

Chaotic Contagion: The Idealized Hipster Effect

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Complex Networks | @networksvox
CSYS/MATH 303, Spring, 2019

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Outline

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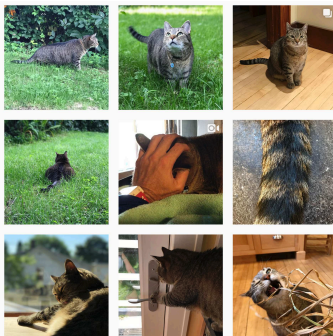


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Chaotic Contagion on Networks:



"Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability" [↗](#)
Dodds, Harris, and Danforth, Phys. Rev. Lett., **110**, 158701, 2013. [1]



"Dynamical influence processes on networks: General theory and applications to social contagion" [↗](#)
Harris, Danforth, and Dodds, Phys. Rev. E, **88**, 022816, 2013. [2]

A. Mandel, conference at Urbana-Champaign, 2007:

"If I was a younger man, I would have stolen this from you."

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Chaotic contagion:

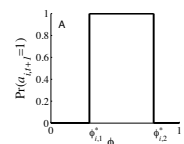
What if individual response functions are not monotonic?

Consider a simple deterministic version:

Node i has an 'activation threshold' $\phi_{i,1}$

...and a 'de-activation threshold' $\phi_{i,2}$

Nodes like to imitate but only up to a limit—they don't want to be like everyone else.



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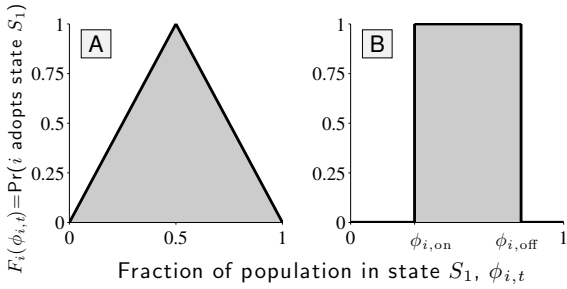
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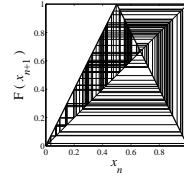
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Chaotic behavior

Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

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Chaotic contagion

Definition of the tent map:

$$F(x) = \begin{cases} rx & \text{for } 0 \leq x \leq \frac{1}{2}, \\ r(1-x) & \text{for } \frac{1}{2} \leq x \leq 1. \end{cases}$$

- The usual business: look at how F iteratively maps the unit interval $[0, 1]$.

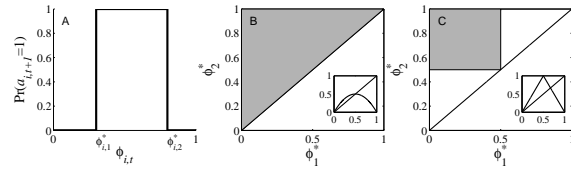
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Two population examples:



- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

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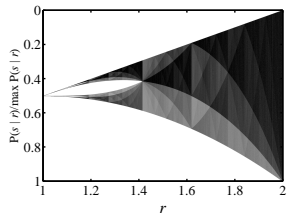
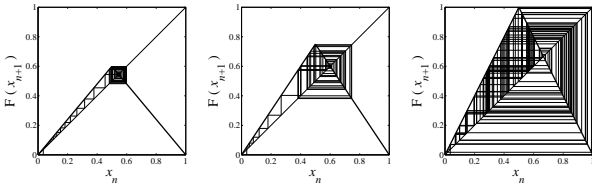
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The tent map

Effect of increasing r from 1 to 2.



Orbit diagram:
Chaotic behavior increases as map slope r is increased.

