# Random Networks Nutshell

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Complex Networks | @networksvox CSYS/MATH 303, Spring, 2018

## Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont





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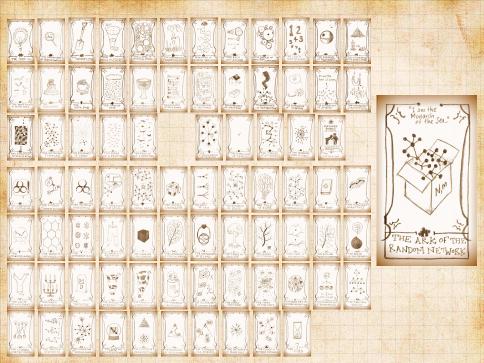
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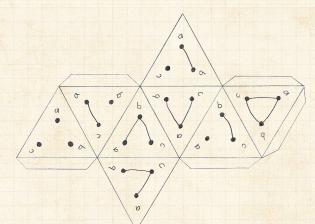




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## Random network generator for N = 3:



Set your own exciting generator here  $\mathbb{Z}$ . As  $N \nearrow$ , polyhedral die rapidly becomes a ball...

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## Pure random networks Definitions

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# Pure, abstract random networks:

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## Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
  - Standard random network = one randomly chosen network from this set. To be clear: each network is equally probable. Sometimes equiprobability is a good assumptio but it is always an assumption.
  - Known as Erdős-Rényi random networks or graphs.

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A Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

Limit of m = 0: empty graph. Limit of  $m = \binom{N}{2}$ : complete or fully-connected graph. Number of possible networks with N labelled nodes:

Given m edges, there are  $\binom{l_2 l}{m}$  different possib networks. Crazy factorial explosion for  $1 \ll m \ll \binom{N}{2}$ . Real world. links are usually costly so real networks are almost always sparse.

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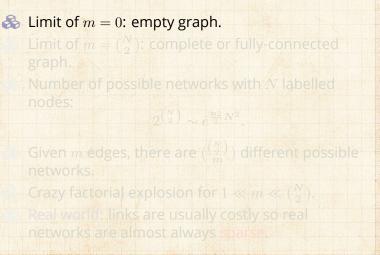




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- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Given m edges, there are  $\binom{\binom{N}{2}}{m}$  different possible networks. Crazy factorial explosion for  $1 \ll m \ll \binom{N}{2}$ . Real work links are usually costly so real networks are almost always sparse.

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# How to build standard random networks:



 $\leq$  Given N and m.

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How to build standard random networks:

- $\bigotimes$  Given N and m.
- 🗞 Two probablistic methods (we'll see a third later

Connect each of the  $\binom{N}{2}$  pairs with appropriate probability p.

Take N nodes and add exactly m links by selectir edges without replacement.

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  - 2. Take N nodes and add exactly m links by selecting edges without replacement.

Algorithm: Randomly choose a pair of nodes *i* an *j*,  $i \neq j$ , and connect if unconnected; repeat until all *m* edges are allocated. Best for adding relatively small numbers of links (most cases).

and 2 are effectively equivalent for large I

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    - 1 and 2 are effectively equivalent for large N.

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## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2}$$

So the expected or average degree

Which is what it should be... If we keep ( ) constant then  $p\propto 1/2$ 

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## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

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## A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

## So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

Which is what it should be... If we keep  $\langle k 
angle$  constant then  $p \propto$ 

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Which is what it should be... If we keep  $\langle k \rangle$  constant then p

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$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

Which is what it should be... If we keep  $\langle k \rangle$  constant then  $p \propto 1/N \rightarrow 0$ 

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# Random networks

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$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

🚳 Which is what it should be...

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So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$= \frac{2}{N} p \frac{1}{2} N(N-1) = \frac{2}{N} p \frac{1}{2} \mathcal{N}(N-1) = p(N-1)$$

Which is what it should be...
If we keep (k) constant then  $p \propto 1/N \rightarrow 0$  as  $N \rightarrow \infty$ .

