# **Curious and Interesting Things**

Last updated: 2019/01/14, 22:50:59

Complex Networks | @networksvox CSYS/MATH 303, Spring, 2019

### Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont













Licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License.

### COcoNuTS @networksvox Interesting Things

Random Randomness References

## Outline

### Random

Randomness

### References



COcoNuTS @networksvox

Randomness

References

Interesting Things





# 2 9 0 4 of 47

COcoNuTS @networksvox

Random

References

Interesting Things

# These slides are brought to you by:



## COcoNuTS @networksvox Interesting Things

少 Q (~ 1 of 47

IVM S

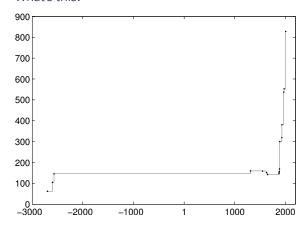
Random Randomness References





•9 Q (№ 2 of 47

### What's this?







少∢<a>○ 5 of 47</a>

COcoNuTS

Random

Randomness

References

Interesting Things

# These slides are also brought to you by:

# **Special Guest Executive Producer**



On Instagram at pratchett\_the\_cat

# COcoNuTS Interesting Things

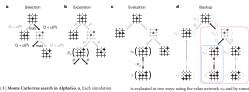
Random Randomness References

114 | S

◆) q (~ 3 of 47

# Advances in sociotechnical algorithms: "Mastering the game of Go with deep

neural networks and tree search" Silver and Silver, Nature, **529**, 484-489, 2016. [6]





💫 Nature News (2016): Digital Intuition 🗹 Network Science of the game of



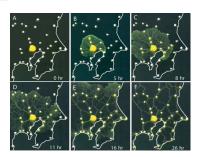
v (s)

Go ☑



"Rules for Biologically Inspired Adaptive 

Tero et al., Science, **327**, 439-442, 2010. [7]



### Urban deslime in action:

https://www.youtube.com/watch?v=GwKuFREOgmo



"Citations to articles citing Benford's law: A Benford analysis"

Tariq Ahmad Mir, Preprint available at http://arxiv.org/abs/1602.01205, 2016. [4]

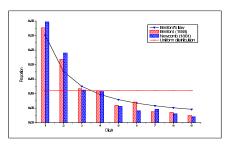
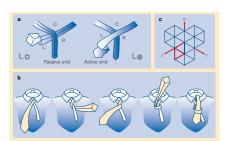


Fig. 1: The observed proportions of first digits of citations received by the articles citing FB and SN on September 30, 2012. For comparison the proportions expected from BL and uniform distributions are also shown.

# Applied knot theory:



"Designing tie knots by random walks" Fink and Mao, Nature, **398**, 31–32, 1999. [1]



### COcoNuTS @networksvox Interesting Things

Random Randomness





•9 q (~ 7 of 47

COcoNuTS @networksvox Interesting Things

Random Randomness References





少 Q (~ 8 of 47

COcoNuTS Interesting Things

Random Randomness References





少 Q (~ 9 of 47

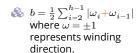
# Applied knot theory:

h	γ	γ/h	$K(h, \gamma)$	S	b	Name	Sequence
3	1	0.33	1	0	0		L₀R⊗C₀T
1	1	0.25	1	-1	1	Four-in-hand	L <sub>∞</sub> R <sub>☉</sub> L <sub>∞</sub> C <sub>☉</sub> T
5	2	0.40	2	-1	0	Pratt knot	L₀C₀R₀L₀C₀T
6	2	0.33	4	0	0	Half-Windsor	$L_{\otimes}R_{\circ}C_{\otimes}L_{\circ}R_{\otimes}C_{\circ}T$
7	2	0.29	6	-1	1		$L_{\circ}R_{\circ}L_{\circ}C_{\circ}R_{\circ}L_{\circ}C_{\circ}T$
7	3	0.43	4	0	1		$L_{\circ}C_{\otimes}R_{\circ}C_{\otimes}L_{\circ}R_{\otimes}C_{\circ}T$
3	2	0.25	8	0	2		L <sub>o</sub> R <sub>o</sub> L <sub>o</sub> C <sub>o</sub> R <sub>o</sub> L <sub>o</sub> R <sub>o</sub> C <sub>o</sub> T
3	3	0.38	12	-1	0	Windsor	$L_{\odot}C_{\circ}R_{\circ}L_{\circ}C_{\circ}R_{\circ}L_{\odot}C_{\circ}T$
9	3	0.33	24	0	0		$L_{\circ}R_{\circ}C_{\circ}L_{\circ}R_{\circ}C_{\circ}L_{\circ}R_{\circ}C_{\circ}$
9	4	0.44	8	-1	2		LoCoRoCoLoCoRoLoCo



