CocoNuTs
Complex Networks
@networksvox
Everything is connected

Complex Networks, CSYS/MATH 303
University of Vermont, Spring 2018
Assignment 2 • code name: All the bacon and eggs

Dispersed: Thursday, January 18, 2018.
Due: Friday, February 2, by 11:59 pm, 2018.
Some useful reminders:
Deliverator: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 10:05 am to 12:00 pm, Tuesday and Thursday
Course website: http://www.uvm.edu/pdodds/teaching/courses/2018-01UVM-303

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

Email submission: PDF only! Please name your file as follows (where the number is to be padded by a 0 if less than 10 and names are all lowercase):
`CSYS303assignment%02d$firstname-$lastname.pdf` as in
`CSYS303assignment06michael-palin.pdf`

Please submit your project's current draft in pdf format via email. Please use this file name format (all lowercase after CSYS):
`CSYS303project-$firstname-$lastname-YYYY-MM-DD.pdf` as in
`CSYS303project-lisa-simpson-1989-12-17.pdf`

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1. Tokunaga's law is statistical but we can consider a rigid version. Take $T_1 = 2$ and $R_T = 2$ and draw an example network of order $\Omega = 4$ with these parameters.

   Please take some effort to make your network look somewhat like a river network.

2. Show $R_s = R_\ell$. In other words show that Horton's law of stream segments matches that of main stream lengths, and do this by showing they imply each other.

3. Tokunaga's law implies Horton's laws:

   In lectures, we established the following:

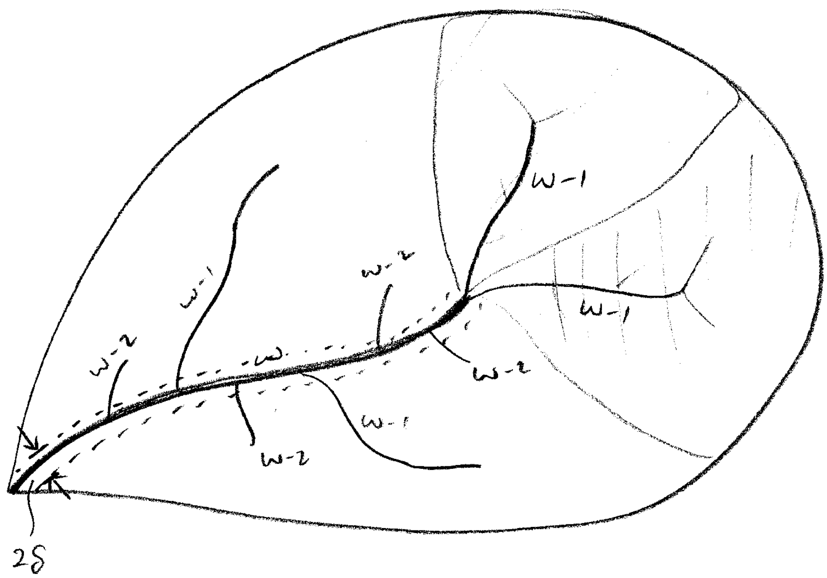
$$n_\omega = \underbrace{2\, n_{\omega+1}}_{\text{generation}} + \sum_{\omega'=\omega+1}^{\Omega} \underbrace{T_{\omega'-\omega}\, n_{\omega'}}_{\text{absorption}}$$

From here, derive Horton's law for stream numbers: $n_\omega/n_{\omega+1} = R_n$, where $R_n > 1$ and is independent of ω, and find R_n in terms of Tokunaga's two parameters T_1 and R_T.

4. Show $R_n = R_a$ by using Tokunaga's law to find the average area of an order ω basin, $\bar{a}_\omega$, in terms of the average area of basins of order 1 to $\omega - 1$.

   (In lectures, we use Horton's laws to roughly demonstrate this result.)

   Here's the set up:



Using the Tokunaga picture, we see a basin of order ω can be broken down into non-overlapping sub-basins.

Connect $\bar{a}_\omega$ to the average areas of basins of lower orders as follows:

$$\bar{a}_\omega = 2\bar{a}_{\omega-1} + \sum_{\omega'=1}^{\omega-1} T_{\omega,\omega'}\bar{a}_{\omega'} + 2\delta\bar{s}_\omega.$$

The first term on the right hand side corresponds to the two 'generating' streams of order $\omega - 1$. The second term (the sum) accounts for side streams entering the sole order ω stream segment in the basin. And the last term gives the contribution of 'overland flow,' i.e., flow that does not arrive in the main stream segment through a stream. The length scale δ is the typical distance from stream to ridge.