Social Contagion Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2017

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont











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References

Outline

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Background Granovetter's model Network version Final size Spreading success Groups

DAVID

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From the Atlantic 🗹

From the Atlantic 🗹

DAVID

MICHAEL

DAVID

DAVID

1960: **DAVID**

1960: MARY

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These slides are also brought to you by:

Special Guest Executive Producer: Pratchett

On Instagram at pratchett_the_cat

Things that spread well:

buzzfeed.com **□**:



















Dangerously self aware: 11 Elements that make a perfect viral video.

+ News ...

LOL + cute + fail + wtf:

Dopsie!



Please try reloading this page. If the problem persists let us know.

The whole lolcats thing:



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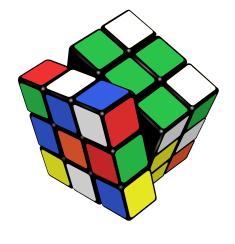


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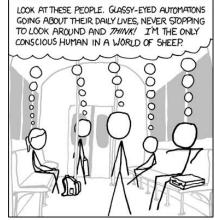
Some things really stick:



wtf + geeky + omg:



Why social contagion works so well:



http://xkcd.com/610/☑

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Market much?

Advertisement enjoyed during "Herstory of Dance" , Community S4E08, April 2013.

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Examples abound

- 🚓 fashion
- 🚓 striking
- residential segregation [22]
- iPhones and iThings

countries, ...

SIR and SIRS type contagion possible

Classes of behavior versus specific behavior :

dieting, horror movies, getting married, invading

obesity
 obesity

- 🙈 Harry Potter
- voting
- 🚓 gossip
- 🙈 Rubik's cube 💗
- areligious beliefs
- & school shootings
- leaving lectures

Framingham heart study:

Evolving network stories (Christakis and Fowler):

- Also: happiness
 ☐ [11], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (2)

Controversy:

- Are your friends making you fat? (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious —Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).





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Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [12] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [19]





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Mixed messages: Please copy, but also, don't copy ...

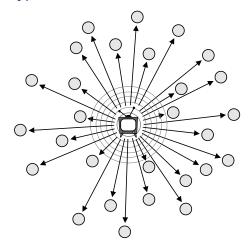
- Cindy Harrell appeared
 in the (terrifying) music video for Ray Parker Jr.'s Ghostbusters
 .
- In Stranger Things 2 ☑, Steve Harrington reveals his Fabergé secret ☑.





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The hypodermic model of influence



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Why do things spread socially?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers:
 homo narrativus ♂.
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

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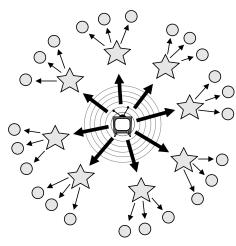
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The two step model of influence [19]



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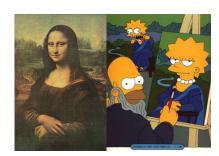
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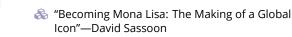
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The Mona Lisa



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Not the world's greatest painting from the start...

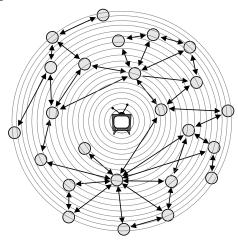
See Escalation through theft, vandalism, parody, ...





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The general model of influence: the Social Wild



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Tattooed Guy' Was Pivotal in Armstrong Case [nytimes] 2



"... Leogrande's doping sparked a series of events ..."

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The completely unpredicted fall of Eastern Europe:



Timunr Kuran: [20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"

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Drafting success in the NFL: ☑

Top Players by Round, 1995-2012







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The dismal predictive powers of editors...





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Messing with social connections

Ads based on message content (e.g., Google and email)

- Harnessing of BzzAgents to directly market through social ties.
- Generally: BzzAgents did not reveal their BzzAgent status and did not want to be paid.
- NYT, 2004-12-05: "The Hidden (in Plain Sight)
 Persuaders" ☑
- $\ \, \& \ \,$ One of Facebook's early advertising attempts: Beacon $\ensuremath{\mathbb{Z}}$
- All of Facebook's advertising attempts.
- & Seriously, Facebook. What could go wrong?





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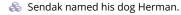
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From a 2013 Believer Magazine interview with Maurice Sendak ::

BLVR: Did the success of Where the Wild Things Are ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention Moby-Dick. They're all going to talk about my first book, about ****ing maidens in Tahiti." He was right. No mention of Moby-Dick then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came Moby-Dick and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. Moby-Dick didn't get famous until 1930.