h = number ofmoves

center moves



COcoNuTS

Random

Randomness References

@networksvox

Interesting Things



•9 q (~ 10 of 47

COcoNuTS @networksvox Interesting Things



# Irregular verbs

# Cleaning up the code that is English:



"Quantifying the evolutionary dynamics of language" 🖸

Lieberman et al., Nature, **449**, 713–716, 2007. [2]



Irregular verbs

60

50

10

10-5

10-4

Number of irregular 40 30 20

- Exploration of how verbs with irregular conjugation gradually become regular over time.
- Comparison of verb behavior in Old, Middle, and Modern English.

Old English
Middle English
Modern English





少 Q ← 11 of 47

COcoNuTS Interesting Things

# Random

Randomness References







Rare verbs tend to be regular in the first place

10-2

10-

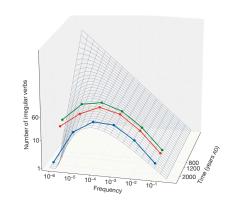
10-3

Frequency Universal tendency towards regular conjugation

IVM S

•9 q (~ 12 of 47

# Irregular verbs b Old to Mo Regularization rate ency The more common a verb is, the more is to change.



Projecting back in time to proto-Zipf story of many

COcoNuTS @networksvox Interesting Things

Random Randomness

References





•9 q (> 16 of 47

# Irregular verbs

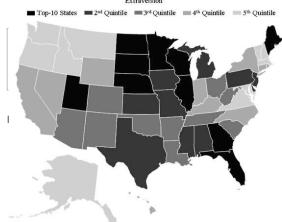
Frequency	Verbs	Regularization (%)	Half-life (yr)
10-1-1	be, have	0	38,800
10-2-10-1	come, do, find, get, give, go, know, say, see, take, think	0	14,400
10-3-10-2	begin, break, bring, buy, choose, draw, drink, drive, eat, fall, flight, forget, grow, hang, help, hold, leave, let, lie, lose, reach, rise, run, seek, set, shake, sit, sleep, speak, stand, teach, throw, understand, walk, win, work, write	10	5,400
10-4-10-3	arise, bake, bear, beat, bind, bite, blow, bow, burn, burst, carve, chew, climb, cling, creep, dare, dig, ding, flee, float, flow, fly, fold, freeze, grind, leap, lend, lock, melt, reckon, ride, rush, shape, shine, shoot, shirvis, ship, sing, sink, slide, slp, smoke, spin, sping, stave, stead, step, shretch, trifiee, slp, smoke, spin, sping, stave, stead, step, shretch, trifiee, slp, smoke, spin, seem, weak, with weak, weak, weak, with seem, which, with, vall, vield	43	2,000
10-5-10-4	wase, wash, weeve, weep, weep, way, ver, yee, yee, bab, bah, bablow, bd, blend, rand, brew, desec, cringe, crow, bah, crow, bal, crow, and rail, mourn, move, prescribe, redden, reek, row, scrape, seethe, shear, shed, show, sally, all, smile, soow, span, spurn, sting, stink, strew, stride, swell, tread, uproot, wade, warp, way, weld, viring, writhe	72	700
10-6-10-5	bide, chide, delve, flay, hew, rue, shrive, slink, snip, spew,	91	300