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#### Next slides: Example realizations of random networks

Vary *m*, the number of edges from 100 to 1000 Average degree  $\langle k \rangle$  runs from 0.4 to 4. Look at full network plus the largest component



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# Next slides: Example realizations of random networks $\gg N = 500$

Vary *µ*, the number of edges from 100 to 1000
Average degree (*k*) runs from 0.4 to 4,
Look at full network plus the largest component



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# Next slides: Example realizations of random networks N = 500 Vary m, the number of edges from 100 to 1000. Average degree (/) runs from 0.4 to 4. Look at full network plus the largest component



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#### Next slides: Example realizations of random networks $\Im N = 500$ $\Im$ Vary *m*, the number of edges from 100 to 1000. $\Im$ Average degree $\langle k \rangle$ runs from 0.4 to 4.

Look at full network plus the largest component.

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#### Next slides:

#### Example realizations of random networks

- N = 500
  Vary *m*, the number of edges from 100 to 1000.
  Average degree (k) runs from 0.4 to 4.
- look at full network plus the largest component.



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#### Random networks: examples for N=500

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m = 100

(k) = 0.4



m = 200

 $\langle k \rangle = 0.8$ 



*m* = 300

 $\langle k \rangle = 1.2$ 

m = 230

 $\langle k \rangle = 0.92$ 



m = 500 $\langle k \rangle = 2$ 

m = 240

 $\langle k \rangle = 0.96$ 

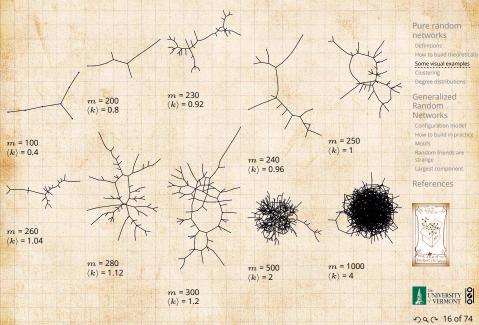
m = 1000 $\langle k \rangle = 4$ 

m = 250

 $\langle k \rangle = 1$ 

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#### Random networks: largest components



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#### Random networks: examples for N=500

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m = 250 $\langle k \rangle = 1$ 

m = 250

 $\langle k \rangle = 1$ 



m = 250

 $\langle k \rangle = 1$ 



m = 250

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m = 250

 $\langle k \rangle = 1$ 

### Random networks: largest components

*m* = 250

m = 250 $\langle k \rangle = 1$ 

> m = 250 $\langle k \rangle = 1$

 $\langle k \rangle = 1$ 



m = 250 $\langle k \rangle = 1$ 



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m = 250 $\langle k \rangle = 1$ 

*m* = 250

 $\langle k \rangle = 1$ 

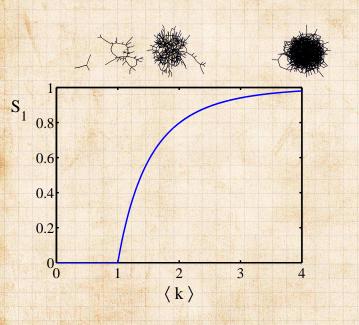
m = 250

m = 250

 $\langle k \rangle = 1$ 

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# Giant component



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#### Clustering in random networks: For construction method 1, what is the clustering coefficient for a finite network?

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: <sup>[6]</sup>

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$ 

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: <sup>[6]</sup>

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$ 

\*

Recall:  $C_2$  = probability that two friends of a node are also friends. COcoNuTS

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: <sup>[6]</sup>

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$ 

Recall:  $C_2$  = probability that two friends of a node are also friends.

Or:  $C_2$  = probability that a triple is part of a triangle.



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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: <sup>[6]</sup>

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$ 

Recall:  $C_2$  = probability that two friends of a node are also friends.

- Or:  $C_2$  = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p_1$$

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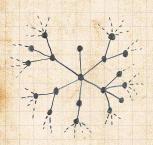
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So for large random networks  $(N \rightarrow \infty)$ , clustering drops to zero.

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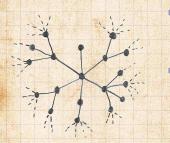
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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks

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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks
 No small loops.

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# Outline

#### Pure random networks

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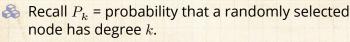
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Consider method 1 for constructing random networks: each possible link is realized with probability *p*.

Now consider one node: there are 'N - 1 choose ways the node can be connected to k of the other N - 1 nodes.

Each connection occurs with probability p, eac non-connection with probability (1-p).

