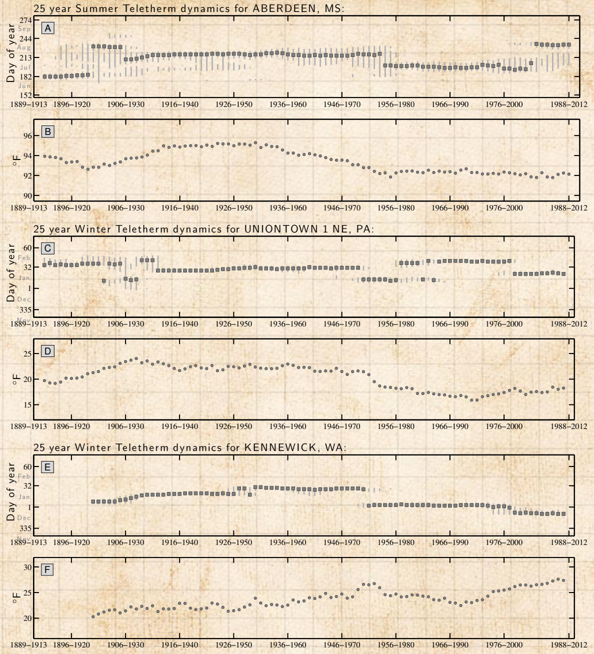
A. Earliest Summer Teletherm:

B. Earliest Winter Teletherm:

C. Latest Summer Teletherm:

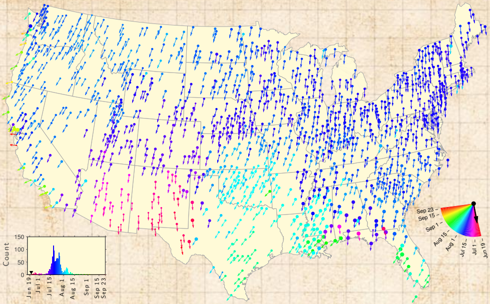
D. Latest Winter Teletherm:



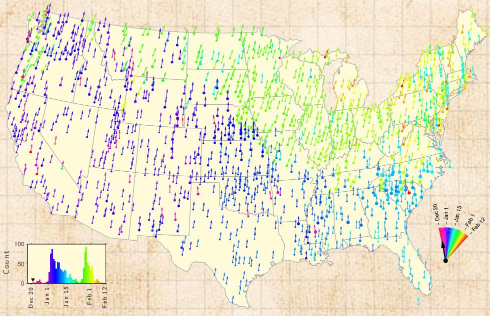

25 year Summer Teletherm dynamics for ABERDEEN, MS:
A
Day of year
274
Sep
244
Aug
213
Jul
182
Jun
152
°F
B
96
94
92
90
25 year Winter Teletherm dynamics for UNIONTOWN 1 NE, PA:
C
60
Feb
32
Jan
1
Dec
335
Nov
D
25
20
15
25 year Winter Teletherm dynamics for KENNEWICK, WA:
E
F
30
25
20
1889–1913
1896–1920
1906–1930
1916–1940
1926–1950
1936–1960
1946–1970
1956–1980
1966–1990
1976–2000
1988–2012

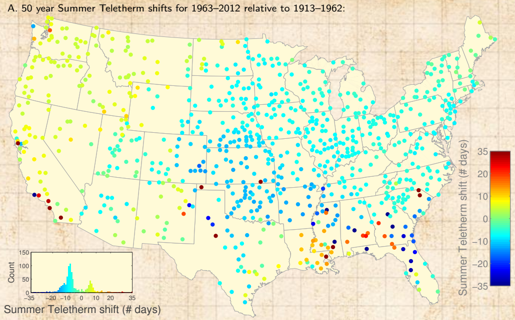
Random
Randomness
References

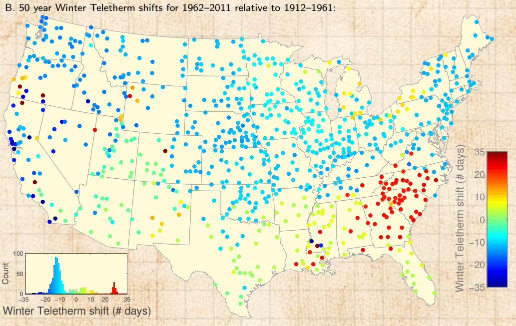
A. Summer Teletherms for 1853–2012:




B. Winter Teletherms for 1853–2011:




A. 50 year Summer Teletherm shifts for 1963–2012 relative to 1913–1962:
Count
Summer Teletherm shift (# days)
Summer Teletherm shift (# days)


B. 50 year Winter Teletherm shifts for 1962–2011 relative to 1912–1961:
Count
Winter Teletherm shift (# days)
Winter Teletherm shift (# days)

# Homo nonprobabilisticus, continued:

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- Important detour: The final digits of primes are not entirely random (how did we not know this?).

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References

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- Two tosses: What are the probabilities of flipping (1) $HH$ and (2) $HT$?