Red = regularized

 $\clubsuit$  Estimates of half-life for regularization ( $\propto f^{1/2}$ )

# Personality distributions:

夕 Q № 13 of 47



COcoNuTS @networksvox Interesting Things

Random References





ൗ.എം 17 of 47

COcoNuTS @networksvox Interesting Things

Random References



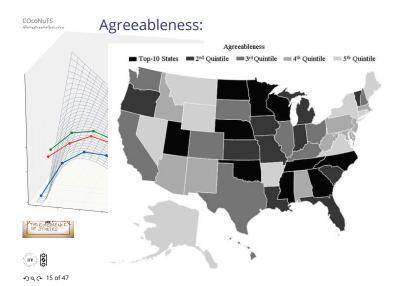


10-3 10-4 10-3 Frequency 10-5

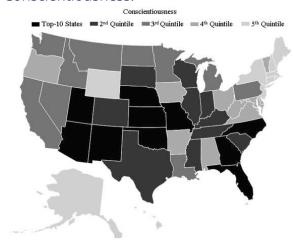
& 'Wed' is next to go.

Number of irregular verbs

& -ed is the winning rule...



# Conscientiousness:



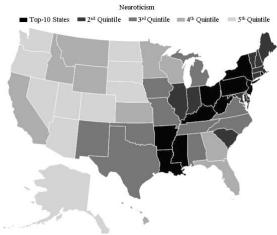
COcoNuTS @networksvox Interesting Things

Random Randomness









COcoNuTS @networksvox Interesting Things

Random Randomness

References



IVN S

•9 q (№ 22 of 47

COcoNuTS @networksvox Interesting Things

Random Randomness References







少∢ № 23 of 47

COcoNuTS Interesting Things

Random Randomness References

Limits of testability and happiness in Science:

From A Fight for the soul of Science **☑** in Quanta

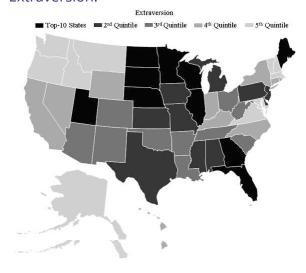
se over a vast range of scales (white area), but many modern physics theories

Magazine (2016/02):

The Ends of Evidence Humans can probe the universe over a va involve scales outside of this range (grey).

## Extraversion:

Openness



Openness ■ Top-10 States ■ 2<sup>nd</sup> Quintile ■ 3<sup>rd</sup> Quintile ■ 4<sup>th</sup> Quintile ■ 5<sup>th</sup> Quintile

# COcoNuTS @networksvox Interesting Things

Random References





COcoNuTS Interesting Things

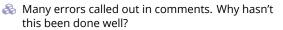
Random Randomness





夕 Q № 21 of 47

### Europe:







∙0 q (~ 24 of 47

# John Conway's Doomsday rule for determining a date's day of the week:

Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.	Mon.	Tue.	Wed.	Thu.	Fri.	Sat.	Sun.
1898	1899	1900	1901	1902	1903	-	1904	1905	1906	1907	-	1908	1909
1910	1911	-+	1912	1913	1914	1915	-	1916	1917	1918	1919	-	1920
1921	1922	1923	-	1924	1925	1926	1927	-	1928	1929	1930	1931	-+
1932	1933	1934	1935	-+	1936	1937	1938	1939	-	1940	1941	1942	1943
-	1944	1945	1946	1947	-	1948	1949	1950	1951	-	1952	1953	1954
1955	-	1956	1957	1958	1959	-	1960	1961	1962	1963	-	1964	1965
1966	1967	-	1968	1969	1970	1971	-+	1972	1973	1974	1975	-	1976
1977	1978	1979	-	1980	1981	1982	1983	-	1984	1985	1986	1987	-+
1988	1989	1990	1991	-	1992	1993	1994	1995	-	1996	1997	1998	1999
-	2000	2001	2002	2003	-	2004	2005	2006	2007	-	2008	2009	2010
2011	-	2012	2013	2014	2015	-	2016	2017	2018	2019	-	2020	2021
2022	2023	-	2024	2025	2026	2027	-	2028	2029	2030	2031	-	2032
2033	2034	2035	-	2036	2037	2038	2039	-	2040	2041	2042	2043	-+
2044	2045	2046	2047	-	2048	2049	2050	2051	-	2052	2053	2054	2055
-	2056	2057	2058	2059	-	2060	2061	2062	2063		2064	2065	2066
2067	-	2068	2069	2070	2071	-	2072	2073	2074	2075	-	2076	2077
2078	2079		2080	2081	2082	2083	-	2084	2085	2086	2087	-	2088
2089	2090	2091		2092	2093	2094	2095		2096	2097	2098	2099	2100