The essential Colbert interview: Pt. 1
 and Pt. 2
 .

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WHERE THE WILD THINGS ARE



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Getting others to do things for you

A very good book: 'Influence' [8] by Robert Cialdini 🗗

Six modes of influence:

- 1. Reciprocation: *The Old Give and Take... and Take*; e.g., Free samples, Hare Krishnas.
- 2. Commitment and Consistency: Hobgoblins of the Mind; e.g., Hazing.
- 3. Social Proof: *Truths Are Us*; e.g., Jonestown ☑, Kitty Genovese ☑ (contested).
- 4. Liking: *The Friendly Thief*; e.g., Separation into groups is enough to cause problems.
- Authority: Directed Deference;
 e.g., Milgram's obedience to authority experiment.
- 6. Scarcity: *The Rule of the Few*; e.g., Prohibition.





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- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)



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Some possible origins of thresholds:

- Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [
- & Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
 - Externalities = Effects on others not directly involved in a transaction
 - Examples: telephones, fax machine, Facebook, operating systems
 - An individual's utility increases with the adoption level among peers and the population in general



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Some important models:

- Tipping models—Schelling (1971) [22, 23, 24]
 - Simulation on checker boards
 - ldea of thresholds
 - Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)
 - Explore the Netlogo online implementation [29]
- Threshold models—Granovetter (1978) [15]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
 - Social learning theory, Informational cascades,...

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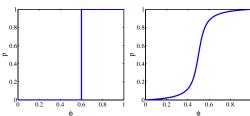
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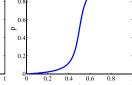
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Threshold models—response functions





- Example threshold influence response functions:
- ϕ = fraction of contacts 'on' (e.g., rioting)

deterministic and stochastic

Two states: S and I.

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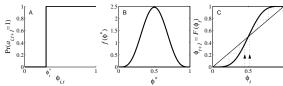
Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).

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Social Contagion Models Action based on perceived behavior of others:



- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model





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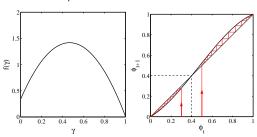




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Threshold models

Another example of critical mass model:



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Threshold models—Nutshell

Implications for collective action theory:

- 1. Collective uniformity ⇒ individual uniformity
- 2. Small individual changes ⇒ large global changes
- 3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
- 4. System stories live in left null space of our stories—we can't even see them.
- 5. But we happily impose simplistic, individual-centric stories—we can't help ourselves .

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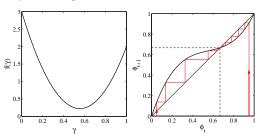
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Threshold models

Threshold models

Chaotic behavior possible [17, 16, 9, 18]

Example of single stable state model:



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Many years after Granovetter and Soong's work:

- "A simple model of global cascades on random networks"
 - D. J. Watts. Proc. Natl. Acad. Sci., 2002 [26]
 - Mean field model → network model
 - Individuals now have a limited view of the world

We'll also explore:

- "Seed size strongly affects cascades on random networks" [14] Gleeson and Cahalane, Phys. Rev. E, 2007.
- 🖚 "Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks" [10] Dodds, Harris, and Payne, Phys. Rev. E, 2011
- "Influentials, Networks, and Public Opinion Formation" [27] Watts and Dodds, J. Cons. Res., 2007.
- "Threshold models of Social Influence" [28]

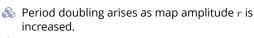
Thre Watts and Dodds The Oxford Handbook of Analytical



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All nodes have threshold $\phi = 0.2$.



Synchronous update assumption is crucial





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Threshold model on a network

- Interactions between individuals now represented by a network.
- Network is sparse.
- \mathbb{A} Individual i has k_i contacts.
- Influence on each link is reciprocal and of unit weight.
- & Each individual i has a fixed threshold ϕ_i .
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individuals remain active when switched (no recovery = SI model).

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Snowballing

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.

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Snowballing

First study random networks:

- & Start with N nodes with a degree distribution P_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

Dirig

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \ge \phi_i$$

- $\mbox{\&}$ Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [26]
- Cluster of vulnerables = critical mass
- Network story: 1 node \rightarrow critical mass \rightarrow everyone.





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The Cascade Condition: 1. If one individual is init

1

- If one individual is initially activated, what is the probability that an activation will spread over a network?
- 2. What features of a network determine whether a cascade will occur or not?





