CocoNuTs
Complex Networks
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Everything is connected

Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2018
Assignment 5 • code name: I don't understand how all this happens

Dispersed: Friday, February 23, 2018.
Due: Friday, March 9, by 11:59 pm, 2018.
Last updated: Wednesday, March 21, 2018, 11:59 am
Some useful reminders:
Deliverator: Peter Dodds
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Office hours: 10:05 am to 12:00 pm, Tuesday and Thursday
Course website: http://www.uvm.edu/pdodds/teaching/courses/2018-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

Email submission: PDF only! Please name your file as follows (where the number is to be padded by a 0 if less than 10 and names are all lowercase):
`CSYS303assignment%02d$firstname-$lastname.pdf` as in
`CSYS303assignment06michael-palin.pdf`

Please submit your project's current draft in pdf format via email. Please use this file name format (all lowercase after CSYS):
`CSYS303project-$firstname-$lastname-YYYY-MM-DD.pdf` as in
`CSYS303project-lisa-simpson-1989-12-17.pdf`

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1. For each of our main six networks, compute and present distributions of the shortest path length between all pairs of nodes. Notation: $d_{i,j}$ is the shortest distance between i and j.

   Also compute the average shortest path length, $\langle d \rangle$.

2. Generate ensembles of random networks of the same 'size' as the six networks. Process 1 random network and then scale up as computing power/time/sanity permits. 1000 random networks would be good.

   Size here means having the same number of nodes and the same number of edges.

   As for the real networks, compute the shortest path lengths for these random networks and present frequency distributions.

3. Determine how well/poorly random networks produce the shortest path distributions of real world networks.

   Using whatever tests you like, show how well both the average shortest path length and the full distributions compare between the real network and their random counterparts.

4. (3 + 3 + 3)

   (a) Determine all possible three node motifs for undirected networks and sketch them. (Do not consider motifs for which one or more nodes are disconnected.)

   (b) Do the same as above for directed networks, allowing that a pair of nodes may have two edges traveling opposite ways behind them.

   (c) For our base six networks, use the ensembles of random networks generated in assignment 3, question 2 to test for three node motifs in the manner of [2].

   Note: You should be able to process the four smaller networks, and you should feel free to abandon the larger ones if your computing friend explodes.

5. For a uniformly distributed population, to minimize the average distance between individuals and their nearest facility, we've made a claim that facilities would be placed at the centres of the tiles on a hexagonal lattice (or the vertices of a triangular lattice). Why is this?

6. In two dimensions, the size-density law for distributed source density $D(\vec{x})$ given a sink density $\rho(\vec{x})$ states that $D \propto \rho^{2/3}$. We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent ([1]; also see Supply Networks lecture notes).

   Repeat this argument for the d-dimensional case and find the general form of the exponent μ in $D \propto \rho^{\mu}$.

7. Following Um et al.'s approach [3], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the cost function:

   $$c_i = n_i \langle r_i \rangle^{\beta} \text{ with } 0 \leq \beta \leq 1,$$

   where, respectively, n_i and $\langle r_i \rangle$ are population and the average 'source to sink' distance for the population of the ith Voronoi cell (which surrounds the ith facility).

   Note that $\beta = 0$ corresponds to purely commercial facilities, and $\beta = 1$ to strongly social ones.

References

[1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. *Phys. Rev. E*, 74:016117, 2006. pdf

[2] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon. Network motifs in the transcriptional regulation network of *Escherichia coli*. *Nature Genetics*, 31:64–68, 2002. pdf

[3] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. *Proc. Natl. Acad. Sci.*, 106:14236–14240, 2009. pdf