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References

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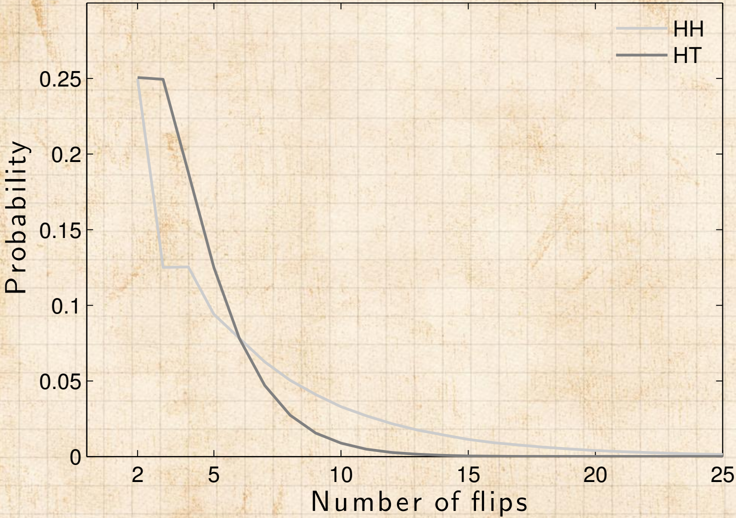
- What's the probability of first flipping two heads in a row ($HH$) on the $(n-1)$th and $n$th flips?

# Homo nonprobabilisticus, continued:





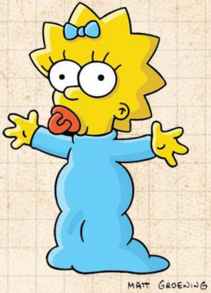

Average number of flips: 4 and 6.

# Universal numbers


From here.

# Universal numbers

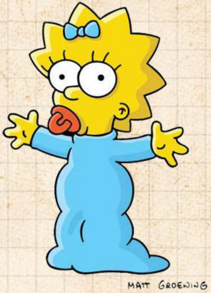





From here.

Accidents of evolution[1] give us 5 + 5 = 10 fingers and hence base 10.

# Universal numbers

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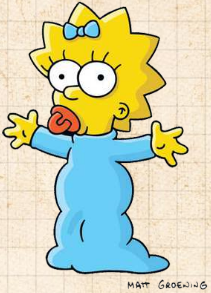


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- We could be happy with base 6, 8, 12, ...
- We like these:
  - 60 seconds in a minute
  - 60 minutes in an hour.
  - $2 \times 12 = 24$ hours in a day.
  - 360 degrees in a circle.

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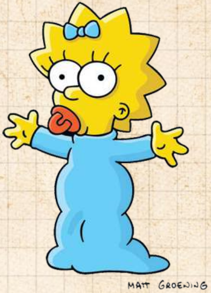


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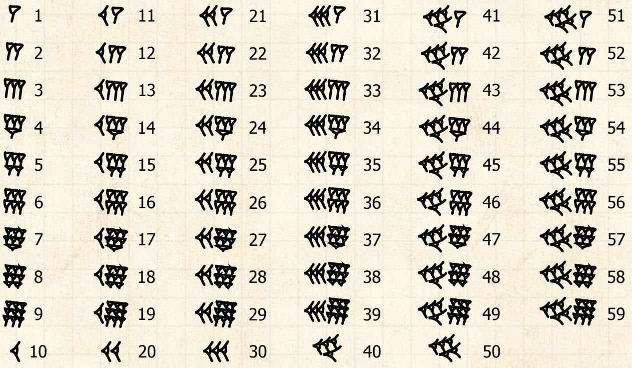
  - 360 degrees in a circle.

[1] Maybe 5 fingers are not an accident

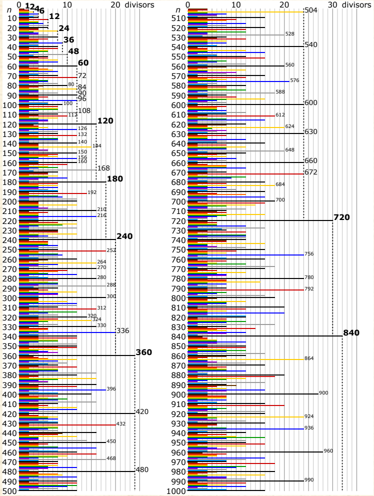
# We've liked these kinds of numbers for a long time:




- 2000 BC: Babylonian base 60/Sexagesimal system.
- Other bases (or radices): 2, 10, 12 (duodecimal/dozenal), 6 (senary), 8, 16, 20 (vigesimal), 60.

# Highly composite numbers:






HCN = natural number with more divisors than any smaller natural number.

