Random Networks Nutshell

Last updated: 2018/03/23, 12:08:15

Complex Networks | @networksvox CSYS/MATH 303, Spring, 2018

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont





Licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 1 of 74

These slides are brought to you by:

Sealie & Lambie Productions

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



UNIVERSITY SVERMONT

990 2 of 74

These slides are also brought to you by:

Special Guest Executive Producer



On Instagram at pratchett_the_cat

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 3 of 74

Outline

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

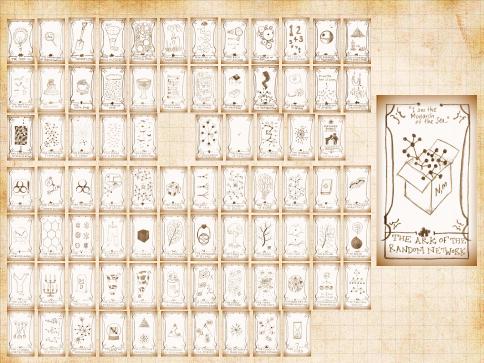
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

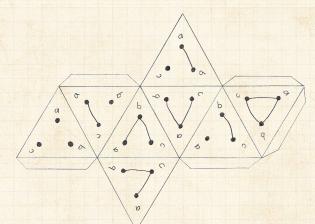




Dac 4 of 74



Random network generator for N = 3:



Set your own exciting generator here \mathbb{Z} . As $N \nearrow$, polyhedral die rapidly becomes a ball...

COcoNuTS

Pure random networks

How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 6 of 74

Outline

Pure random networks Definitions

Some visual examples Clustering Degree distributions

Configuration model How to build in practice Motifs Random friends are strang Largest component COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 7 of 74

Pure, abstract random networks:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
 - Standard random network = one randomly chosen network from this set. To be clear: each network is equally probable. Sometimes equiprobability is a good assumptio but it is always an assumption.
 - Known as Erdős-Rényi random networks or graphs.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
 - To be clear: each network is equally probable. Sometimes equiprobability is a good assumption but it is always an assumption. Known as Erdős-Rényi random networks of ER

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- least the clear: each network is equally probable.
 - Sometimes equiprobability is a good assumption, but it is always an assumption. Known as Erdős-Rényi random networks or ER graphs.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- lot be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.

Pure random networks Definitions How to build theoretic Some visual examples Clustering Decree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- lear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

COcoNuTS

Pure random networks Definitions How to build theoretic. Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





A Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

Limit of m = 0: empty graph. Limit of $m = \binom{N}{2}$: complete or fully-connected graph. Number of possible networks with N labelled nodes:

Given m edges, there are $\binom{l_2 l}{m}$ different possib networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real world. links are usually costly so real networks are almost always sparse.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

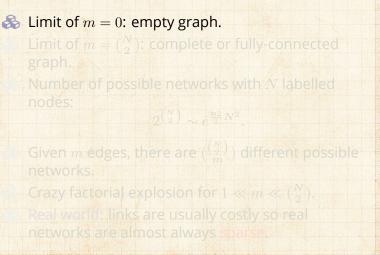




9 a @ 9 of 74

A Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$



COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 9 of 74

A Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

3 Limit of m = 0: empty graph. \bigotimes Limit of $m = \binom{N}{2}$: complete or fully-connected graph.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





9 a @ 9 of 74

Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

 \bigotimes Limit of m = 0: empty graph.

- Solution Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real work links are usually costly so real networks are almost always sparse.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

 \bigotimes Limit of m = 0: empty graph.

- Solution Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

Siven m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.

Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$ Real world links are usually costly so real networks are almost always sparse.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange Largest component





🚳 Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

 \bigotimes Limit of m = 0: empty graph.

- Solution Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

- Siven *m* edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- \bigotimes Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.

Real world: links are usually costly so real networks are almost always sparse.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Number of possible edges:

$$0 \le m \le \binom{N}{2} = \frac{N(N-1)}{2}$$

 \bigotimes Limit of m = 0: empty graph.

- Solution Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

 $2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}.$

- Siven *m* edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- So Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.
- Real world: links are usually costly so real networks are almost always sparse.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange Largest component





Outline

Pure random networks

How to build theoretically

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 10 of 74

How to build standard random networks:



 \leq Given N and m.

COCONUTS

Pure random networks How to build theoretically

Clustering Degree distributions

Generalized Networks How to build in practice Motifs





DQ @ 11 of 74

How to build standard random networks:

- \bigotimes Given N and m.
- 🗞 Two probablistic methods (we'll see a third later

Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.

Take N nodes and add exactly m links by selectir edges without replacement.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
 - Take N nodes and add exactly m links by selectine dges without replacement.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.

Take N nodes and add exactly m links by selectin edges without replacement.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.

Algorithm: Randomly choose a pair of nodes *i* an *j*, $i \neq j$, and connect if unconnected; repeat until all *m* edges are allocated. Best for adding relatively small numbers of links (most cases).

and 2 are effectively equivalent for large I

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 11 of 74

How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.
 - Useful for theoretical work.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i an $j, i \neq j$, and connect if unconnected; repeat until all m edges are allocated. Best for adding relatively small numbers of links (most cases).
 - and 2 are effectively equivalent for large I

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.
 - Useful for theoretical work.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - Best for adding relatively small numbers of links (most cases)
 - d^2 are effectively equivalent for large N.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 11 of 74

How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.
 - Useful for theoretical work.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - Best for adding relatively small numbers of links (most cases).

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





How to build standard random networks:

- \bigotimes Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability *p*.
 - Useful for theoretical work.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - Best for adding relatively small numbers of links (most cases).
 - 1 and 2 are effectively equivalent for large N.

COcoNuTS

Pure random metworks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p \binom{N}{2}$$

So the expected or average degree

Which is what it should be... If we keep () constant then $p\propto 1/2$

COcoNuTS

Pure random networks Definitions How to build theoretically

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 12 of 74

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions Generalized Random Networks

Configuration model How to build in practice Motifs Random friends are strange Largest component

References



UNIVERSITY SVERMONT

og@ 12 of 74

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

Which is what it should be... If we keep $\langle k
angle$ constant then $p \propto$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





nac 12 of 74

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)$$

Which is what it should be...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{\cancel{2}}{\cancel{N}}p\frac{1}{\cancel{2}}\cancel{N}(N-1)$$

Which is what it should be... If we keep $\langle k \rangle$ constant then p

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 12 of 74

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

Which is what it should be... If we keep $\langle k \rangle$ constant then $p \propto 1/N \rightarrow 0$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 12 of 74

Random networks

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k
angle = rac{2 \langle m
angle}{N}$$

$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

🚳 Which is what it should be...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 12 of 74

Random networks

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$= \frac{2}{N} p \frac{1}{2} N(N-1) = \frac{2}{N} p \frac{1}{2} \mathcal{N}(N-1) = p(N-1)$$

Which is what it should be...
If we keep (k) constant then $p \propto 1/N \rightarrow 0$ as $N \rightarrow \infty$.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Outline

Pure random networks

Some visual examples

Configuration model How to build in practice Motifs Random mends are strange Largest component

COcoNuTS

Pure random networks Definitions How to build theoretically

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





990 13 of 74

Next slides: Example realizations of random networks

Vary *m*, the number of edges from 100 to 1000 Average degree $\langle k \rangle$ runs from 0.4 to 4. Look at full network plus the largest component



Pure random networks Definitions How to build theore

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Next slides: Example realizations of random networks $\gg N = 500$

Vary *µ*, the number of edges from 100 to 1000
Average degree (*k*) runs from 0.4 to 4,
Look at full network plus the largest component



Pure random networks Definitions How to build theore

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Next slides: Example realizations of random networks N = 500 Vary m, the number of edges from 100 to 1000. Average degree (/) runs from 0.4 to 4. Look at full network plus the largest component



Pure random networks Definitions How to build theorem

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Next slides: Example realizations of random networks $\Im N = 500$ \Im Vary *m*, the number of edges from 100 to 1000. \Im Average degree $\langle k \rangle$ runs from 0.4 to 4.