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Example random network structure:

 $\begin{array}{ll} & \Omega_{\rm crit} = \Omega_{\rm vuln} = \\ & {\rm critical\ mass} = \\ & {\rm global} \\ & {\rm vulnerable} \\ & {\rm component} \end{array}$

- $\Omega_{\rm trig}$ = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

 $\Omega_{\mathrm{crit}} \subset \Omega_{\mathrm{trig}}; \ \Omega_{\mathrm{crit}} \subset \Omega_{\mathrm{final}}; \ \mathrm{and} \ \Omega_{\mathrm{trig}}, \Omega_{\mathrm{final}} \subset \Omega.$

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Cascade condition

Back to following a link:

- $\ \, \& \ \,$ A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k.$
- $\ensuremath{\mathfrak{S}}$ Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} k P_k = \langle k \rangle$$

备 So

 $P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$





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Cascade condition

Next: Vulnerability of linked node

& Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'=0}^{1/k} f(\phi'_*) \mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces k-1 new outgoing active links
- If linked node is not vulnerable, it produces no active links.

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Cascade condition

Two special cases:

 $\mbox{\ensuremath{\&}}$ (1) Simple disease-like spreading succeeds: $\beta_k=\beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

& (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$





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Cascade condition

Putting things together:

Expected number of active edges produced by an active edge:

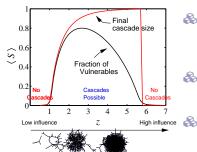
$$\begin{split} R = \left[\sum_{k=1}^{\infty} \underbrace{\frac{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}}} \right. \\ + & \underbrace{\frac{\mathbf{0} \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}} \right] \\ = & \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} \end{split}$$





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Cascades on random networks



Cascades occur only if size of max vulnerable cluster > 0.

System may be 'robust-yetfragile'.







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Cascade condition

So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

 $\beta_k = \text{probability a degree } k \text{ node is vulnerable.}$

 $\Re P_k = \text{probability a node has degree } k.$

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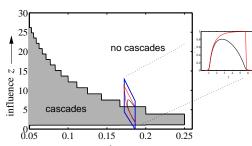
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Cascade window for random networks



 ϕ = uniform individual threshold

- & 'Cascade window' widens as threshold ϕ decreases.
- Lower thresholds enable spreading.

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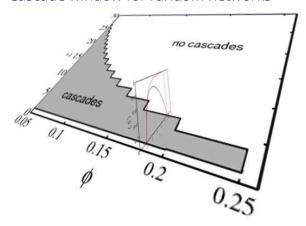
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Cascade window for random networks



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Threshold contagion on random networks

Next: Find expected fractional size of spread.

- Not obvious even for uniform threshold problem.
- & Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem beautifully solved for infinite seed case by Gleeson and Cahalane: "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. [14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [13]

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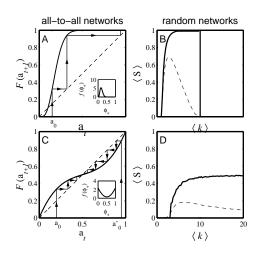
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All-to-all versus random networks



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Determining expected size of spread:

- $\ \,$ Randomly turn on a fraction ϕ_0 of nodes at time t=0
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node *i* to become active at time *t*:
 - t=0: i is one of the seeds (prob = ϕ_0)
- t=1: i was not a seed but enough of i's friends switched on at time t=0 so that i's threshold is now exceeded.
- t=2: enough of i's friends and friends-of-friends switched on at time t=0 so that i's threshold is now exceeded.
- t=n: enough nodes within n hops of i switched on at t=0 and their effects have propagated to reach i.

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Cascade window—summary

For our simple model of a uniform threshold:

- Low \(\lambda k\rangle\): No cascades in poorly connected networks.
 No global clusters of any kind.
- 2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
- 3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists.

Cascades are possible in "Cascade window."

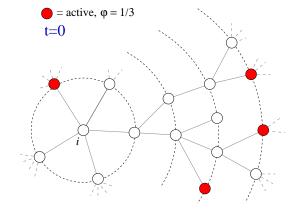
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Expected size of spread



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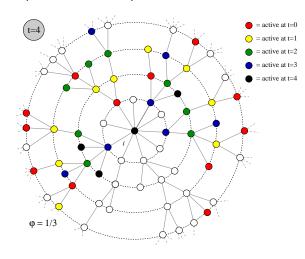
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Expected size of spread



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Expected size of spread

Notation:

 $\phi_{k,t} = \mathbf{Pr}(\mathbf{a} \text{ degree } k \text{ node is active at time } t).$