Therefore have a

 $P(k;p,N) = \binom{N}{k}$ 

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- Recall  $P_k$  = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
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Dac 24 of 74

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- Therefore have a binomial distribution

$$P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

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🚳 Our degree distribution:  $P(k; p, N) = {\binom{N-1}{k}} p^k (1-p)^{N-1-k}.$ 

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Solution:  $P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$ What happens as  $N \to \infty$ ?

right? If p is fixed, then we would end up with a Gaussia with average degree  $\langle k \rangle \simeq pN \rightarrow \infty$ . But we want to keep  $\langle k \rangle$  fixed... So examine limit of P(k; p, N) when p = 0 and with  $\langle k \rangle = pN \rightarrow \infty$ .

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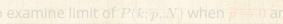
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Networks

So examine limit of P(k; p, N) when  $p \rightarrow 0$  and

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 $\mathfrak{F}$  This is a Poisson distribution  $\mathfrak{T}$  with mean  $\langle k \rangle$ .

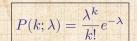
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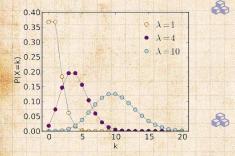
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λ > 0
k = 0, 1, 2, 3, ...
Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.

e.g.: phone calls/minute, horse-kick deaths. 'Law of small numbers' COcoNuTS

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The variance of degree distributions for random networks turns out to be very important.

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The variance of degree distributions for random networks turns out to be very important.
Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$ 



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 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2} = \langle k \rangle.$ 

So standard deviation  $\sigma$  is equal to  $\sqrt{\langle k \rangle}$ .

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So standard deviation *σ* is equal to √⟨k⟩.
 Note: This is a special property of Poisson distribution and can trip us up...

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Generalized Random Networks Configuration model

So... standard random networks have a Poisson degree distribution

 Randomly wiring up (and rewiring) already exit nodes with fixed degrees.

Examining mechanisms that lead to network certain degree distributions.

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So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P<sub>k</sub>.

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 Generalize to arbitrary degree distribution P<sub>k</sub>.
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 Assign each node a weight w from some

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 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$ 

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But we'll be more interested in

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### Coming up:

Example realizations of random networks with power law degree distributions:

 $P_k \propto k^{-\gamma}$  for  $k \ge 1$ . Set  $P_0 = 0$  (no isolated nodes). Vary exponent  $\gamma$  between 2.10 and 2.91. Again, look at full network plus the largest component. COcoNuTS

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### Coming up:

Example realizations of random networks with power law degree distributions:

 $\implies N = 1000.$ 

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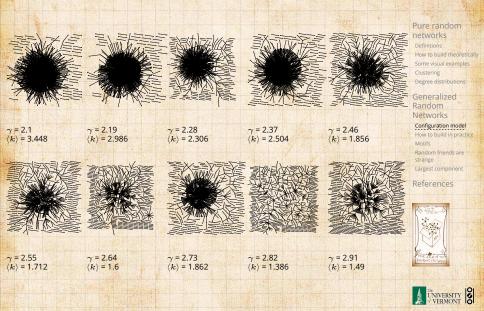
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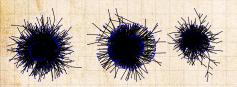


## Random networks: examples for N=1000

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## Random networks: largest components





 $\gamma = 2.19$ (k) = 2.986  $\gamma = 2.28$ (k) = 2.306



 $\gamma = 2.46$ (k) = 1.856





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 $\gamma = 2.55$ (k) = 1.712





 $\gamma = 2.64$  $\langle k \rangle = 1.6$ 

 $\gamma = 2.73$ (k) = 1.862  $\gamma = 2.82$ (k) = 1.386  $\gamma = 2.91$ (k) = 1.49

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# Generalized random networks:

Arbitrary degree distribution  $P_k$ . Create (unconnected) nodes with degrees sampled from  $P_k$ . Wire nodes together randomly.

randomness.

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Generalized random networks:

& Arbitrary degree distribution  $P_k$ .

Create (unconnected) nodes with degrees sampled from  $P_k$ . Wire nodes together randomly. Create ensemble to test deviations from randomness.

### Generalized random networks:

 $\clubsuit$  Arbitrary degree distribution  $P_k$ .

