## Curious and Interesting Things

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Complex Networks｜＠networksvox CSYS／MATH 303，Spring， 2019

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＂Rules for Biologically Inspired Adaptive Network Design＂${ }^{\circ}$
Tero et al．，
Science，327，439－442，2010．${ }^{[7]}$


Urban deslime in action：
https：／／www．youtube．com／watch？v＝GwKuFREOgmo■

＂Citations to articles citing Benford＇s law：A Benford analysis＂
Táriq Āhmad Mir，
Preprint available at
http：／／arxiv．org／abs／1602．01205，2016．${ }^{[4]}$


Fig．1：The observed proportions of first digits of citations received by the articles citing FB and SN on September 30，2012．For comparison the proportions expected
from BL and uniform distributions are also shown．

## Applied knot theory：


＂Designing tie knots by random walks＂ $\mathbb{Z}$ Fink and Mao， Nature，398，31－32，1999．${ }^{[1]}$


Figure 1 All diagrams are drawn in the frame of reference of the mirror image of the actual tie． a，The two ways of beginning a knot，$L_{0}$ and $L_{\infty}$ ．For knots beginning with $L_{0}$ the tie must begin
inside－out． $\mathbf{b}$ ，The fourin－hand，denoted by the sequence $L_{0} R L_{8} L_{0} T$ ． $\mathbf{e}, A$ knot may be represented by a persisitent random walk on a triangular lattice．The example shown is the fourin－hand，indicated by the
walk $\hat{\imath} \hat{\imath} c$ ．

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Applied knot theory：

| Table 1 Aesthetic tie knots |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\stackrel{\text { n }}{ }$ | $\gamma$ | $\gamma / \mathrm{h}$ | $K(n, \gamma)$ | $s$ | b | Name | Sequence |
| 3 | 1 | 0.33 | 1 | 0 | 0 |  | $\mathrm{L}_{0} \mathrm{R}_{\odot} \mathrm{C}_{0} \mathrm{~T}$ |
| 4 | 1 | 0.25 | 1 | $-1$ | 1 | Four－in－hand | $\mathrm{L}_{\mathrm{B}}^{\mathrm{R}} \mathrm{L}_{0} \mathrm{C}$ |
| 5 | 2 | 0.40 | 2 | －1 | 0 | Pratt knot | $\mathrm{L}_{0} \mathrm{C}_{8} \mathrm{R}_{0} L_{0} \mathrm{C}_{0} \top$ |
| 6 | 2 | 0.33 | 4 | 0 | 0 | Hali－Windsor | $L_{s} R_{0} C_{S} L_{0} \mathrm{R}_{\infty} \mathrm{C}_{0} T$ |
| 7 | 2 | 0.29 | 6 | －1 | 1 |  | $\mathrm{L}_{0} \mathrm{R}_{0} L_{0} \mathrm{C}_{8} \mathrm{R}_{0} L_{8} \mathrm{C}_{0} \mathrm{~T}$ |
| 7 | 3 | 0.43 | 4 | 0 | 1 |  | $L_{0} C_{0} R_{0} C_{0} L_{0} \mathrm{R}_{9} \mathrm{CO}_{0} T$ |
| 8 | 2 | 0.25 | 8 | 0 | 2 |  | $L_{\otimes} R_{0} L_{0} C_{0} R_{\infty} L_{0} R_{\odot} C_{T} T$ |
| 8 | 3 | 0.38 | 12 | －1 | 0 | Windsor | $L_{0} C_{0} R_{\infty} L_{0} C_{0} R_{0} L_{0} C_{0} T$ |
| 9 | 3 | 0.33 | 24 | 0 | 0 |  | $L_{0} R_{0} C_{0} L_{0} R_{0} C_{6} L_{0} R_{0} C_{0} T$ |
| 9 | 4 | 0.44 | 8 | －1 | 2 |  | $\mathrm{L}_{0} \mathrm{C}_{8} \mathrm{R}_{0} \mathrm{C}_{\Phi} \mathrm{L}_{0} \mathrm{C}_{\Phi} \mathrm{R}_{0} \mathrm{~L}_{8} \mathrm{C}_{\circ} \mathrm{T}$ |

㽞 $h=$ number of moves

婉 $\gamma=$ number of center moves
\＆ $\begin{aligned} & K(h, \gamma)= \\ & 2^{\gamma-1}\binom{h-\gamma-2}{\gamma-1}\end{aligned}$
$s=\sum_{i=1}^{h} x_{i}$ where $x=-1$ for $L$ and +1 for $R$ ．
－$b=\frac{1}{2} \sum_{i=2}^{h-1}\left|\omega_{i}+\omega_{i-1}\right|$ where $\omega \stackrel{i=2}{=} \pm 1$ represents winding direction．