Look at full network plus the largest component.

Pure random networks Definitions How to build theorem

COCONUTS

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Next slides:

Example realizations of random networks

- N = 500
 Vary *m*, the number of edges from 100 to 1000.
 Average degree (k) runs from 0.4 to 4.
- look at full network plus the largest component.



COCONUTS

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 14 of 74

Random networks: examples for N=500

COcoNuTS



Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

References



UNIVERSITY





m = 100

(k) = 0.4



m = 200

 $\langle k \rangle = 0.8$



m = 300

 $\langle k \rangle = 1.2$

m = 230

 $\langle k \rangle = 0.92$



m = 500 $\langle k \rangle = 2$

m = 240

 $\langle k \rangle = 0.96$

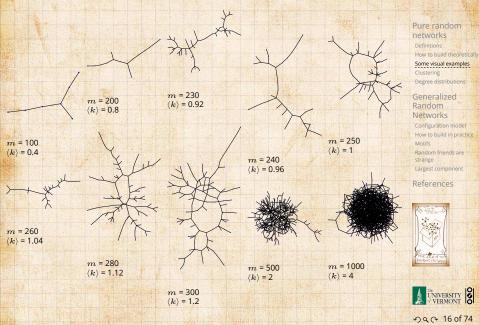
m = 1000 $\langle k \rangle = 4$

m = 250

 $\langle k \rangle = 1$

Dac 15 of 74

Random networks: largest components



COcoNuTS

Random networks: examples for N=500

COcoNuTS



Clustering Degree distributions

Generalized Random Networks How to build in practice Motifs Random friends are





References



m = 250 $\langle k \rangle = 1$

m = 250

 $\langle k \rangle = 1$



m = 250

 $\langle k \rangle = 1$



m = 250

 $\langle k \rangle = 1$

m = 250 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$

m = 250 $\langle k \rangle = 1$

m = 250 $\langle k \rangle = 1$

m = 250

 $\langle k \rangle = 1$

Random networks: largest components

m = 250

m = 250 $\langle k \rangle = 1$

> m = 250 $\langle k \rangle = 1$

 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$



Pure random networks Definitions How to build theoretically

COcoNuTS

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





m = 250 $\langle k \rangle = 1$

m = 250

 $\langle k \rangle = 1$

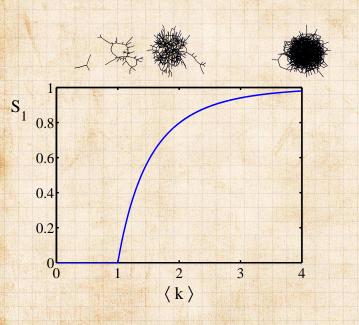
m = 250

m = 250

 $\langle k \rangle = 1$

Dac 18 of 74

Giant component



COcoNuTS

Pure random networks Definitions How to build theoretically

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 19 of 74

Outline

Pure random networks

Clustering

Configuration model How to build in practice Motifs Random friends are strange Largest component

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





20 of 74

Clustering in random networks: For construction method 1, what is the clustering coefficient for a finite network?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 21 of 74

For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[6]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distribution

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 21 of 74

For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[6]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

*

Recall: C_2 = probability that two friends of a node are also friends. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





29 c 21 of 74

For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[6]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

Recall: C_2 = probability that two friends of a node are also friends.

Or: C_2 = probability that a triple is part of a triangle.



Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distribution

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 21 of 74

For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[6]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

Recall: C_2 = probability that two friends of a node are also friends.

- Or: C_2 = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p_1$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples

Clustering Degree distribution

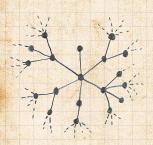
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 21 of 74



So for large random networks $(N \rightarrow \infty)$, clustering drops to zero.

COCONUTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

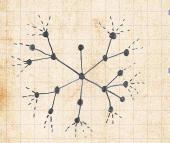
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 22 of 74



So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distribution

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





na @ 22 of 74



So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks
 No small loops.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distribution

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





na @ 22 of 74

Outline

Pure random networks

Degree distributions

Configuration model How to build in practice Motifs Random friends are strange Largest component

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

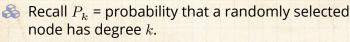
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





na @ 23 of 74



Consider method 1 for constructing random networks: each possible link is realized with probability *p*.

Now consider one node: there are 'N - 1 choose ways the node can be connected to k of the other N - 1 nodes.

Each connection occurs with probability p, eac non-connection with probability (1-p).

Therefore have a

 $P(k;p,N) = \binom{N}{k}$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



VINVERSITY S

24 of 74

- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
 - Now consider one node: there are N 1 choose ways the node can be connected to k of the other N 1 nodes.
 - Each connection occurs with probability p, each non-connection with probability (1 p).
 - Therefore have a



Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Solution Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.

Each connection occurs with probability p, each non-connection with probability (1 - p). Therefore have a connection of the probability p.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 24 of 74

- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Solution Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).

Pure random networks Definitions How to build theoretic Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 24 of 74

- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Solution Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1 p).
- Therefore have a binomial distribution

$$P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



VERMONT 8

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



UNIVERSITY S

DQ @ 25 of 74

🚳 Our degree distribution: $P(k; p, N) = {\binom{N-1}{k}} p^k (1-p)^{N-1-k}.$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



UNIVERSITY S

25 of 74

Solution: $P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$ What happens as $N \to \infty$?

right? If p is fixed, then we would end up with a Gaussia with average degree $\langle k \rangle \simeq pN \rightarrow \infty$. But we want to keep $\langle k \rangle$ fixed... So examine limit of P(k; p, N) when p = 0 and with $\langle k \rangle = pN \rightarrow \infty$.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?
- We must end up with the normal distribution right?

If p is fixed, then we would end up with a Gauss with average degree $\langle k \rangle \simeq pN \rightarrow \infty$. But we want to keep $\langle k \rangle$ fixed... So examine limit of P(k; p, N) when $p \rightarrow 0$ and

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?
- We must end up with the normal distribution right?
- Solution If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \rightarrow \infty$.