- Works for Gregorian (1582-, haphazardly) and the increasingly inaccurate Julian calendars (400 and 28 vears cycles).
- Apparently inspired by Lewis Carroll's work on a perpetual calendar.

Interesting Things

Random

Randomness

References



COcoNuTS

•9 q (~ 25 of 47

# Interesting Things

Random References

### Outline:

- Determine "anchor day" for a given century, then find Doomsday for a given year in that century.
- Remember special Doomsday dates and work from there.
- 💫 Naturally: Load this year's Doomsday into brain.

# Century's anchor day (Gregorian, Sunday $\equiv$ 0):

$$5 \times \left( \left\lfloor \frac{YYYY}{100} \right\rfloor \mod 4 \right) \mod 7 + \mathsf{Tuesday}$$

### Offset:

$$\left(365YY + \left\lfloor \frac{YY}{4} \right\rfloor \right) \mod 7 = \left(YY + \left\lfloor \frac{YY}{4} \right\rfloor \right) \mod 7$$





Random

Randomness

•9 a (№ 27 of 47

References

少 Q ← 26 of 47

# COcoNuTS Interesting Things

### Memorable Doomsdays:

Month	Memorable date	Month/Day	Mnemonic <sup>[6]</sup>			
January	January 3 (common years), January 4 (leap years)	1/3 or 1/4	the 3rd 3 years in 4 and the 4th in the 4th			
February	February 28 (common years), February 29 (leap years)	2/28 or 2/29	last day of February			
March	"March 0"	3/0	last day of February			
April	April 4	4/4	4/4, 6/6, 8/8, 10/10, 12/12			
May	May 9	5/9	9-to-5 at 7-11			
June	June 6	6/6	4/4, 6/6, 8/8, 10/10, 12/12			
July	July 11	7/11	9-to-5 at 7-11			
August	August 8	8/8	4/4, 6/6, 8/8, 10/10, 12/12			
September	September 5	9/5	9-to-5 at 7-11			
October	October 10	10/10	4/4, 6/6, 8/8, 10/10, 12/12			
November	November 7	11/7	9-to-5 at <b>7-11</b>			
December	December 12	12/12	4/4, 6/6, 8/8, 10/10, 12/12			

Pi day (March 14), July 4, Halloween, and Boxing Day are always Doomsdays.

# The bissextile year 🗹

"The Julian calendar, which was developed in 46 BC by Julius Caesar, and became effective in 45 BC, distributed an extra ten days among the months of the Roman Republican calendar. Caesar also replaced the intercalary month by a single intercalary day, located where the intercalary month used to be. To create the intercalary day, the existing ante diem sextum Kalendas Martias (February 24) was doubled, producing ante diem bis sextum Kalendas Martias. Hence, the year containing the doubled day was a bissextile (bis sextum, "twice sixth") year. For legal purposes, the two days of the bis sextum were considered to be a single day, with the second half being intercalated; but in common practice by 238, when Censorinus wrote, the intercalary day was followed by the last five days of February, a. d. VI, V, IV, III and pridie Kal. Mart. (the days numbered 24, 25, 26, 27, and 28 from the beginning of February in a common year), so that the intercalated day was the first half of the doubled day. Thus the intercalated day was effectively inserted between the 23rd and 24th days of February."