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Wire nodes together randomly. Create ensemble to test deviations from randomness. COcoNuTS

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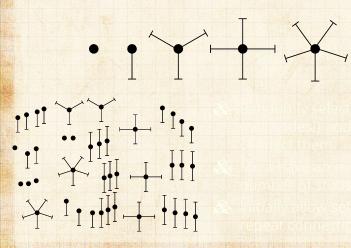




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### Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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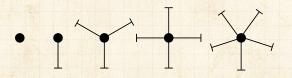


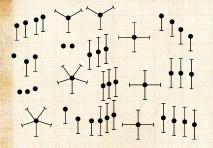
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### Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stut (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- an connections.

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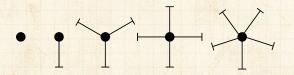


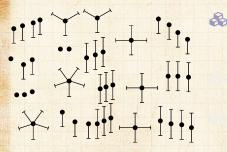


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### Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





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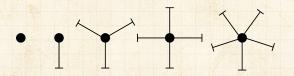


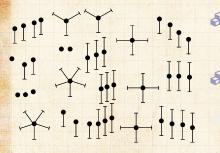


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### Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs.

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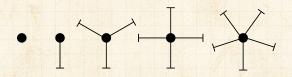


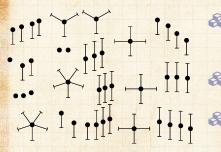


# Building random networks: Stubs

#### Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- and repeat connections. COcoNuTS

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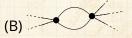


# Building random networks: First rewiring

#### Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful: we can't change the degree of a node, so we can't simply move links around. Simplest solution: randomly rewire two edges time. COcoNuTS

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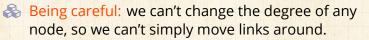




# Building random networks: First rewiring

#### Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



(B)

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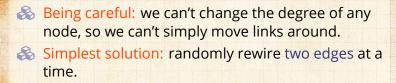


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# Building random networks: First rewiring

#### Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



(R)

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Randomly choose two edges. (Or choose problem edge and a random edge) COcoNuTS

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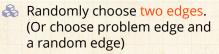
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Check to make sure edges are disjoint. Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

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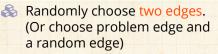
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Check to make sure edges are 3 disjoint.



#### Rewire one end of each edge.

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e1

Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

lacktrian Rewire one end of each edge.

Node degrees do not change.

Works if  $e_1$  is a self-loop or repeated edge. Same as finding on/off/on/of Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

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e1

Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

🗞 Rewire one end of each edge.

- Node degrees do not change.
- Works if  $e_1$  is a self-loop or repeated edge.

Same as finding on/off/on/off 4-cycles, and rotating them. Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

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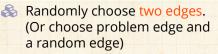




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e2

e1



Check to make sure edges are disjoint.

- Rewire one end of each edge.
  - Node degrees do not change.
  - Works if  $e_1$  is a self-loop or repeated edge.
    - Same as finding on/off/on/off 4-cycles. and rotating them.

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#### Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

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#### Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

#### Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Rule of thumb: # Rewirings  $\simeq 10 \times #$  edge

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#### Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

### Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Solution Relation Re

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#### Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

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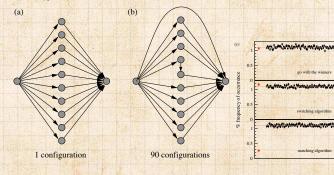




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### Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.
 Example from Milo et al. (2003)<sup>[4]</sup>:



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 $\mathbb{R}$  What if we have  $P_k$  instead of  $N_k$ ?



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 $\mathbb{R}$  What if we have  $P_k$  instead of  $N_k$ ? Must now create nodes before start of the construction algorithm.

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What if we have P<sub>k</sub> instead of N<sub>k</sub>?
 Must now create nodes before start of the construction algorithm.
 Construct N nodes by sampling from degree

Solution  $P_k$ . Generate N nodes by sampling from degree distribution  $P_k$ .

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What if we have P<sub>k</sub> instead of N<sub>k</sub>?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P<sub>k</sub>.

Easy to do exactly numerically since k is discrete.

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What if we have P<sub>k</sub> instead of N<sub>k</sub>?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P<sub>k</sub>.
Easy to do exactly numerically since k is discrete.
Note: not all P<sub>k</sub> will always give nodes that can be wired together.