- Notation: $B_{k,i} = \mathbf{Pr}$ (a degree k node becomes active if j neighbors are active).
- $\mbox{\&}$ Our starting point: $\phi_{k,0} = \phi_0$.
- & $\binom{k}{j}\phi_0^{\ j}(1-\phi_0)^{k-j}$ = **Pr** (j of a degree k node's neighbors were seeded at time t=0).
- \Re Probability a degree k node was a seed at t=0 is ϕ_0 (as above).
- $\ensuremath{\mathfrak{S}}$ Probability a degree k node was not a seed at t=0 is
- Combining everything, we have:

$$\phi_{k,1} = \frac{\phi_0}{\phi_0} + \frac{(1-\phi_0)}{\sum_{i=0}^k {k \choose j}} \phi_0^{\ j} (1-\phi_0)^{k-j} B_{kj}.$$



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Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine **Pr**(node of degree *k* switching on at time *t*).
- Asynchronous updating can be handled too.

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\Re For general t, we need to know the probability an edge coming into a degree k node at time t is active.

- \aleph Notation: call this probability θ_t .
- $\ensuremath{\&}$ We already know $\theta_0 = \phi_0$.
- \mathfrak{S} Story analogous to t = 1 case. For node i:

$$\phi_{i,t+1} = \frac{\phi_0}{0} + \frac{(1-\phi_0)}{\sum_{j=0}^{k_i} {k_j \choose j} \theta_t^{\,j} (1-\theta_t)^{k_i-j} B_{k_i j}}.$$

& Average over all nodes to obtain expression for ϕ_{t+1} :

$$\phi_{t+1} = \frac{\phi_0}{\phi_0} + \frac{(1-\phi_0)}{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^{\ j} (1-\theta_t)^{k-j} B_{kj}}.$$

& So we need to compute θ_t ... massive excitement...





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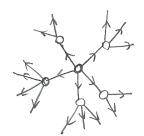


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Expected size of spread

Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.





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Expected size of spread

First connect θ_0 to θ_1 :

$$\theta_1 = \phi_0 +$$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} {k-1 \choose j} \theta_0^{j} (1 - \theta_0)^{k-1-j} B_{kj}$$

- $\frac{kP_k}{lk} = R_k$ = **Pr** (edge connects to a degree k node).
- $\underset{i=0}{\&} \sum_{j=0}^{k-1}$ piece gives **Pr**(degree node k activates) of its neighbors k-1 incoming neighbors are active.
- $\ \ \phi_0$ and $(1-\phi_0)$ terms account for state of node at
- & See this all generalizes to give θ_{t+1} in terms of θ_t ...





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Expected size of spread

Two pieces: edges first, and then nodes

1.
$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1-\phi_0)\underbrace{\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}{k-1\choose j}\theta_t^{\ j}(1-\theta_t)^{k-1-j}B_{kj}}_{\text{social effects}}$$

with
$$\theta_0 = \phi_0$$
.

2.
$$\phi_{t+1} =$$

$$\underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k {k \choose j} \theta_t^{\ j} (1 - \theta_t)^{k-j} B_{kj}}_{\text{social effects}}.$$





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some nodes turn on for free. \mathbb{A} If G has an unstable fixed point at $\theta = 0$, then cascades are also always possible.

 \Re If $G(0; \phi_0) > 0$, spreading must occur because

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Non-vanishing seed case:

Expected size of spread:

In words:

- & Cascade condition is more complicated for $\phi_0 > 0$.
- \Re If G has a stable fixed point at $\theta = 0$, and an unstable fixed point for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- $\ref{eq:continuous}$ Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G.
- A version of a critical mass model again.





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Expected size of spread

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1-\phi_0)\underbrace{\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_t^{\ j}(1-\theta_t)^{k-1-j}B_{kj}}_{\text{social effects}}$$

$$=\underline{G(\theta_t;\phi_0)}$$

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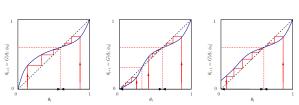
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General fixed point story:



- $\mbox{\&} \mbox{ Given } \theta_0 (=\phi_0) \mbox{, } \theta_\infty \mbox{ will be the nearest stable fixed}$ point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- \mathbb{A} Important: Actual form of G depends on ϕ_0 .
- \mathfrak{S} So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G.





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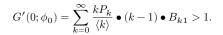
Expected size of spread:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \to 0$.
- $\ensuremath{\mathfrak{S}}$ Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0;\phi_0) = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \ge 1$.