COCONUTS

Pure random

Degree distributions Generalized

How to build in practice

VERMONT

Random friends are

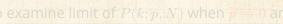
networks Definitions How to build theoretically

Clustering

Networks

So examine limit of P(k; p, N) when $p \rightarrow 0$ and

- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?
- We must end up with the normal distribution right?
- Solution If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \rightarrow \infty$.
- \bigotimes But we want to keep $\langle k \rangle$ fixed...



COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





25 of 74

- Our degree distribution: $P(k; p, N) = {\binom{N-1}{k}} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?
- 🚳 We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$.
- But we want to keep $\langle k \rangle$ fixed...
- So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1)$ = constant.

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \rightarrow \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

COCONUTS

Pure random networks Degree distributions

Generalized Networks How to build in practice Random friends are





- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
- \mathfrak{S} What happens as $N \to \infty$?
- 🚳 We must end up with the normal distribution right?
- If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$.
- But we want to keep $\langle k \rangle$ fixed...
- So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1)$ = constant.

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \to \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

 \mathfrak{F} This is a Poisson distribution \mathfrak{T} with mean $\langle k \rangle$.

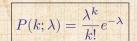
COCONUTS

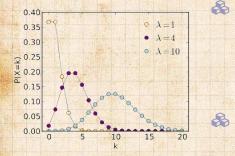
Pure random networks Degree distributions

Generalized Networks How to build in practice











λ > 0
k = 0, 1, 2, 3, ...
Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.

e.g.: phone calls/minute, horse-kick deaths. 'Law of small numbers' COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





The variance of degree distributions for random networks turns out to be very important.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 27 of 74

The variance of degree distributions for random networks turns out to be very important.
Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 27 of 74

The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

🗞 Variance is then

 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$



Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 27 of 74

The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

🗞 Variance is then

 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

🗞 Variance is then

 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2} = \langle k \rangle.$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 27 of 74

The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

🚷 Variance is then

 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2} = \langle k \rangle.$

So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

🗞 Variance is then

 $\sigma^{2} = \langle k^{2} \rangle - \langle k \rangle^{2} = \langle k \rangle^{2} + \langle k \rangle - \langle k \rangle^{2} = \langle k \rangle.$

So standard deviation *σ* is equal to √⟨k⟩.
 Note: This is a special property of Poisson distribution and can trip us up...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Outline

COCONUTS

Pure random networks How to build theoretically Clustering Degree distributions

Configuration model

Motifs Random friends are

References





Generalized Random Networks

How to build in practice



28 of 74

Generalized Random Networks Configuration model

So... standard random networks have a Poisson degree distribution

 Randomly wiring up (and rewiring) already exit nodes with fixed degrees.

Examining mechanisms that lead to network certain degree distributions.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

Largest component

References





na @ 29 of 74

So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.

Pure random hereits of the second sec

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Lacrest component

References





 Randomly wiring up (and rewiring) already exis nodes with fixed degrees.

 Examining mechanisms that lead to network certain degree distributions.

na 29 of 74

- So... standard random networks have a Poisson degree distribution
- Seneralize to arbitrary degree distribution P_k .
- lso known as the configuration model. [6]



COCONUTS

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange Largest component

References





So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model.^[6]
 Can generalize construction method from ER random networks.



Pure random networks Definitions How to build theoretics Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model. ^[6]
 Can generalize construction method from ER random networks.
 Assign each node a weight w from some

distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

nodes with fixed degrees. 2. Examining mechanisms that lead to networks certain degree distributions.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





So... standard random networks have a Poisson degree distribution \mathcal{L} Generalize to arbitrary degree distribution P_k . Also known as the configuration model.^[6] Can generalize construction method from ER random networks. Assign each node a weight w from some distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$



But we'll be more interested in

COCONUTS

Pure random How to build theoretically

Generalized Networks Configuration model How to build in practice





DQ @ 29 of 74

So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model. ^[6]
 Can generalize construction method from ER random networks.
 Assign each node a weight w from some distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

🚳 But we'll be more interested in

 Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





- So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model. ^[6]
 Can generalize construction method from ER random networks.
 Assign each node a weight w from some
 - distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

🚳 But we'll be more interested in

- 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
- 2. Examining mechanisms that lead to networks with certain degree distributions.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





うへへ 29 of 74

Coming up:

Example realizations of random networks with power law degree distributions:

 $P_k \propto k^{-\gamma}$ for $k \ge 1$. Set $P_0 = 0$ (no isolated nodes). Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 30 of 74

Coming up:

Example realizations of random networks with power law degree distributions:

 $\implies N = 1000.$

Set $P_0 = 0$ (no isolated nodes). Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 30 of 74

Coming up:

Example realizations of random networks with power law degree distributions:

- \$ N = 1000.

Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component.

Apart from degree distribution, wiring is random

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Coming up:

Example realizations of random networks with power law degree distributions:

 $\implies N = 1000.$

4

$$P_k \propto k^{-\gamma}$$
 for $k \ge 1$.

 $rac{2}{3}$ Set $P_0 = 0$ (no isolated nodes).

Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component.

Apart from degree distribution, wiring is random

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Coming up:

Example realizations of random networks with power law degree distributions:

- \$ N = 1000.
- ${}_{k} {}_{k} \propto k^{-\gamma}$ for $k \geq 1$.
- $rac{2}{3}$ Set $P_0 = 0$ (no isolated nodes).
- & Vary exponent γ between 2.10 and 2.91.

Again, look at full network plus the largest component. Apart from degree distribution, wiring is rai

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 30 of 74

Coming up:

Example realizations of random networks with power law degree distributions:

- $\implies N = 1000.$
- ${}_{k} {}_{k} \propto k^{-\gamma}$ for $k \geq 1$.
- $rac{2}{3}$ Set $P_0 = 0$ (no isolated nodes).
- & Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.

Apart from degree distribution, wiring is random

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Coming up:

Example realizations of random networks with power law degree distributions:

- $\implies N = 1000.$
- ${\clubsuit} P_k \propto k^{-\gamma}$ for $k \ge 1$.
- $rac{2}{3}$ Set $P_0 = 0$ (no isolated nodes).
- & Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

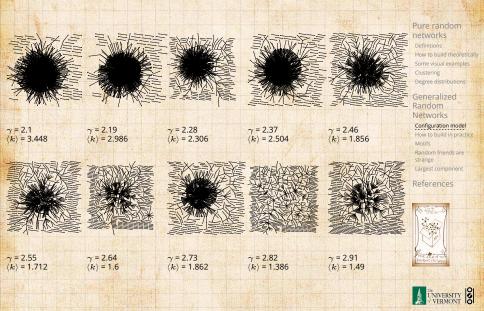
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component



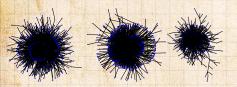


Random networks: examples for N=1000

COcoNuTS



Random networks: largest components





 $\gamma = 2.19$ (k) = 2.986 $\gamma = 2.28$ (k) = 2.306



 $\gamma = 2.46$ (k) = 1.856





COcoNuTS

How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

References





Da @ 32 of 74



 $\gamma = 2.55$ (k) = 1.712





 $\gamma = 2.64$ $\langle k \rangle = 1.6$

 $\gamma = 2.73$ (k) = 1.862 $\gamma = 2.82$ (k) = 1.386 $\gamma = 2.91$ (k) = 1.49

Outline

COCONUTS

Pure random How to build theoretically Clustering Degree distributions

Generalized Random Networks How to build in practice Random friends are

References





networks



Generalized Random Networks

How to build in practice

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Generalized random networks:

Arbitrary degree distribution P_k . Create (unconnected) nodes with degrees sampled from P_k . Wire nodes together randomly.

randomness.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Generalized random networks:

& Arbitrary degree distribution P_k .