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Random

Motifs



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Generalized Random Networks

Motifs

# Idea of motifs<sup>[7]</sup> introduced by Shen-Orr, Alon et al. in 2002.

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- Idea of motifs<sup>[7]</sup> introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

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References





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- Idea of motifs<sup>[7]</sup> introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.



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- Idea of motifs<sup>[7]</sup> introduced by Shen-Orr, Alon et al. in 2002.
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- 🚳 Specific example of Escherichia coli.
  - Directed network with 577 interactions (edges) and 424 operons (nodes).



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- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency  $N_k$ .

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- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency  $N_k$ .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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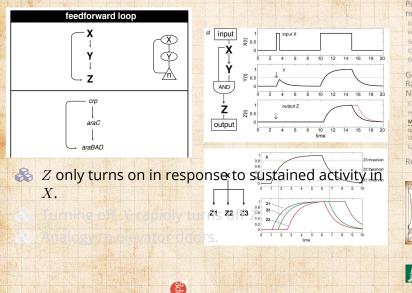
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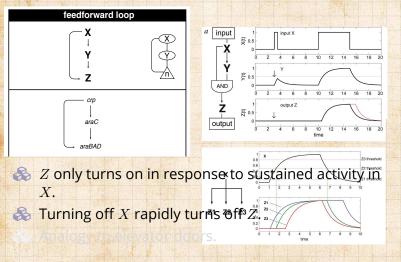
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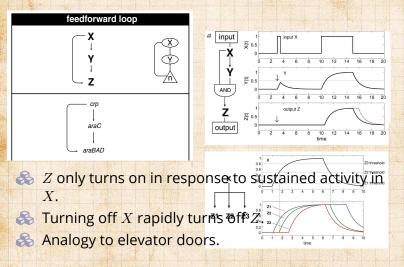


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Pure random



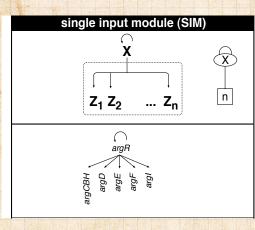
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#### 🚳 Master switch.

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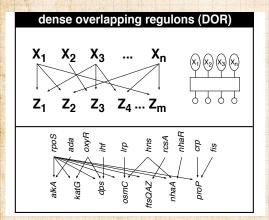
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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
 For more, see work carried out by Wiggins *et al.* at Columbia.



### Outline

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Generalized Random Networks

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#### The edge-degree distribution:

The degree distribution  $P_k$  is fundamental for our description of many complex networks

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The degree distribution  $P_k$  is fundamental for our description of many complex networks

Solution Again:  $P_k$  is the degree of randomly chosen node.

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- The degree distribution  $P_k$  is fundamental for our description of many complex networks
- Solution Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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- The degree distribution  $P_k$  is fundamental for our description of many complex networks
- Solution Again:  $P_k$  is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Solution  $Q_k$  to be the probability the node at a random end of a randomly chosen edge has degree k.



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- The degree distribution  $P_k$  is fundamental for our description of many complex networks
- Solution Again:  $P_k$  is the degree of randomly chosen node.
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- Befine  $Q_k$  to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$ 

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- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$ 

Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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- Define Q<sub>k</sub> to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

 $Q_k \propto k P_k$ 

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- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$ 

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

Big deal: Rich-get-richer mechanism is built into this

selection process.

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Probability of randomly selecting a node of degree k by choosing from nodes:  $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$  $P_6 = 1/7.$ 

> Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:  $Q_1 = 3/16, Q_2 = 4/16,$  $Q_3 = 3/16, Q_6 = 6/16,$

> Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

 $R_0 = 3/16 R_1 = 4/10$  $R_2 = 3/16, R_5 = 6/1$ 

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Probability of randomly selecting a node of degree k by choosing from nodes:  $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$  $P_6 = 1/7.$ 

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> Probability of finding # outgoing edges = & after randomly selecting an edge and then randomly choosing one direction to travel:

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Probability of randomly selecting a node of degree k by choosing from nodes:  $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$  $P_6 = 1/7.$ 

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:  $Q_1 = 3/16, Q_2 = 4/16,$  $Q_3 = 3/16, Q_6 = 6/16.$ 

> Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$\begin{split} R_0 &= 3/16 \; R_1 = 4/16, \\ R_2 &= 3/16, \; R_5 = 6/16. \end{split}$$

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

 $\bigotimes$  Useful variant on  $Q_k$ :

 $R_k$  = probability that a friend of a random node has k other friends. COcoNuTS

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

 $\bigotimes$  Useful variant on  $Q_k$ :

3

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

 $\bigotimes$  Useful variant on  $Q_k$ :

2

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

 $\bigotimes$  Useful variant on  $Q_k$ :

3

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Solution Equivalent to friend having degree k + 1.