Irregular verbs
Cleaning up the code that is English：

＂Quantifying the evolutionary dynamics of language＂
Lieberman et al．，
Nature，449，713－716，2007．${ }^{[2]}$


Exploration of how verbs with irregular conjugation gradually become regular over time．
－Comparison of verb behavior in Old，Middle，and Modern English．

## Irregular verbs


－Universal tendency towards regular conjugation
Rare verbs tend to be regular in the first place




## Irregular verbs



R Rates are relative.
The more common a verb is, the more resilient it is to change.

## Irregular verbs

| Frequency | Verbs | Reguarization (\%) | Hall-life (yr) |
| :---: | :---: | :---: | :---: |
| ${ }^{10^{-1}-1}$ | be, have | 0 | 38,800 |
| $\begin{aligned} & 10^{-2}-2 \\ & 10^{-2}-1 \\ & \hline 0-1 \\ & \hline-2 \end{aligned}$ | come, do, find, get, give, go, know, say, see, take, think | 0 | 14,400 |
|  | begin, break, bring, buy, choose, draw, drink, drive, eat, fall, fight, forget, grow, hang, help, hold, leave, let, lie, lose, | 10 | 5,400 |
|  | reach, rise, run, seek, set, shake, sit, sleep, speak, stand, teach, throw, understand, walk, win, work, write |  |  |
| $10^{-4}-10^{-3}$ | arise, bake, bear, beat, bind, bite, blow, bow, burn, burst, | ${ }^{43}$ | 2,000 |
|  | carve, chee, cilime, cling, creep, dare, dig, drag, tiee, float, filow, fly, fold, freeze, gind, leap, lend, lock, melt, reckon, |  |  |
|  | ride, rush, shape, shine, shoot, shrink, sigh, sing, sink, slide, slip, smoke, spin, spring, starve, steal, step, stretch, strike, |  |  |
|  | stroke, suck, swallow, swear, sweep, swim, swing, tear, wake, wash, weave, weep, weigh, wind, yell, yield |  |  |
| $10^{-5-10-4}$ | bark, bellow, bid, blend, braid, brew, cleave, cringe, crow, | 72 | 700 |
|  | dive, drip, fare, fret, glide, gnaw, grip, heave, knead, low, milk, mourn, mow, prescribe, redden, reek, row, scrape seethe, shear, shed, shove, slay, slit, smite, sow, span, |  |  |
|  | spurn, sting, stink, strew, stride, swell, tread, uproot, wade, warp, wax, wield, wring, writhe |  |  |
| $10^{-6-10^{-5}}$ | bide, chide, delve, flay, hew, rue, shrive, slink, snip, spew, sup, wreak | 91 | 300 |

\& Red = regularized
Estimates of half-life for regularization ( $\propto f^{1 / 2}$ )

\& 'Wed' is next to go.
-ed is the winning rule...

- But 'snuck' is sneaking up on sneaked. [CT [3]


Personality distributions:

"A Theory of the Emergence, Persistence, and Expression of Geographic Variation in Psychological Co
Rentfrow, Gosling, and Potter, Perspectives on Psychological Science, 3, 339-369, 2008. ${ }^{[5]}$

Five Factor Model (FFM):
\& Extraversion [E]
8 Agreeableness [A]
, Conscientiousness [C]
R Neuroticism [N]
? Openness [O]
"...a robust and widely accepted framework for conceptualizing the structure of personality... Although the FFM is not universally accepted in the field..." ${ }^{[5]}$
A concern: self-reported data.

Agreeableness:


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## Conscientiousness：



Extraversion：


## Openness



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Neuroticism：


Limits of testability and happiness in Science：
From A Fight for the soul of Science $]$ in Quanta
Magazine（2016／02）：


## Europe：

Many errors called out in comments．Why hasn＇t this been done well？
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John Conway＇s Doomsday rule［］for determining a date＇s day of the week：

\＆Works for Gregorian（1582－，haphazardly）and the increasingly inaccurate Julian calendars（400 and 28 years cycles）．
\＆Apparently inspired by Lewis Carroll＇s work on a perpetual calendar．

## Outline：

Determine＂anchor day＂for a given century，then find Doomsday for a given year in that century．
Remember special Doomsday dates and work from there．
Naturally：Load this year＇s Doomsday into brain．