Create (unconnected) nodes with degrees sampled from P_k . Wire nodes together randomly. Create ensemble to test deviations from randomness.

Generalized random networks:

 \clubsuit Arbitrary degree distribution P_k .

Solution Create (unconnected) nodes with degrees sampled from P_k .

Wire nodes together randomly. Create ensemble to test deviations from randomness. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

References





Dac 34 of 74

Generalized random networks:

- & Arbitrary degree distribution P_k .
- Solution Create (unconnected) nodes with degrees sampled from P_k .
- 🚳 Wire nodes together randomly.

Create ensemble to test deviations from randomness.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 34 of 74

Generalized random networks:

- \clubsuit Arbitrary degree distribution P_k .
- Solution Create (unconnected) nodes with degrees sampled from P_k .
- 🚳 Wire nodes together randomly.
- Create ensemble to test deviations from randomness.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

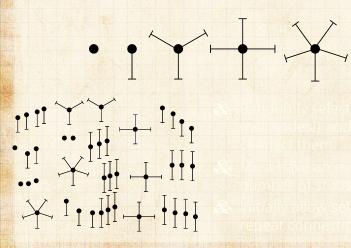




200 34 of 74

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References

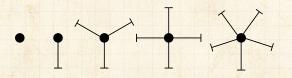


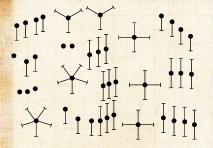
VIVERSITY S

うへで 35 of 74

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stut (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- an connections.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

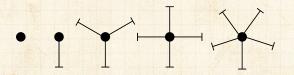


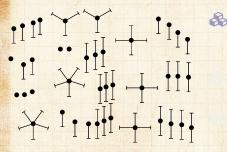


200 35 of 74

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them.

Must have an even number of stubs. Initially allow self- ar connections. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

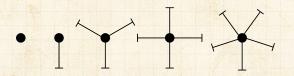


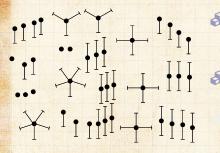


200 35 of 74

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs.

COcoNuTS

Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

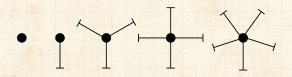


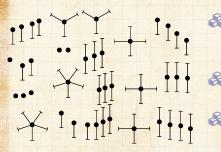


Building random networks: Stubs

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- and repeat connections. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



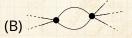


Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful: we can't change the degree of a node, so we can't simply move links around. Simplest solution: randomly rewire two edges time. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

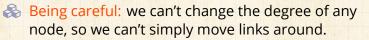




Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



(B)

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



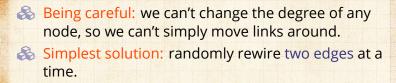


20 36 of 74

Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



(R)

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





20 36 of 74

Randomly choose two edges. (Or choose problem edge and a random edge) COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

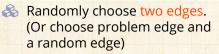
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Da @ 37 of 74



Check to make sure edges are disjoint. Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

COCONUTS

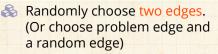
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





うへで 37 of 74



Check to make sure edges are 3 disjoint.



Rewire one end of each edge.

Pure random networks How to build theoretically Clustering Degree distributions

COCONUTS

Generalized Networks How to build in practice Random friends are

References





DQ @ 37 of 74

e1

Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

lacktrian Rewire one end of each edge.

Node degrees do not change.

Works if e_1 is a self-loop or repeated edge. Same as finding on/off/on/of Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





うへで 37 of 74

e1

Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges are disjoint.

🗞 Rewire one end of each edge.

- Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.

Same as finding on/off/on/off 4-cycles, and rotating them. Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

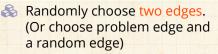




Dac 37 of 74

e2

e1



Check to make sure edges are disjoint.

- Rewire one end of each edge.
 - Node degrees do not change.
 - Works if e_1 is a self-loop or repeated edge.
 - Same as finding on/off/on/off 4-cycles. and rotating them.

Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





20 37 of 74

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

References





DQ @ 38 of 74

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Rule of thumb: # Rewirings $\simeq 10 \times #$ edge

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 38 of 74

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Solution Relation Re

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

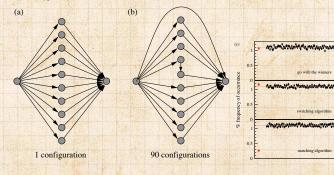




990 39 of 74

Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.
 Example from Milo et al. (2003)^[4]:



COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 39 of 74



 \mathbb{R} What if we have P_k instead of N_k ?



Pure random networks How to build theoretically Clustering Degree distributions

Generalized Random Networks How to build in practice Random friends are

References







 \mathbb{R} What if we have P_k instead of N_k ? Must now create nodes before start of the construction algorithm.

COCONUTS

Pure random networks How to build theoretically Clustering Degree distributions

Generalized Networks How to build in practice Random friends are

References





2 a a 40 of 74

What if we have P_k instead of N_k?
 Must now create nodes before start of the construction algorithm.
 Construct N nodes by sampling from degree

Solution P_k . Generate N nodes by sampling from degree distribution P_k .

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





20 A 40 of 74

What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.

Easy to do exactly numerically since k is discrete.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

References





うへへ 40 of 74

What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.
Easy to do exactly numerically since k is discrete.
Note: not all P_k will always give nodes that can be wired together.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





うへへ 40 of 74

Outline

COCONUTS

Pure random networks How to build theoretically Clustering Degree distributions

Generalized Networks How to build in practice

Random friends are

References





Random

Motifs



DQ @ 41 of 74

Generalized Random Networks

Motifs

Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange Largest component

References





DQC 42 of 74

- Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

COcoNuTS

Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





20 A 42 of 74

- Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.



COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 42 of 74

- Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
 - Directed network with 577 interactions (edges) and 424 operons (nodes).



Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice <u>Motifs</u> Random friends are

Largest component

References





Dac 42 of 74

- Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
 - Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice <u>Motifs</u> Random friends are strange

Largest component

References





うへへ 42 of 74

- Idea of motifs^[7] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

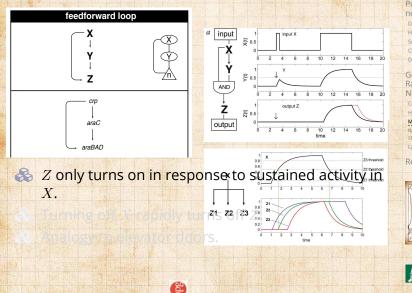
References





うへへ 42 of 74





Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice <u>Motifs</u> Random friends are strange Largest component

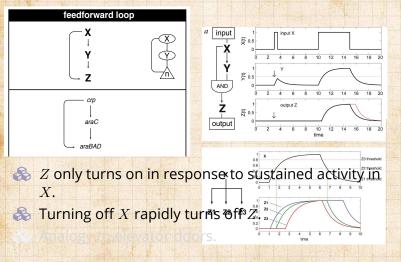
References



UNIVERSITY S

200 43 of 74





Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice <u>Motifs</u> Random friends are strange Largest component

References

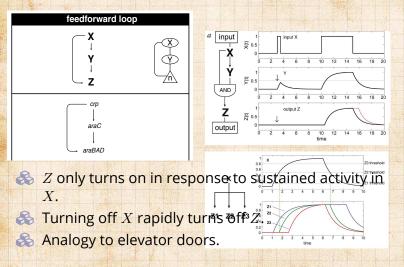


VERMONT 8

DQC 43 of 74



Pure random



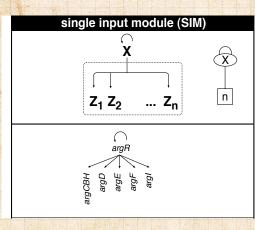
networks Definitions How to build theoretically Some visual examples Clustering Degree distributions Generalized

Random Networks Configuration model How to build in practice <u>Motifs</u> Random friends are strange Largest component

References



VERMONT



🚳 Master switch.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

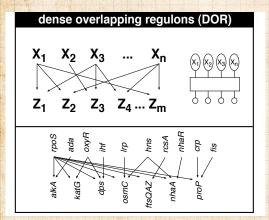
Random friends are strange Largest component

References





DQC 44 of 74



COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





DQ @ 45 of 74

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
 For more, see work carried out by Wiggins *et al.* at Columbia.



Outline

COCONUTS

Pure random networks How to build theoretically Clustering Degree distributions

Generalized Random Networks How to build in practice Motifs







DQ @ 47 of 74

Generalized Random Networks

Random friends are strange

The edge-degree distribution:

The degree distribution P_k is fundamental for our description of many complex networks

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





20 A 48 of 74

The degree distribution P_k is fundamental for our description of many complex networks

Solution Again: P_k is the degree of randomly chosen node.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQC 48 of 74

- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





うへへ 48 of 74

- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Solution Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.



Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Befine Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

 $Q_k \propto k P_k$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





Dac 48 of 74

- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

Normalized form:

$$Q_{k} = \frac{kP_{k}}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_{k}}{\langle k \rangle}$$

Big deal: Rich-get-richer mechanism is built into this

selection process.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 48 of 74



Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

> Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16,$

> Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

 $R_0 = 3/16 R_1 = 4/10$ $R_2 = 3/16, R_5 = 6/1$

COCONUTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References



VERMONT

Dac 49 of 74



Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

Robability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16.$

> Probability of finding # outgoing edges = & after randomly selecting an edge and then randomly choosing one direction to travel:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange

References



UNIVERSITY S

200 49 of 74



Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16, Q_2 = 4/16,$ $Q_3 = 3/16, Q_6 = 6/16.$

> Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$\begin{split} R_0 &= 3/16 \; R_1 = 4/16, \\ R_2 &= 3/16, \; R_5 = 6/16. \end{split}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange

References



VERMONT 8

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





na ~ 50 of 74

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends. COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





20 0 50 of 74

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

3

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





20 0 50 of 74

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

2

 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

3

 R_k = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Solution Equivalent to friend having degree k + 1.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

3

 R_k = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k \right\rangle_R = \sum_{k=0}^\infty k R_k$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQC 51 of 74

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 51 of 74

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left< k \right>_R = \sum_{k=0}^\infty k R_k = \sum_{k=0}^\infty k \frac{(k+1) P_{k+1}}{\left< k \right>}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^\infty k(k+1) P_{k+1}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 51 of 74

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

$$=\frac{1}{\langle k\rangle}\sum_{k=1}^{\infty}k(k+1)P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad \text{(using j = k+1)}$$

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 51 of 74

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

$$=rac{1}{\langle k
angle}\sum_{j=0}^{\infty}(j^2-j)P_j$$
 (using j = k+1)

$$=rac{1}{\langle k
angle}\left(\langle k^2
angle-\langle k
angle
ight)$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 51 of 74

8

Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





na ~ 52 of 74

Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





nac 52 of 74

Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



to

$$\left< k \right>_R = rac{1}{\left< k \right>} \left(\left< k \right>^2 + \left< k \right> - \left< k \right> \right)$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Note: our result, $\langle k \rangle_{R} = \frac{1}{\langle k \rangle} (\langle k^{2} \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



-Sa

A Therefore:

$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

COCONUTS

Pure random

Generalized Networks How to build in practice

Random friends are strange

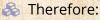




Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



$$\left\langle k \right\rangle_{R} = rac{1}{\left\langle k \right\rangle} \left(\left\langle k \right\rangle^{2} + \left\langle k \right\rangle - \left\langle k \right\rangle \right) = \left\langle k \right\rangle$$

Again, neatness of results is a special property of the Poisson distribution.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

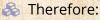




Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$



$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

 Again, neatness of results is a special property of the Poisson distribution.
 So friends on average have (l) other friends and

So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 52 of 74

The edge-degree distribution: In fact, R_k is rather special for pure random networks ...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References

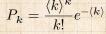




na ~ 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

into



 $R_{k}=\frac{(k+1)P_{k+1}}{\langle k\rangle}$

COcoNuTS

Pure random in networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





nac 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_{k} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_k = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle} e^{-\langle k \rangle} = \frac{(k+1)}{(k+1)k!} e^{-\langle k \rangle} e^{-\langle k$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_k = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle} e^{-\langle$$

$$=\frac{\langle k\rangle^k}{k!}e^{-\langle k\rangle}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





na 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_{k} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle}$$

$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





na ~ 53 of 74

In fact, R_k is rather special for pure random networks ...
 Substituting

 $P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_{k} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle} e^{-$$

$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

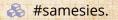
Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References







200 53 of 74

Two reasons why this matters

Reason #1:

Average # friends of friends per node is

Key: Average depends on the 1st and 2nd moments P_k and not just the 1st moment. Three peculiarities:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 54 of 74

Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

Key: Average depends on the 1st and 2nd moments P_k and not just the 1st moment. Three peculiarities:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$$

Key: Average depends on the 1st and 2nd P_k and not just the 1st moment. Three peculiarities:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

Key: Average depends on the 1st and 2nd m P_k and not just the 1st moment. Three peculiarities:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right) = \langle k^2 \rangle - \langle k \rangle.$$

Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

Pure random

COCONUTS

networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 54 of 74

Reason #1:

Average # friends of friends per node is

$$\langle k_2
angle = \langle k
angle imes \langle k
angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
angle - \langle k
angle
ight) = \langle k^2
angle - \langle k
angle.$$

Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

Three peculiarities:

1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle - 1)$ but it's actually $\langle k(k-1) \rangle$.

Your friends really are different from you... See also: class size paradoxes (nod to: Gelm

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





Reason #1:

Average # friends of friends per node is

$$\langle k_2
angle = \langle k
angle imes \langle k
angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
angle - \langle k
angle
ight) = \langle k^2
angle - \langle k
angle.$$

Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- 2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big.