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For random networks,  $Q_k$  is also the probability that a friend (neighbor) of a random node has  $\frac{k}{k}$  friends.

 $\bigotimes$  Useful variant on  $Q_k$ :

3

 $R_k$  = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?

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Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k \right\rangle_R = \sum_{k=0}^\infty k R_k$$

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Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

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Given R<sub>k</sub> is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left< k \right>_R = \sum_{k=0}^\infty k R_k = \sum_{k=0}^\infty k \frac{(k+1) P_{k+1}}{\left< k \right>}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^\infty k(k+1) P_{k+1}$$

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$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} \left( (k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

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Note: our result,  $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$ , is true for all random networks, independent of degree distribution.

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🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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to

$$\left< k \right>_R = rac{1}{\left< k \right>} \left( \left< k \right>^2 + \left< k \right> - \left< k \right> \right)$$

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-Sa

A Therefore:

$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left( \langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

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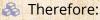




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Again, neatness of results is a special property of the Poisson distribution.

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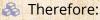




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 So friends on average have (l) other friends and

So friends on average have  $\langle k \rangle$  other friends, and  $\langle k \rangle + 1$  total friends...

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# The edge-degree distribution: In fact, $R_k$ is rather special for pure random networks ...

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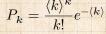




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In fact, R<sub>k</sub> is rather special for pure random networks ...
 Substituting

into



 $R_{k}=\frac{(k+1)P_{k+1}}{\langle k\rangle}$ 

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In fact, R<sub>k</sub> is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$ 

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_{k} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle}$$

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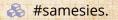
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#### Two reasons why this matters

#### Reason #1:

Average # friends of friends per node is

Key: Average depends on the 1st and 2nd moments  $P_k$  and not just the 1st moment. Three peculiarities:

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### Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

Key: Average depends on the 1st and 2nd moments  $P_k$  and not just the 1st moment. Three peculiarities:

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### Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left( \langle k^2 \rangle - \langle k \rangle \right)$$

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#### Three peculiarities:

1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$  but it's actually  $\langle k(k-1) \rangle$ .

Your friends really are different from you... See also: class size paradoxes (nod to: Gelm

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Random friends are strange Largest component





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- 1. We might guess  $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$  but it's actually  $\langle k(k-1) \rangle$ .
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(e.g., in the case of a power-law distribution Your friends really are different from you...

See also: class size paradoxes (nod to: Gelman)

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- 3. Your friends really are different from you...<sup>[3, 5]</sup>

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### More on peculiarity #3:

- $\mathfrak{A}$  A node's average # of friends:  $\langle k \rangle$ 
  - Friend's average # of friends: 4

So only if everyone has the same degree (variance =  $\sigma^2 = 0$ ) can a node be the same as friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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A node's average # of friends:  $\langle k \rangle$ Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$ 

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### More on peculiarity #3:

- A node's average # of friends:  $\langle k \rangle$ Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
  - $\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$

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 $|k\rangle$ 





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### More on peculiarity #3:

- A node's average # of friends:  $\langle k \rangle$ Friend's average # of friends:  $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
  - $\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left( 1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge \langle k \rangle$
- So only if everyone has the same degree (variance=  $\sigma^2 = 0$ ) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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Your friends really are monsters #winners:<sup>1</sup> Go on the met Friends have more coauthors citations, and publications. Other homfic studies, your connections on Twitter have more followers than you, your se partners more partners than you, ... The hope: Maybe they have more enemies an diseases too.