Century＇s anchor day（Gregorian，Sunday $\equiv 0$ ）：

$$
5 \times\left(\left\lfloor\frac{Y Y Y Y}{100}\right\rfloor \bmod 4\right) \quad \bmod 7+\text { Tuesday }
$$

Offset：

$$
\left(365 Y Y+\left\lfloor\frac{Y Y}{4}\right\rfloor\right) \quad \bmod 7=\left(Y Y+\left\lfloor\frac{Y Y}{4}\right\rfloor\right) \quad \bmod 7
$$

## Memorable Doomsdays：

| Month | Memorable date | Month／Day | Mnemonic ${ }^{[6]}$ |
| :---: | :---: | :---: | :---: |
| January | January 3 （common years），January 4 （leap years） | $1 / 3$ or $1 / 4$ | the 3 rd 3 years in 4 and the 4th in the 4th |
| February | February 28 （common years），February 29 （leap years） | 2128 or 2／29 | last day of February |
| March | ＂March 0＂ | $3 / 10$ | last day of February |
| April | April 4 | $4 / 4$ | 4／4，6／6，8／8，10／10，12／12 |
| May | May 9 | $5 / 9$ | 9－to－5 at 7－11 |
| June | June 6 | 6／6 | 4／4，6／6，8／8，10／10，12／12 |
| July | July 11 | $7 / 11$ | 9－to－5 at 7－11 |
| August | August 8 | 8／8 | 4／4，6／6，8／8，10／10，12／12 |
| September | September 5 | 9／5 | 9－to－5 at 7－11 |
| October | October 10 | 10／10 | 4／4，6／6，8／8，10／10，12／12 |
| November | November 7 | 11／7 | 9－to－5 at 7－11 |
| December | December 12 | $12 / 12$ | 4／4，6／6，8／8，10／10，12／12 |

\＆Pi day（March 14），July 4，Halloween，and Boxing Day are always Doomsdays．


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## The bissextile year■ $\quad$ ォ

＂The Julian calendar，which was developed in 46 BC by Julius Caesar，and became effective in 45 BC ，distributed an extra ten days among the months of the Roman Republican calendar．Caesar also replaced the intercalary month by a single intercalary day，located where the intercalary month used to be．To create the intercalary day，the existing ante diem sextum Kalendas Martias（February 24）was doubled， producing ante diem bis sextum Kalendas Martias．Hence， the year containing the doubled day was a bissextile（bis sextum，＂twice sixth＂）year．For legal purposes，the two days of the bis sextum were considered to be a single day， with the second half being intercalated；but in common practice by 238 ，when Censorinus wrote，the intercalary day was followed by the last five days of February，a．d．VI，V，IV， III and pridie Kal．Mart．（the days numbered 24，25，26，27， and 28 from the beginning of February in a common year）， so that the intercalated day was the first half of the doubled day．Thus the intercalated day was effectively inserted between the 23rd and 24th days of February．＂

The Teletherm，an early conception：

．Hibernal Teletherm $\approx$ February 4.
Halfway between Winter Solstice and Spring Equinox
\＆Bonus：Groundhog Day［ $\mathcal{B}$ ，Imbolc［ $\mathcal{B}, \ldots$
A Aesteval Teletherm $\approx$ July 19 （164 days later）．

In review：＂Tracking the Teletherms：The spatiotemporal dynamics of the hottest and coldest days of the year＂©＂， Dodds，Mitchell，Reagan，and Danforth．

． $2 \times 1218$ similar figures for the US．
6000ish pages of Supplementary Information（all figures）
\＆Interactive website．©

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Homo nonprobabilisticus，continued：
－Important detour：The final digits of primes are not entirely random（how did we not know this？）．
Start flipping a coin ．．．
Two tosses：What are the probabilities of flipping（1） $H H$ and（2）$H T$ ？
的 Flip a coin $n \geq 2$ times：What are the probabilities that the last two tosses are（1）$H H$ or（2）$H T$ ？
Estimate：On average，how many flips does it take to first see the sequence $H T$ ？
Estimate：On average，how many flips does it take to first see the sequence $H H$ ？
What＇s the probability of first flipping a $H T$ sequence on the $n-1$ th and $n$th flips？
What＇s the probability of first flipping two heads in a row（ $H H$ ）on the（ $n-1$ ）th and $n$th flips？

Homo nonprobabilisticus，continued：


Average number of flips： 4 and 6 ．

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－Accidents of evolution ${ }^{1}$ give us $5+5=10$ fingers and hence base 10 ．
We could be happy with base 6， $8,12, \ldots$
细 We like these：
（7． 60 seconds in a minute （－7） 60 minutes in an hour． （7） $2 \times 12=24$ hours in a day． （8） 360 degrees in a circle．