By Cmglee - Own work, CC BY-SA 3.0,

https://commons.wikimedia.org/w/index.php?curid=31684018

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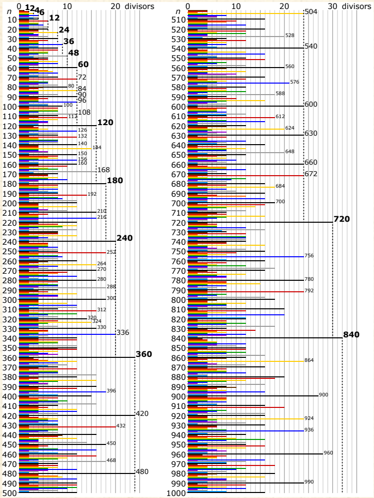


- HCN = natural number with more divisors than any smaller natural number.
- 2, 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720, 840, 1260, 1680, 2520, 5040 (Plato's optimal city population), ...

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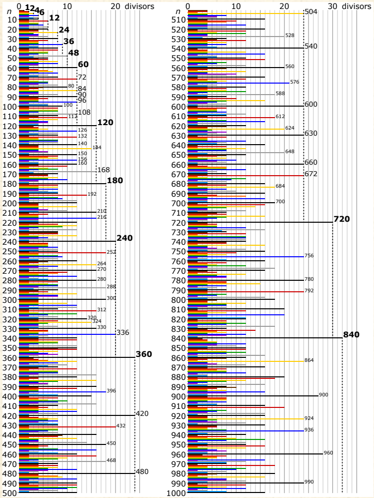
References

# Highly composite numbers:

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- OEIS sequence A002182

# Superior highly composite numbers:

| # prime factors | SHCN n | prime factorization | prime exponents | # divisors d(n) | | primorial factorization |
|---|---|---|---|---|---|---|
| 1 | 2 | $2$ | 1 | 2 | 2 | $2$ |
| 2 | 6 | $2 \cdot 3$ | 1,1 | $2^2$ | 4 | $6$ |
| 3 | 12 | $2^2 \cdot 3$ | 2,1 | 3×2 | 6 | $2 \cdot 6$ |
| 4 | 60 | $2^2 \cdot 3 \cdot 5$ | 2,1,1 | $3\times2^2$ | 12 | $2 \cdot 30$ |
| 5 | 120 | $2^3 \cdot 3 \cdot 5$ | 3,1,1 | $4\times2^2$ | 16 | $2^2 \cdot 30$ |
| 6 | 360 | $2^3 \cdot 3^2 \cdot 5$ | 3,2,1 | 4×3×2 | 24 | $2 \cdot 6 \cdot 30$ |
| 7 | 2520 | $2^3 \cdot 3^2 \cdot 5 \cdot 7$ | 3,2,1,1 | $4\times3\times2^2$ | 48 | $2 \cdot 6 \cdot 210$ |
| 8 | 5040 | $2^4 \cdot 3^2 \cdot 5 \cdot 7$ | 4,2,1,1 | $5\times3\times2^2$ | 60 | $2^2 \cdot 6 \cdot 210$ |
| 9 | 55440 | $2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$ | 4,2,1,1,1 | $5\times3\times2^3$ | 120 | $2^2 \cdot 6 \cdot 2310$ |
| 10 | 720720 | $2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$ | 4,2,1,1,1,1 | $5\times3\times2^4$ | 240 | $2^2 \cdot 6 \cdot 30030$ |

- SHCN = natural number $n$ whose number of divisors exceeds that of any other number when scaled relative to itself in a sneaky way:

$$\frac{d(n)}{n^\epsilon} \geq \frac{d(j)}{j^\epsilon} \text{ and } \frac{d(n)}{n^\epsilon} > \frac{d(k)}{k^\epsilon}$$

for $j < n < k$ and some $\epsilon > 0$.

## There's more: Superabundant numbers

- $n$ is superabundant if:

$$\frac{\sigma_1(n)}{n} > \frac{\sigma_1(j)}{j}$$

for $j < n$ and where $\sigma_x(n) = \sum_{d|n} d^x$ is the divisor function.

- 449 numbers are both superabundant and highly composite.

## Yet more: Colossally abundant numbers:

- $n$ is colossally abundant if for all $j$ and some $\epsilon > 0$:

$$\frac{\sigma_1(n)}{n^{1+\epsilon}} \geq \frac{\sigma_1(j)}{j^{1+\epsilon}}$$

- Infinitely many but only 22 less than $10^{18}$.

## Some very, very silly units of measurement courtesy of the Imperial system:

- 22 yards in a chain = 1 cricket pitch, 100 links in a chain, 10 chains in a furlong, 80 chains in a mile.

## Also:

## Some very, very silly units of measurement courtesy of the Imperial system:

- 22 yards in a chain = 1 cricket pitch, 100 links in a chain, 10 chains in a furlong, 80 chains in a mile.
- 1 acre = 1 furlong $\times$ 1 chain = 43,560 square feet.

## Also:

Random

Randomness

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## Also:

- Fahrenheit, Celcius, and Kelvin.
- The entire metric system.

# Training with stories as fuel:

# Randomness:

Random

Randomness

References

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