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### (Big) Reason #2:

- $\langle k \rangle_R$  is key to understanding how well random networks are connected together.
  - e.g., we'd like to know what's the size of the larg component within a network. As  $N \rightarrow \infty$ , does our network have a giant component?
  - Defo: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
  - Defn: Giant component = component that comprises a non-zero fraction of a network as
  - Note: Component = Cluster

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# Outline

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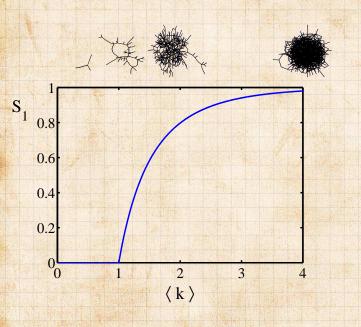
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# Giant component



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# Structure of random networks Giant component:

A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

Equivalently, expect exponential growth in nod number as we move out from a random node. All of this is the same as requiring  $\langle k \rangle_R > 1$ . Giant-component condition (or percolation condition):

Again, see that the second moment is an essenti part of the story. Equivalent statement:  $\langle k^2 \rangle > 2 \langle k \rangle$ 

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Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 2$$

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- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
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Equivalent statement:  $\langle k^2 \rangle > 2 \langle k \rangle$ 

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# Spreading on Random Networks

For random networks, we know local structure is pure branching.

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For random networks, we know local structure is pure branching. 🚳 Successful spreading is 🛛 contingent on single

edges infecting nodes.

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- For random networks, we know local structure is pure branching.
- Successful spreading is ... contingent on single edges infecting nodes.

Success Failure:

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Success

For random networks, we know local structure is pure branching.

Failure:

Successful spreading is ... contingent on single edges infecting nodes.

Focus on binary case with edges and nodes either infected or not.

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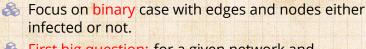




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- For random networks, we know local structure is pure branching.
- Successful spreading is .. contingent on single edges infecting nodes.

Success Failure:



First big question: for a given network and contagion process, can global spreading from a single seed occur? COcoNuTS

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We need to find: <sup>[1]</sup>
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

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 $R = \sum_{i=1}^{\infty}$ 

2

 $kP_k$  $\langle k \rangle$ 

prob. of connecting to a degree k node

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3

 $\frac{kP_k}{\langle k\rangle}$ 

prob. of connecting to a degree k node

(k - 1)

# outgoing infected edges COcoNuTS

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(k - 1)

# outgoing infected edges

Prob. of infection

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 Define B<sub>k1</sub> as the probability that a node of

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 $R = \sum$ 

3

 $\langle k \rangle$ prob. of connecting to a degree k node

 $kP_k$ 

$$+\sum_{k=0}^\infty \frac{\widehat{kP_k}}{\langle k\rangle}$$

(k - 1)

# outgoing infected edges

Prob. of infection

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# outgoing infected edges

 $\mathbf{R} = \sum_{k=0}^{\infty}$ 

3

3

prob. of connecting to a degree k node

 $\frac{kP_k}{\langle k \rangle}$ 

(k - 1)

# outgoing

infected edges

 $B_{k1}$ 

Prob. of infection

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3

🙈 We need to find: [1] R = the average # of infected edges that one random infected edge brings about. 🚳 Call **R** the gain ratio. Define  $B_{k_1}$  as the probability that a node of degree k is infected by a single infected edge.

 $\mathbf{R} = \sum_{k=0}^{\infty}$ (k - 1) $B_{k1}$ Prob. of # outgoing prob. of infected infection connecting to edges a degree k node  $+\sum_{k=0}^{\infty}\frac{\hat{k}P_k}{\langle k\rangle}$  $(1 - B_{k1})$ # outgoing Prob. of infected no infection edges

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lour global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

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🙈 Case 1–Rampant spreading:

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Our global spreading condition is then:

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Solution Case 1–Rampant spreading: If  $B_{k1} = 1$ 

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Solution Case 1–Rampant spreading: If  $B_{k1} = 1$  then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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Good: This is just our giant component condition again.

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Case 2—Simple disease-like:

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#### Case 2—Simple disease-like: If $B_{k1} = \beta < 1$

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So Case 2—Simple disease-like: If  $B_{k1} = \beta < 1$  then

$$\mathbf{R} = \sum_{k=0}^\infty \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

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### $\Im$ A fraction (1- $\beta$ ) of edges do not transmit infection.

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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.

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 Aka bond percolation C.

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A fraction (1-β) of edges do not transmit infection.
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Aka bond percolation .