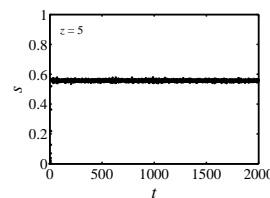
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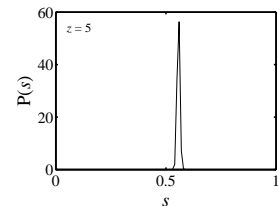


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Invariant densities—stochastic response functions



activation time series



activation density

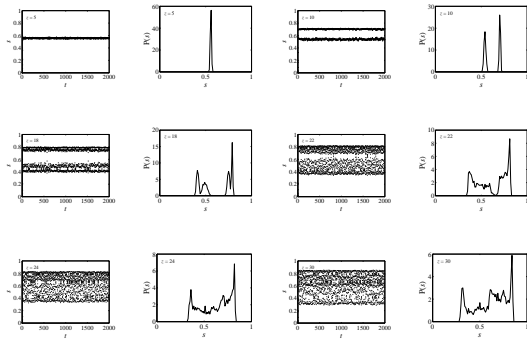
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Invariant densities—stochastic response functions



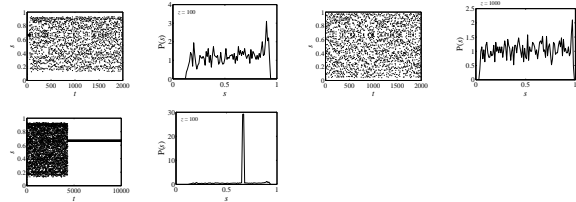
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Invariant densities—deterministic response functions



Trying out higher values of $\langle k \rangle$...

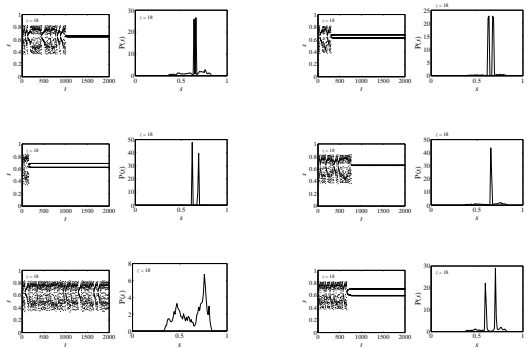
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Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



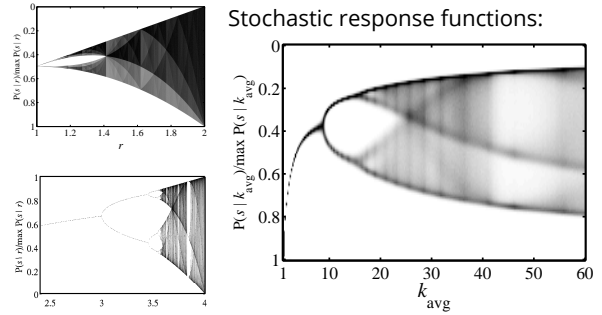
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Connectivity leads to chaos:



Stochastic response functions:

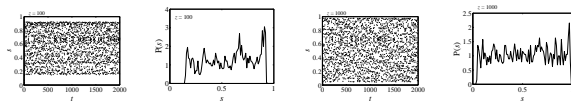
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Invariant densities—stochastic response functions



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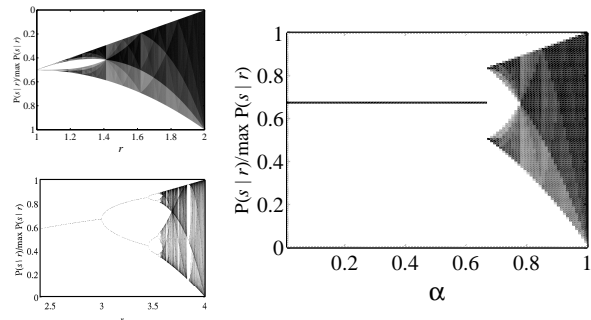
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Bifurcation diagram: Asynchronous updating



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Bifurcation diagram: Asynchronous updating