 $\begin{cases} \& \end{cases}$ If $\theta=0$ is a fixed point of G (i.e., $G(0;\phi_0)=0$) then spreading occurs if



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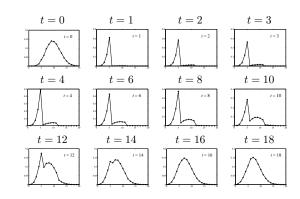
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Early adopters—degree distributions



 $P_{k,t}$ versus k

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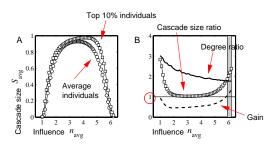
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The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

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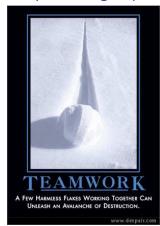
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The power of groups...



despair.com

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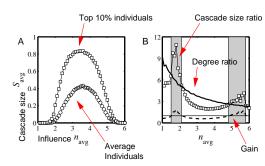
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flakes working together can unleash an avalanche of destruction."

"A few harmless



The multiplier effect:



Skewed influence distribution example.

Extensions



"Threshold Models of Social Influence" Watts and Dodds,

The Oxford Handbook of Analytical Sociology, , 475–497, 2009. [28]

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure

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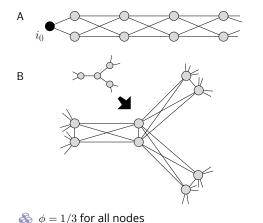
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Special subnetworks can act as triggers



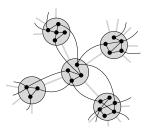
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Group structure—Ramified random networks



p = intergroup connection probability q = intragroup connection probability.

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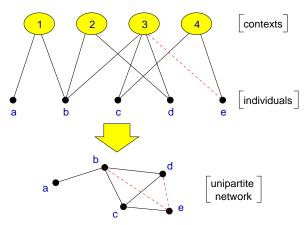
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Bipartite networks



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Generalized affiliation model networks with triadic closure

- - α = homophily parameter and
 - d = distance between nodes (height of lowest common ancestor)
- \mathfrak{F}_1 = intergroup probability of friend-of-friend connection
- $\approx au_2$ = intragroup probability of friend-of-friend connection

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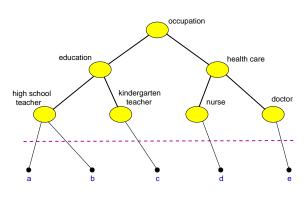
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Context distance



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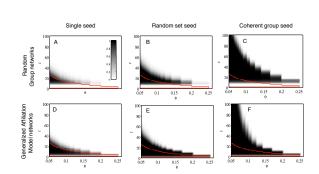
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Cascade windows for group-based networks



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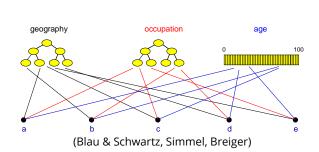
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Generalized affiliation model



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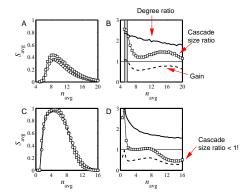
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Multiplier effect for group-based networks:



Multiplier almost always below 1.

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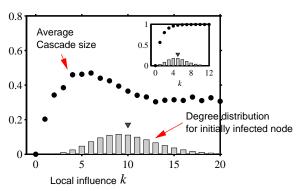
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Assortativity in group-based networks



- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

Social contagion

Summary

- & 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

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References

References I

[1] A. Bentley, M. Earls, and M. J. O'Brien. I'll Have What She's Having: Mapping Social Behavior.

MIT Press, Cambridge, MA, 2011.

- S. Bikhchandani, D. Hirshleifer, and I. Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. J. Polit. Econ., 100:992-1026, 1992.
- S. Bikhchandani, D. Hirshleifer, and I. Welch. Learning from the behavior of others: Conformity, fads, and informational cascades. J. Econ. Perspect., 12(3):151-170, 1998. pdf

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Social Contagion Models

References





ჟიდ 101 of 110

References II

- [4] I. M. Carlson and I. Doyle. Highly optimized tolerance: A mechanism for power laws in designed systems. Phys. Rev. E, 60(2):1412–1427, 1999. pdf
- J. M. Carlson and J. Doyle. Highly optimized tolerance: Robustness and design in complex systems. Phys. Rev. Lett., 84(11):2529-2532, 2000. pdf
- N. A. Christakis and J. H. Fowler. The spread of obesity in a large social network over 32 years. New England Journal of Medicine, 357:370-379, 2007. pdf ☑