Resulting degree distribution  $\tilde{P}_k$ :

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

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### Giant component for standard random networks: Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$ .

Determine condition for giant component:

Therefore when  $\langle k \rangle > 1$ , standard random networks have a giant component. When  $\langle k \rangle < 1$ , all components are finite. Fine example of a continuous phase of the say  $\langle k \rangle = 1$  marks the critical point of the system. COcoNuTS

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Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\left\langle k \right\rangle_R = rac{\left\langle k^2 \right\rangle - \left\langle k \right\rangle}{\left\langle k \right\rangle}$$

Therefore when  $\langle k \rangle > 1$ , standard random networks have a giant component. When  $\langle k \rangle < 1$ , all components are finite. Fine example of a continuous phase We say  $\langle k \rangle = 1$  marks the critical point of th system.

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Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

4

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

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6

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Recall 
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Solution
 Solution

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Solution (k) > 1, standard random networks have a giant component.
 When (k) < 1, all components are finite.</li>
 Fine example of a continuous phase transition C.
 We say (k) = 1 marks the critical point of the system.

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$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

So giant component always exists for these kinds of networks. Cutoff scaling is  $k^{-3}$ : if  $\gamma > 3$  then we have to look harder at  $\langle k \rangle_R$ . How about  $P_k \neq \delta_{kk_0}$ ?

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$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

So giant component always exists for these kinds of networks. Cutoff scaling is  $k^{+3}$ : if  $\gamma > 3$  then we have to look harder at  $\langle k \rangle_{R}$ . How about  $P_k \neq \delta_{kk_0}$ ?

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$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

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$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty}$$

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$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

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### Random networks with skewed $P_k$ : e.g. if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$ , $k \ge 1$ , then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

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6

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- $\Im$  Define  $S_1$  as the size of the largest component.
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  - Let's find  $S_1$  with a back-of-the-envelope argumen Define  $\delta$  as the probability that a randomly chosen node does not belong to the largest component. Simple connection:  $\delta = 1 - S_1$ .

Dirty trick: If a randomly chosen node is not part of th largest component, then none of its neighbors are.

Substitute in Poisson distribution

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 $\kappa = 0$ 

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So

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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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VERMONT 8

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💑 So

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Substitute in Poisson distribution...

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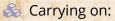
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# ${\color{black} \delta} = \sum_{k=0}^{\infty} P_k \delta^k$

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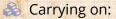
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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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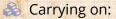
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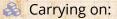
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$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta}$$

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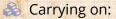
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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1-\delta)}$$

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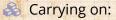
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$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$=e^{-\langle k
angle}e^{\langle k
angle\delta}=e^{-\langle k
angle(1-\delta)}$$

Solution Now substitute in  $\delta = 1 - S_1$  and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}$$

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We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ .

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We can figure out some limits and details for  $S_1 = 1 - e^{-\langle k \rangle S_1}$ . First, we can write  $\langle k \rangle$  in terms of  $S_1$ :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

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As 
$$\langle k \rangle \to 0$$
,  $S_1 \to 0$ .

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$$\begin{split} & \clubsuit \quad \mathsf{As} \ \langle k \rangle \to 0, \, S_1 \to 0. \\ & \clubsuit \quad \mathsf{As} \ \langle k \rangle \to \infty, \, S_1 \to 1. \end{split}$$

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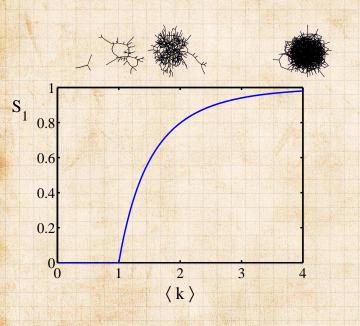
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### Turns out we were lucky...



### Our dirty trick only works for ER random networks.

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Turns out we were lucky...

Our dirty trick only works for ER random networks.
 The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.

But we know our friends are different from us. Works for ER random networks because

We need a separate probability  $\delta'$  for the chance that an edge leads to the giant (infinite) component.

We can sort many things out with sensible probabilistic arguments...

More detailed investigations will profit from a s of General ingrunctionology.

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- Solution Works for ER random networks because  $\langle k \rangle = \langle k \rangle_R$ .

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