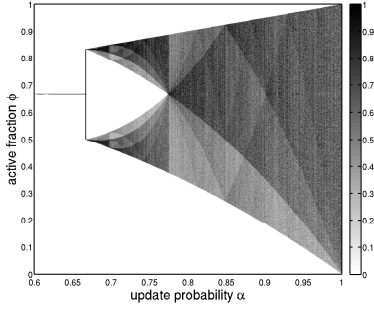


FIG. 3. Bifurcation diagram for the dense map $\Phi(\phi; \alpha)$, Eqn. (18). This was generated by iterating the map at 1000 α values between 0 and 1. The iteration was carried out with 3 random initial conditions for 10000 time steps each, discarding the first 1000. The ϕ -axis contains 1000 bins and the invariant density, shown by the grayscale value, is normalized by the maximum for each α . With $\alpha < 2/3$, all trajectories go to the fixed point at $\phi = 2/3$.

<https://www.youtube.com/watch?v=7JHrZyyq870?rel=0>
How the bifurcation diagram changes with increasing average degree $\langle k \rangle$ as a function of the synchronicity parameter α for the stochastic response (tent map) case.

https://www.youtube.com/watch?v=_zwK6polBvc?rel=0
How the bifurcation diagram changes with increasing α , the synchronicity parameter as a function of average degree $\langle k \rangle$ for the stochastic response (tent map) case.

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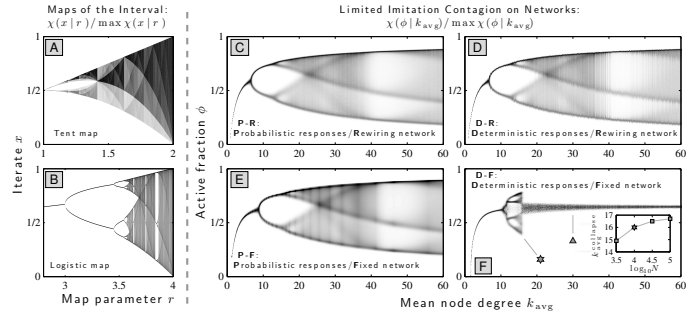
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<https://www.youtube.com/watch?v=3bo4fzp4Snw?rel=0>
LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 6, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is period-1, plus noisy fluctuations.

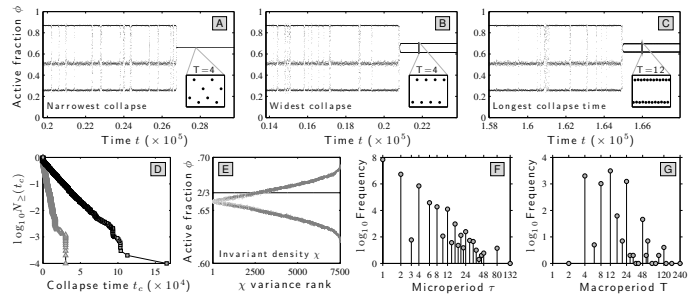
https://www.youtube.com/watch?v=7UCula_ktmw?rel=0
LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 11, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is period-2, plus noisy fluctuations.

<https://www.youtube.com/watch?v=oWkt8Zj1Ccw?rel=0>
LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. $\langle k \rangle = 30$, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is chaotic.

<https://www.youtube.com/watch?v=AfhUkIOiOU?rel=0>
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter $\alpha = 1$. Shown are nodes which continue changing (703/1000) after the transient chaotic behavior has "collapsed."



<https://www.youtube.com/watch?v=ZwY0hTstj2M?rel=0>
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter $\alpha = 1$. The dynamics exhibit transient chaotic behavior before collapsing to a fixed point.



<https://www.youtube.com/watch?v=YDhjmFyBSn4?rel=0>
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 17, update synchronicity parameter $\alpha = 1$. The dynamics exhibit transient chaotic behavior before collapsing to a period-4 orbit.

References

- [1] P. S. Dodds, K. D. Harris, and C. M. Danforth. Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability. *Phys. Rev. Lett.*, 110:158701, 2013. [pdf](#)
- [2] K. D. Harris, C. M. Danforth, and P. S. Dodds. Dynamical influence processes on networks: General theory and applications to social contagion. *Phys. Rev. E*, 88:022816, 2013. [pdf](#)