### COcoNuTS @networksvox Interesting Things

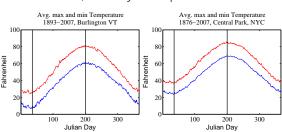
Random





•28 of 47

### The Teletherm, an early conception:



COcoNuTS Interesting Things

Random References

- A Hibernal Teletherm ≈ February 4.
- Halfway between Winter Solstice and Spring Equinox
- Bonus: Groundhog Day ☑, Imbolc ☑, ...
- Aesteval Teletherm  $\approx$  July 19 (164 days later).

In review: "Tracking the Teletherms: The spatiotemporal dynamics of the hottest and coldest days of the year" 2,



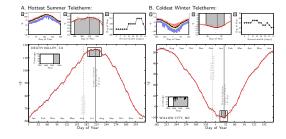


• റ q С№ 29 of 47

COcoNuTS Interesting Things

Random

Randomness References



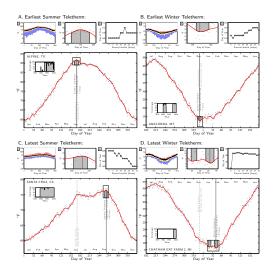
Dodds, Mitchell, Reagan, and Danforth.

- 8 6000ish pages of Supplementary Information (all figures)
- Interactive website.



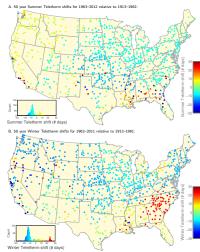


少 Q (~ 30 of 47





# Random Randomness



COcoNuTS @networksvox Interesting Things

Random

Randomness

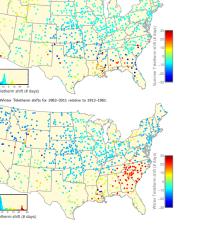


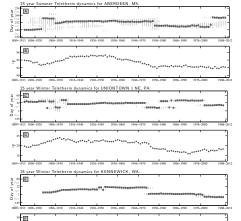
•2 a c ≥ 34 of 47

COcoNuTS @networksvox

Interesting Things

Random References





## COcoNuTS @networksvox Interesting Things

•9 q (~ 31 of 47

# Random References

# Homo nonprobabilisticus, continued:

- Start flipping a coin ...
- Two tosses: What are the probabilities of flipping (1) HH and (2) HT?
- $\Re$  Flip a coin  $n \geq 2$  times: What are the probabilities that the last two tosses are (1) HH or (2) HT?
- 🙈 Estimate: On average, how many flips does it take to first see the sequence HT?
- & Estimate: On average, how many flips does it take to first see the sequence *HH*?
- & What's the probability of first flipping a HT sequence on the n-1th and nth flips?
- Nhat's the probability of first flipping two heads in a row (HH) on the (n-1)th and nth flips?



◆) Q (~ 35 of 47

COcoNuTS

Random

Interesting Things

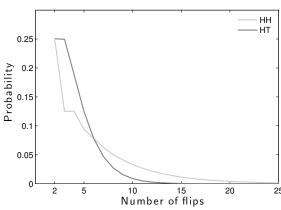


少 Q (~ 32 of 47

# Interesting Things

# Random Randomness

# Homo nonprobabilisticus, continued:











Average number of flips: 4 and 6.

# Universal numbers



From here .

- Accidents of evolution<sup>1</sup> give us 5 + 5 = 10 fingers and hence base 10.
- We could be happy with base 6, 8, 12, ...
- We like these:
  - 60 seconds in a minute
  - 60 minutes in an hour.

  - 360 degrees in a circle.