(e.g., in the case of a power-law distribution Your friends really are different from you...

See also: class size paradoxes (nod to: Gelman)

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





Dac 54 of 74

Reason #1:

Average # friends of friends per node is

$$\langle k_2
angle = \langle k
angle imes \langle k
angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
angle - \langle k
angle
ight) = \langle k^2
angle - \langle k
angle.$$

Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- 2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big. (e.g., in the case of a power-law distribution)

friends really are different from you...

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





Reason #1:

Average # friends of friends per node is

$$\langle k_2
angle = \langle k
angle imes \langle k
angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
angle - \langle k
angle
ight) = \langle k^2
angle - \langle k
angle.$$

Key: Average depends on the 1st and 2nd moments of P_{k} and not just the 1st moment.

Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- 2. If P_k has a large second moment, then $\langle k_2 \rangle$ will be big. (e.g., in the case of a power-law distribution)
- 3. Your friends really are different from you...^[3, 5]

COCONUTS

Pure random networks

Generalized Networks How to build in practice

Random friends are strange





Reason #1:

Average # friends of friends per node is

$$\langle k_2
angle = \langle k
angle imes \langle k
angle_R = \langle k
angle rac{1}{\langle k
angle} \left(\langle k^2
angle - \langle k
angle
ight) = \langle k^2
angle - \langle k
angle.$$

Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

Three peculiarities:

- 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
- If P_k has a large second moment, then ⟨k₂⟩ will be big. (e.g., in the case of a power-law distribution)
 Your friends really are different from you...^[3, 5]
- 4. See also: class size paradoxes (nod to: Gelman)

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 54 of 74

More on peculiarity #3:

- \mathfrak{A} A node's average # of friends: $\langle k \rangle$
 - Friend's average # of friends: 4

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





うへで 55 of 74

More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as i friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
 - $\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as i friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





うへで 55 of 74

More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2}$$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as i friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right)$$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as if friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge$$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as it friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References

 $|k\rangle$





More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
 - $\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge \langle k \rangle$

So only if everyone has the same degree
(variance=
$$\sigma^2 = 0$$
) can a node be the same as its
friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 55 of 74

More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
 - $\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right) \ge \langle k \rangle$
- So only if everyone has the same degree (variance= $\sigma^2 = 0$) can a node be the same as its friends.
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange







Your friends really are monsters #winners:¹ Go on the met Friends have more coauthors citations, and publications. Other homfic studies, your connections on Twitter have more followers than you, your se partners more partners than you, ... The hope: Maybe they have more enemies an diseases too.

¹Some press here C [MIT Tech Review].

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References







Your friends really are monsters #winners:¹
 Go on, hurt me: Friends have more coauthors, citations, and publications.

Other homitic studies: your connections on Twitter have more followers than you, your sexua partners more partners than you, ... The hope. Maybe they have more enemies and diseases too.

¹Some press here C [MIT Tech Review].

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References







Your friends really are monsters #winners:¹

- Go on, hurt me: Friends have more coauthors, citations, and publications.
- Other horrific studies: your connections on Twitter have more followers than you, your sexual partners more partners than you, ...

the hope. Maybe they have more enemies and diseases too.

¹Some press here C [MIT Tech Review].



Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References







Your friends really are monsters #winners:¹

- Go on, hurt me: Friends have more coauthors, citations, and publications.
- Other horrific studies: your connections on Twitter have more followers than you, your sexual partners more partners than you, ...
- The hope: Maybe they have more enemies and diseases too.

¹Some press here C [MIT Tech Review].

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
 - e.g., we'd like to know what's the size of the larg component within a network. As $N \rightarrow \infty$, does our network have a giant component?
 - Defo: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
 - Defn: Giant component = component that comprises a non-zero fraction of a network as
 - Note: Component = Cluster

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 57 of 74

(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.

- Defor Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component = component that comprises a non-zero fraction of a network as
- Note: Component = Cluster

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





DQ @ 57 of 74

(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- Solution As $N \to \infty$, does our network have a giant component?
 - Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
 - Defn: Giant component = component that comprises a non-zero fraction of a network as
 - Note: Component = Cluster

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





nac 57 of 74

(Big) Reason #2:

- k > k > R is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- Solution As $N \to \infty$, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.

Defn: Giant component = component that comprises a non-zero fraction of a network as $N \rightarrow \infty$.

Note: Component = Cluster

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





(Big) Reason #2:

- k > k > k is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- Solution As $N \to \infty$, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that ∃ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Solution Defn: Giant component = component that comprises a non-zero fraction of a network as $N \rightarrow \infty$.

Note: Component = Cluster

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange Largest component





(Big) Reason #2:

- $\langle k \rangle_{R}$ is key to understanding how well random networks are connected together.
- 🚓 e.g., we'd like to know what's the size of the largest component within a network.
- $R \to \infty$, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that \exists path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component = component that comprises a non-zero fraction of a network as $N \to \infty$.
- 🚳 Note: Component = Cluster

COCONUTS

Pure random

Generalized Networks How to build in practice Random friends are

strange





Outline

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



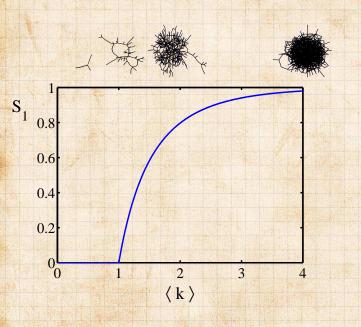
VERMONT

How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice Motifs Random friends are strang Largest component

Giant component



COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





Structure of random networks Giant component:

A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

Equivalently, expect exponential growth in nod number as we move out from a random node. All of this is the same as requiring $\langle k \rangle_R > 1$. Giant-component condition (or percolation condition):

Again, see that the second moment is an essenti part of the story. Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





990 60 of 74

Structure of random networks Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.

Again, see that the second moment is an essentipart of the story. Equivalent statement: $\langle k^2
angle > 2 \langle k
angle$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





990 60 of 74

Structure of random networks Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.

Again, see that the second moment is an essentipart of the story. Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





990 60 of 74

Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.

Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 2$$

Again, see that the second moment is an essentia part of the story. Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.

Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 1$$

Again, see that the second moment is an essential part of the story.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- line contractions and the second seco number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_B > 1$.

🚳 Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 1$$



Again, see that the second moment is an essential part of the story.

Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

COCONUTS

Pure random How to build theoretically

Generalized Networks How to build in practice Random friends are Largest component





2 a a 60 of 74

Spreading on Random Networks

For random networks, we know local structure is pure branching.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





990 61 of 74

For random networks, we know local structure is pure branching. 🚳 Successful spreading is 🛛 contingent on single

edges infecting nodes.

COCONUTS

Pure random networks How to build theoretically Clustering

Generalized Networks How to build in practice Random friends are

Largest component





29 C 61 of 74

- For random networks, we know local structure is pure branching.
- Successful spreading is ... contingent on single edges infecting nodes.

Success Failure:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





うへへ 61 of 74

Success

For random networks, we know local structure is pure branching.

Failure:

Successful spreading is ... contingent on single edges infecting nodes.