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|  |  |  |  |  |

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捔 $2,4,6,12,24,36$ ， $48,60,120,180$, 240，360，720，840， 1260，1680，2520， 5040 （Plato＇s optimal city population（〒），．．．
－OEIS sequence A002 $182 \overline{2}$

[^1]https：／／commons．wikimedia．org／w／index．php？curid＝31684018

Superior highly composite numbers： $\mathcal{B}$

| \＃prime factors | $\begin{gathered} \text { SHCN } \\ \mathrm{n} \end{gathered}$ | $\begin{gathered} \text { prime } \\ \text { factorization } \end{gathered}$ | $\begin{array}{\|c\|} \hline \text { prime } \\ \text { exponents } \end{array}$ | $\begin{gathered} \text { \#diviso } \\ d(n) \end{gathered}$ |  | $\begin{gathered} \text { primorial } \\ \text { factorization } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 2 | 1 | 2 | 2 | 2 |
| 2 | 6 | $2 \cdot 3$ | 1，1 | $2^{2}$ | 4 | 6 |
| 3 | 12 | $2^{2} \cdot 3$ | 2，1 | 3x2 | 6 | $2 \cdot 6$ |
| 4 | 60 | $2^{2} \cdot 3 \cdot 5$ | 2，1，1 | $3 \times 2^{2}$ | 12 | $2 \cdot 30$ |
| 5 | 120 | $2^{3} \cdot 3 \cdot 5$ | 3，1，1 | $4 \times 2^{2}$ | 16 | $2^{2} \cdot 30$ |
| 6 | 360 | $2^{3} \cdot 3^{2} \cdot 5$ | 3，2，1 | $4 \times 3 \times 2$ | 24 | 2．6．30 |
| 7 | 2520 | $2^{3} \cdot 3^{2} \cdot 5 \cdot 7$ | 3，2，1，1 | $4 \times 3 \times 2^{2}$ | 48 | 2．6．210 |
| 8 | 5040 | $2^{4} \cdot 3^{2} \cdot 5 \cdot 7$ | 4，2，1，1 | $5 \times 3 \times 2^{2}$ | 60 | $2^{2} \cdot 6 \cdot 210$ |
| 9 | 55440 | $2^{4} \cdot 3^{2} \cdot 5 \cdot 7 \cdot 11$ | 4，2，2，1，1 | $5 \times 3 \times 2^{3}$ | 120 | $2^{2} \cdot 6 \cdot 2310$ |
| 10 | 720720 | $2^{4} \cdot 3^{2} \cdot 5 \cdot 7 \cdot 11 \cdot 13$ | 4，2，1，1，1，1 | $5 \times 3 \times 2^{4}$ | 240 | $2^{2} \cdot 6 \cdot 30030$ |

SHCN＝natural number $n$ whose number of divisors exceeds that of any other number when scaled relative to itself in a sneaky way：

$$
\frac{d(n)}{n^{\epsilon}} \geq \frac{d(j)}{j^{\epsilon}} \text { and } \frac{d(n)}{n^{\epsilon}}>\frac{d(k)}{k^{\epsilon}}
$$

for $j<n<k$ and some $\epsilon>0$ ．

There＇s more：Superabundant numbers $\pi$
，$n$ is superabundant if：

$$
\frac{\sigma_{1}(n)}{n}>\frac{\sigma_{1}(j)}{j}
$$

for $j<n$ and where $\sigma_{x}(n)=\sum_{d \mid n} d^{x}$ is the divisor function．
449 numbers are both superabundant and highly composite．

Yet more：Colossally abundant numbers：$\pi$
$n$ is colossally abundant if for all $j$ and some $\epsilon>0$ ：

$$
\frac{\sigma_{1}(n)}{n^{1+\epsilon}} \geq \frac{\sigma_{1}(j)}{j^{1+\epsilon}}
$$

Infinitely many but only 22 less than $10^{18}$ ．

Some very，very silly units of measurement courtesy of the Imperial system［
22 yards in a chain $=1$ cricket pitch， 100 links in a chain， 10 chains in a furlong， 80 chains in a mile．
1 acre $=1$ furlong $\times 1$ chain $=43,560$ square feet．
160 fluid ounces in a gallon．
\＆ 14 pounds in a stone．
Hundredweight $=112$ pounds．

Also：
Fahrenheit，Celcius，and Kelvin．
The entire metric system．



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[^1]:    By Cmglee－Own work，CC BY－SA 3．0，