COcoNuTS

Random

Randomness

Interesting Things



少 Q (~ 37 of 47

COcoNuTS @networksvox

Random

Randomness References

少 Q (~ 38 of 47

Interesting Things

COcoNuTS

Random

Randomness

References

Interesting Things

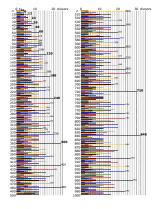
<sup>1</sup>Maybe 5 fingers are not an accident 🗷

# We've liked these kinds of numbers for a long time: 🗹

<b>7</b> 1	<b>∢7</b> 11	. <b>∜?</b> 2	21 <b>(((7</b>	31	<b>₹</b> \$	41	€ 7	51
<b>77</b> 2	<b>∢77</b> 12	<b>₹₹77</b> 2	22 <b>4(17)</b>	32	15 77	42	<b>₹</b> 77	52
<b>үүү</b> з	<b>(777</b> 13	<b>4(777</b> 2	23 <b>((()))</b>	33	<b>₹₹</b> 799	43	<b>₹</b> ₹	53
<b>Ø</b> 4	<b>(87</b> 14	<b>₩</b> \$\$\$ 2	4 <b>(#127</b>	34	低血	44	存を	54
<b>777</b> 5	<b>₹</b> ₩ 15	<b>₩</b> ₩ 2	25 <b>(#\$7</b> )	35	⋘数	45	续链	55
₩ 6	<b>18</b> 16	<b>₩</b> 2	26 <b>₩₩</b>	36	袋器	46	奏器	56
7	<b>( )</b> 17	<b>₩</b> 2	27 <b>₩₩</b>	37	⋘₩	47	袋盘	57
<b>3</b> 8	<b>₹</b> 18	<b>₩</b> 2	28 <b>###</b>	38	袞₩	48	袋雞	58
<b># 9</b>	<b>4</b> 19	<b>代解</b> 2	9 ##	39	核雜	49	<b>核雜</b>	59
<b>4</b> 10	<b>∜(</b> 20	<b>+++</b> 3	30	40	€	50		

- 🗞 2000 BC: Babylonian base 60/Sexagesimal system.
- Source Other bases (or radices): 2, 10, 12 (duodecimal/dozenal (2), 6 (senary), 8, 16, 20 (vigesimal), 60.

# Highly composite numbers: ☑



- HCN = natural number with more divisors than any smaller natural number.
- 2, 4, 6, 12, 24, 36, 48, 60, 120, 180, 240, 360, 720, 840, 1260, 1680, 2520, 5040 (Plato's optimal city population ☑), ...
- OEIS sequence A002182 ☑

# ୍ଞା | <mark>ଖି</mark> ୬୧୯ 39 of 47

Superior highly composite numbers:

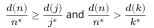
# prime factors	SHCN	prime factorization	prime exponents	# divisors d(n)		primorial factorization	
1	2	2	1	2	2	2	
2	6	2 · 3	1,1	22	4	6	
3	12	$2^2 \cdot 3$	2,1	3×2	6	2 · 6	
4	60	$2^2 \cdot 3 \cdot 5$	2,1,1	3×2 <sup>2</sup>	12	2 · 30	
5	120	$2^3 \cdot 3 \cdot 5$	3,1,1	4×2 <sup>2</sup>	16	$2^2 \cdot 30$	
6	360	$2^3 \cdot 3^2 \cdot 5$	3,2,1	4×3×2	24	2 · 6 · 30	
7	2520	$2^3 \cdot 3^2 \cdot 5 \cdot 7$	3,2,1,1	4x3x2 <sup>2</sup>	48	2 · 6 · 210	
8	5040	$2^4 \cdot 3^2 \cdot 5 \cdot 7$	4,2,1,1	5×3×2 <sup>2</sup>	60	$2^2 \cdot 6 \cdot 210$	
9	55440	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$	4,2,1,1,1	5x3x2 <sup>3</sup>	120	$2^2 \cdot 6 \cdot 2310$	
10	720720	$2^4 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11 \cdot 13$	4,2,1,1,1,1	5×3×2 <sup>4</sup>	240	22 · 6 · 30030	

COcoNuTS @networksvox Interesting Things

Random

Randomness References

 $\Re$  SHCN = natural number n whose number of divisors exceeds that of any other number when scaled relative to itself in a sneaky way:



for j < n < k and some  $\epsilon > 0$ .