Focus on binary case with edges and nodes either infected or not.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References

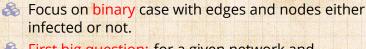




200 61 of 74

- For random networks, we know local structure is pure branching.
- Successful spreading is .. contingent on single edges infecting nodes.

Success Failure:



First big question: for a given network and contagion process, can global spreading from a single seed occur? COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





うへで 61 of 74

We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





うへへ 62 of 74

- We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.
- Befine B_{k1} as the probability that a node of degree k is infected by a single infected edge.



Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component





We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.

 $R = \sum_{i=1}^{\infty}$

2

 kP_k $\langle k \rangle$

prob. of connecting to a degree k node

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





na 62 of 74

We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.

 $R = \sum_{i=1}^{\infty}$

3

 $\frac{kP_k}{\langle k\rangle}$

prob. of connecting to a degree k node

(k - 1)

outgoing infected edges COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





nac 62 of 74

We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.

Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.

 $R = \sum$

3

 $\frac{kP_k}{\langle k \rangle}$

prob. of connecting to a degree k node

(k - 1)

outgoing infected edges

Prob. of infection

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





We need to find: ^[1]
 R = the average # of infected edges that one random infected edge brings about.
 Call R the gain ratio.
 Define B_{k1} as the probability that a node of

degree k is infected by a single infected edge.

 $R = \sum$

3

 $\langle k \rangle$ prob. of connecting to a degree k node

 kP_k

$$+\sum_{k=0}^\infty \frac{\widehat{kP_k}}{\langle k\rangle}$$

(k - 1)

outgoing infected edges

Prob. of infection

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component





🐣 We need to find: [1] **R** = the average # of infected edges that one random infected edge brings about. 🚳 Call **R** the gain ratio. Define B_{k_1} as the probability that a node of

degree k is infected by a single infected edge.

outgoing infected edges

 $\mathbf{R} = \sum_{k=0}^{\infty}$

3

3

prob. of connecting to a degree k node

 $\frac{kP_k}{\langle k \rangle}$

(k - 1)

outgoing

infected edges

 B_{k1}

Prob. of infection

COCONUTS

Pure random How to build theoretically

Generalized Networks How to build in practice Random friends are

Largest component





DQ @ 62 of 74



1

3

🙈 We need to find: [1] R = the average # of infected edges that one random infected edge brings about. 🚳 Call **R** the gain ratio. Define B_{k_1} as the probability that a node of degree k is infected by a single infected edge.

 $\mathbf{R} = \sum_{k=0}^{\infty}$ (k - 1) B_{k1} Prob. of # outgoing prob. of infected infection connecting to edges a degree k node $+\sum_{k=0}^{\infty}\frac{\hat{k}P_k}{\langle k\rangle}$ $(1 - B_{k1})$ # outgoing Prob. of infected no infection edges

COCONUTS

Pure random How to build theoretically

Generalized Networks How to build in practice Random friends are

Largest component





DQ @ 62 of 74

lour global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





990 63 of 74

lour global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

🙈 Case 1–Rampant spreading:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 63 of 74

Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

Solution Case 1–Rampant spreading: If $B_{k1} = 1$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 63 of 74

Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

Solution Case 1–Rampant spreading: If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





lour global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

Solution Case 1–Rampant spreading: If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

Good: This is just our giant component condition again.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

teferences





Case 2—Simple disease-like:

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Case 2—Simple disease-like: If $B_{k1} = \beta < 1$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^\infty \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

\Im A fraction (1- β) of edges do not transmit infection.

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





DQ @ 64 of 74

So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.
 Aka bond percolation C.

Pure random networks Definitions How to build theoretica Some visual examples Clustering

COCONUTS

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





So Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

A fraction (1-β) of edges do not transmit infection.
 Analogous phase transition to giant component case but critical value of (k) is increased.

Aka bond percolation .

Resulting degree distribution \tilde{P}_k :

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Giant component for standard random networks: Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

Determine condition for giant component:

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase of the say $\langle k \rangle = 1$ marks the critical point of the system. COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\left\langle k \right\rangle_R = rac{\left\langle k^2 \right\rangle - \left\langle k \right\rangle}{\left\langle k \right\rangle}$$

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase We say $\langle k \rangle = 1$ marks the critical point of th system.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

4

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase We say $\langle k \rangle = 1$ marks the critical point of th system.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

4

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase We say $\langle k \rangle = 1$ marks the critical point of th system.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$



6

 \leq Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

COCONUTS

Pure random networks Clustering

Generalized Random Networks How to build in practice Random friends are

Largest component





DQ @ 65 of 74

Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Solution
 Solution

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

Largest component





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Solution (k) > 1, standard random networks have a giant component.
 When (k) < 1, all components are finite.
 Fine example of a continuous phase transition C.
 We say (k) = 1 marks the critical point of the system.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

Solution & S

Pure random networks Definitions How to build theoretica Some visual examples Clustering

COCONUTS

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component





$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$. How about $P_k \neq \delta_{kk_0}$?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 66 of 74

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{+3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_{R}$. How about $P_k \neq \delta_{kk_0}$?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





200 66 of 74

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty}$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{+3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$. How about $P_k = \delta_{kk_0}$?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma} \Big|_{x=1}^{\infty} = \infty$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$. How about $P_k \neq \delta_{kk_0}$?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 66 of 74

Random networks with skewed P_k : e.g. if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma} \Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$. How about $P_k = \delta_{kk_0}$?

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 66 of 74

Random networks with skewed P_k : & e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma} \Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

The Action of the Section



So giant component always exists for these kinds of networks.

Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$. Random networks with skewed P_k : & e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



Dec 66 of 74

6

Solution Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

Random networks with skewed P_k : & e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d} x$$

$$\propto x^{3-\gamma}\Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



VERMONT

Solution Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

B How about
$$P_k = \delta_{kk_0}$$

6

- \Im Define S_1 as the size of the largest component.
 - Consider an infinite ER random network with average degree $\langle k \rangle$.
 - Let's find S_1 with a back-of-the-envelope argumen Define δ as the probability that a randomly chosen node does not belong to the largest component. Simple connection: $\delta = 1 - S_1$.

Dirty trick: If a randomly chosen node is not part of th largest component, then none of its neighbors are.

Substitute in Poisson distribution

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



VERMONT

- Define S_1 as the size of the largest component.
- Consider an infinite ER random network with average degree $\langle k \rangle$.

Define δ as the probability that a randomly chosen node does not belong to the largest component. Simple connection: $\delta = 1 - S_1$.

Dirty trick. If a randomly chosen node is not part of th largest component, then none of its neighbors are.

Substitute in Poisson distribution

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



VERMONT

- S Define S_1 as the size of the largest component.
- Source Consider an infinite ER random network with average degree $\langle k \rangle$.
- & Let's find S_1 with a back-of-the-envelope argument.
 - Define δ as the probability that a randomly choser node does not belong to the largest component. Simple connection: $\delta = 1 - S_1$.

Dirty trick. If a randomly chosen node is not part of the largest component, then none of its neighbors are.