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Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

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•୨५० 99 of 110

References III

- N. A. Christakis and J. H. Fowler. The collective dynamics of smoking in a large social network. New England Journal of Medicine, 358:2249-2258, 2008. pdf 🗷
- R. B. Cialdini. Influence: Science and Practice. Allyn and Bacon, Boston, MA, 4th edition, 2000.
- P. S. Dodds, K. D. Harris, and C. M. Danforth. Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability. Phys. Rev. Lett., 110:158701, 2013. pdf



References





ൗ < ॡ 103 of 110

References IV

[10] P. S. Dodds, K. D. Harris, and J. L. Payne. Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks.

Phys. Rev. E, 83:056122, 2011. pdf

[11] J. H. Fowler and N. A. Christakis.

Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.

BMJ, 337:article #2338, 2008. pdf

[12] M. Gladwell.

The Tipping Point.

Little, Brown and Company, New York, 2000.

References V

[13] J. P. Gleeson.

Cascades on correlated and modular random networks.

Phys. Rev. E, 77:046117, 2008. pdf

[14] I. P. Gleeson and D. J. Cahalane. Seed size strongly affects cascades on random networks.

Phys. Rev. E, 75:056103, 2007. pdf ☑

[15] M. Granovetter.

Threshold models of collective behavior. Am. J. Sociol., 83(6):1420-1443, 1978. pdf 2

References VI

[16] M. Granovetter and R. Soong. Threshold models of diversity: Chinese restaurants, residential segregation, and the spiral of silence.

[17] M. S. Granovetter and R. Soong. Threshold models of interpersonal effects in consumer demand.

[18] K. D. Harris, C. M. Danforth, and P. S. Dodds. General theory and applications to social

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Social Contagion Models

References





• വെ 104 of 110

PoCS | @pocsvox

Social Contagion Social Contagion

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Final size Spreading success

References

References VII

[19] E. Katz and P. F. Lazarsfeld. Personal Influence. The Free Press, New York, 1955.

[20] T. Kuran.

Now out of never: The element of surprise in the east european revolution of 1989. World Politics, 44:7–48, 1991. pdf ☑

[21] T. Kuran.

Private Truths, Public Lies: The Social Consequences of Preference Falsification. Harvard University Press, Cambridge, MA, Reprint edition, 1997.

[22] T. C. Schelling. Dynamic models of segregation. J. Math. Sociol., 1:143–186, 1971. pdf 🗹

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Social Contagion

Social Contagion Models

References



ჟიდ 107 of 110

References VIII

[23] T. C. Schelling.

Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.

J. Conflict Resolut., 17:381–428, 1973. pdf

[24] T. C. Schelling.

Micromotives and Macrobehavior. Norton, New York, 1978.

[25] D. Sornette.

Critical Phenomena in Natural Sciences. Springer-Verlag, Berlin, 1st edition, 2003.



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References





ൗ < ॡ 108 of 110

Sociological Methodology, 18:69–104, 1988. pdf

J. Econ. Behav. Organ., 7:83–99, 1986. pdf 2

Dynamical influence processes on networks: contagion.

Phys. Rev. E, 88:022816, 2013. pdf

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• ୨ ৭ № 105 of 110

Social Contagion Models

References



ൗ < ॡ 106 of 110

References IX

[26] D. J. Watts.

A simple model of global cascades on random networks.

Proc. Natl. Acad. Sci., 99(9):5766-5771, 2002. pdf 🖸

[27] D. J. Watts and P. S. Dodds. Influentials, networks, and public opinion Journal of Consumer Research, 34:441–458, 2007. pdf 🛂

[28] D. J. Watts and P. S. Dodds. Threshold models of social influence. In P. Hedström and P. Bearman, editors, The Oxford Handbook of Analytical Sociology, chapter 20, pages 475-497. Oxford University Press, Oxford, UK, 2009. pdf



Social Contagion Models

References





ൗ < ॡ 109 of 110

References X

PoCS | @pocsvox Social Contagion

Social Contagio Models Background Granovetter's model Network version Final size Spreading success

[29] U. Wilensky.

Netlogo segregation model.
http://ccl.northwestern.edu/netlogo/models/Segregation.
Center for Connected Learning and
Computer-Based Modeling, Northwestern
University, Evanston, IL., 1998.





少∢ॡ 110 of 110