UVW |

少 Q (~ 40 of 47

COcoNuTS @networksvox

Random

References

Interesting Things

# There's more: Superabundant numbers

 $\Re n$  is superabundant if:

$$\frac{\sigma_1(n)}{n} > \frac{\sigma_1(j)}{i}$$

for j < n and where  $\sigma_x(n) = \sum_{d \mid n} d^x$  is the divisor function.

449 numbers are both superabundant and highly composite.

# Yet more: Colossally abundant numbers:

n is colossally abundant if for all j and some  $\epsilon > 0$ :

$$\frac{\sigma_1(n)}{n^{1+\epsilon}} \geq \frac{\sigma_1(j)}{j^{1+\epsilon}}$$

 $\red$  Infinitely many but only 22 less than  $10^{18}$ .



∙n q c> 41 of 47 COcoNuTS

Interesting Things

Random

Randomness

References

# Some very, very silly units of measurement courtesy of the Imperial system 2:

- 22 yards in a chain = 1 cricket pitch, 100 links in a chain, 10 chains in a furlong, 80 chains in a mile.
- $\clubsuit$  1 acre = 1 furlong  $\times$  1 chain = 43,560 square feet.
- 160 fluid ounces in a gallon.
- 4 14 pounds in a stone.
- Hundredweight = 112 pounds.

### Also:

- Fahrenheit, Celcius, and Kelvin.
- The entire metric system.





少 Q (~ 42 of 47

By Cmglee - Own work, CC BY-SA 3.0,

https://commons.wikimedia.org/w/index.php?curid=31684018

### Training with stories as fuel:



COcoNuTS Interesting Things

Randomness References

IVM S

少 Q (~ 43 of 47

COcoNuTS @networksvox

Random

Randomness References

Interesting Things

## References II

[4] T. A. Mir.

Citations to articles citing Benford's law: A Benford analysis, 2016.

Preprint available at http://arxiv.org/abs/1602.01205.pdf

[5] P. J. Rentfrow, S. D. Gosling, and J. Potter.

A theory of the emergence, persistence, and expression of geographic variation in psychological characteristics.

Perspectives on Psychological Science, 3:339-369, 2008. pdf 🗹

[6] D. Silver et al.

Mastering the game of Go with deep neural networks and tree search.

Nature, 529:484-489, 2016. pdf 2



COcoNuTS @networksvox

Randomness

References

Interesting Things



少 Q (~ 46 of 47

# References III

COcoNuTS @networks Interesting Things

Random Randomness References

[7] A. Tero, S. Takagi, T. Saigusa, K. Ito, D. P. Bebber, M. D. Fricker, K. Yumiki, R. Kobayashi, and T. Nakagaki.

Rules for biologically inspired adaptive network design.

Science, 327(5964):439-442, 2010. pdf







少 Q ← 44 of 47

COcoNuTS Interesting Things

Random Randomness References

# References I

Randomness:

[1] T. M. Fink and Y. Mao. Designing tie knots by random walks. Nature, 398:31–32, 1999. pdf ☑

[2] E. Lieberman, J.-B. Michel, J. Jackson, T. Tang, and M. A. Nowak. Quantifying the evolutionary dynamics of language.

Nature, 449:713-716, 2007. pdf

[3] J.-B. Michel, Y. K. Shen, A. P. Aiden, A. Veres, M. K. Gray, T. G. B. Team, J. P. Pickett, D. Hoiberg, D. Clancy, P. Norvig, J. Orwant, S. Pinker, M. A. Nowak, and E. A. Lieberman.

Quantitative analysis of culture using millions of digitized books.

Science Magazine, 2010. pdf





少 Q (~ 45 of 47





◆) < (~ 47 of 47