Substitute in Poisson distribution

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



VERMONT

- \mathbb{S} Define S_1 as the size of the largest component.
- Solution Consider an infinite ER random network with average degree $\langle k \rangle$.
- \mathfrak{B} Let's find S_1 with a back-of-the-envelope argument.
- Befine δ as the probability that a randomly chosen node does not belong to the largest component.

Simple connection: $\delta = 1 + S_1$.

Dirty trick. If a randomly chosen node is not part of the largest component, then none of its neighbors are.

Substitute in Poisson distribution

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



VERMONT

- \mathbb{S} Define S_1 as the size of the largest component.
- Solution Consider an infinite ER random network with average degree $\langle k \rangle$.
- \mathfrak{B} Let's find S_1 with a back-of-the-envelope argument.
- Befine δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 S_1$.

Dirty trick: If a randomly chosen node is not part of th largest component, then none of its neighbors are.

 $\kappa = 0$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





- \mathbb{S} Define S_1 as the size of the largest component.
- Solution Consider an infinite ER random network with average degree $\langle k \rangle$.
- \mathfrak{L} Let's find S_1 with a back-of-the-envelope argument.
- Befine δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 S_1$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

cargest component

References





So

And how big is the largest component?

- \Im Define S_1 as the size of the largest component.
- Source consider an infinite ER random network with average degree $\langle k \rangle$.
- \mathfrak{B} Let's find S_1 with a back-of-the-envelope argument.
- Befine δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 S_1$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

COcoNuTS

Pure random networks Definitions How to build theoretical Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

cargest component

References



VERMONT 8

And how big is the largest component?

- \Im Define S_1 as the size of the largest component.
- Solution Consider an infinite ER random network with average degree $\langle k \rangle$.
- \mathfrak{Z} Let's find S_1 with a back-of-the-envelope argument.
- Befine δ as the probability that a randomly chosen node does not belong to the largest component.
- Simple connection: $\delta = 1 S_1$.
- Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

💑 So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

Substitute in Poisson distribution...

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

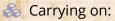
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







${\color{black} \delta} = \sum_{k=0}^{\infty} P_k \delta^k$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

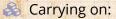
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

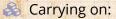
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

COCONUTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

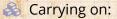
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

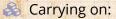
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1-\delta)}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

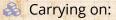
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!}$$

$$=e^{-\langle k
angle}e^{\langle k
angle\delta}=e^{-\langle k
angle(1-\delta)}$$

Solution Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





99 CP 69 of 74

We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

As
$$\langle k \rangle \to 0$$
, $S_1 \to 0$.

6

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





99 CP 69 of 74

We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

$$\begin{split} & \clubsuit \quad \mathsf{As} \ \langle k \rangle \to 0, \, S_1 \to 0. \\ & \clubsuit \quad \mathsf{As} \ \langle k \rangle \to \infty, \, S_1 \to 1. \end{split}$$

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Solution We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

 $\begin{array}{l} & \& \ \ \mathsf{As} \ \langle k \rangle \to 0, \ S_1 \to 0. \\ & \& \ \ \mathsf{As} \ \langle k \rangle \to \infty, \ S_1 \to 1. \\ & \& \ \ \mathsf{Notice that at} \ \langle k \rangle = 1, \ \mathsf{the critical point}, \ S_1 = 0. \end{array}$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Solution We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

 $\begin{array}{l} \underset{k}{\circledast} \quad \text{As } \langle k \rangle \to 0, \, S_1 \to 0. \\ \underset{k}{\circledast} \quad \text{As } \langle k \rangle \to \infty, \, S_1 \to 1. \\ \underset{k}{\circledast} \quad \text{Notice that at } \langle k \rangle = 1, \, \text{the critical point, } S_1 = 0. \\ \underset{k}{\circledast} \quad \text{Only solvable for } S_1 > 0 \text{ when } \langle k \rangle > 1. \end{array}$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Solution We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$. First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}$$

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

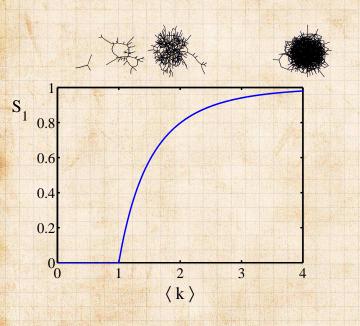
Generalized Random Nétworks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References







COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





200 70 of 74

Turns out we were lucky...



Our dirty trick only works for ER random networks.

COCONUTS

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Networks How to build in practice Motifs Random friends are

Largest component

References





Dac 71 of 74

Turns out we were lucky...

Our dirty trick only works for ER random networks.
 The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.

But we know our friends are different from us. Works for ER random networks because

We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.

We can sort many things out with sensible probabilistic arguments...

More detailed investigations will profit from a s of General ingrunctionology.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

cargest component

References





Turns out we were lucky...

- Our dirty trick only works for ER random networks.
 The problem: We assumed that neighbors have the same probability δ of belonging to the largest
 - the same probability δ of belonging to the la component.
- But we know our friends are different from us...

- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a s of General ing inctionology.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Turns out we were lucky...

- lour dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Solution Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.

We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.

We can sort many things out with sensible probabilistic arguments...

Vore detailed investigations will profit from a s of Generating unctionology.

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Turns out we were lucky...

- lour dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Solution Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
 - We can sort many things out with sensible probabilistic arguments...
 - More detailed investigations will profit from a s

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

Largest component

References





Turns out we were lucky...

- lour dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Solution Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...

More detailed investigations will profit from a spo of General instrumctionology.

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

cargest componen

References





Turns out we were lucky...

- lour dirty trick only works for ER random networks.
- The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Solution Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- Solution We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generatingfunctionology.^[9]

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

cargest componen

References





DQC 71 of 74

References I

[1] P. S. Dodds, K. D. Harris, and J. L. Payne. Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks. Phys. Rev. E, 83:056122, 2011. pdf 2

[2] Y.-H. Eom and H.-H. Jo. Generalized friendship paradox in complex networks: The case of scientific collaboration. Nature Scientific Reports, 4:4603, 2014. pdf

 [3] S. L. Feld.
 Why your friends have more friends than you do. Am. J. of Sociol., 96:1464–1477, 1991. pdf

COcoNuTS

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 72 of 74

References II

[4] R. Milo, N. Kashtan, S. Itzkovitz, M. E. J. Newman, and U. Alon. On the uniform generation of random graphs with prescribed degree sequences, 2003. pdf

[5] M. E. J. Newman. Ego-centered networks and the ripple effect,. Social Networks, 25:83–95, 2003. pdf

[6] M. E. J. Newman. The structure and function of complex networks. <u>SIAM Rev.</u>, 45(2):167–256, 2003. pdf 2

[7] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon. Network motifs in the transcriptional regulation network of *Escherichia coli*. Nature Genetics, 31:64–68, 2002. pdf

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 73 of 74

References III

[8] S. H. Strogatz. Nonlinear Dynamics and Chaos. Addison Wesley, Reading, Massachusetts, 1994.

[9] H. S. Wilf. Generatingfunctionology.

A K Peters, Natick, MA, 3rd edition, 2006. pdf

COcoNuTS

Pure random networks Definitions How to build theoretica Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQ @ 74 of 74