

Social Contagion

Principles of Complex Systems | @pocsvox
 CSYS/MATH 300, Fall, 2017

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
 Vermont Advanced Computing Core | University of Vermont

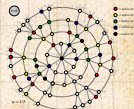
Social Contagion
 Models

Background
 Granovetter's model
 Network version
 Final size
 Spreading success
 Groups

References



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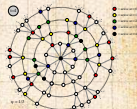
Sealie & Lambie Productions



Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

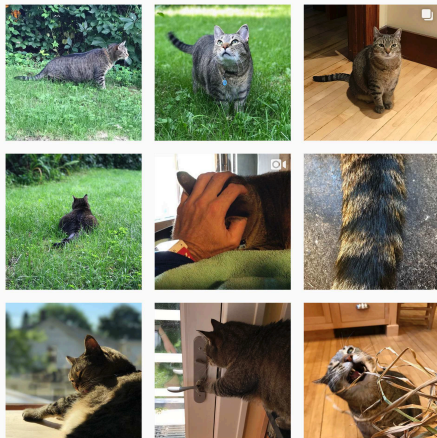
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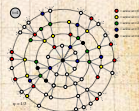
Special Guest Executive Producer: Pratchett





Social Contagion
Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 



Social Contagion Models

Background

Granovetter's model

Network version

Final size

Spreading success

Groups

Social Contagion Models

Background

Granovetter's model

Network version

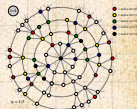
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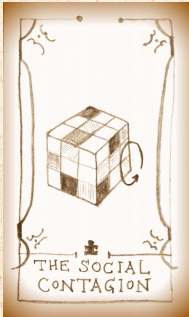
Spreading success

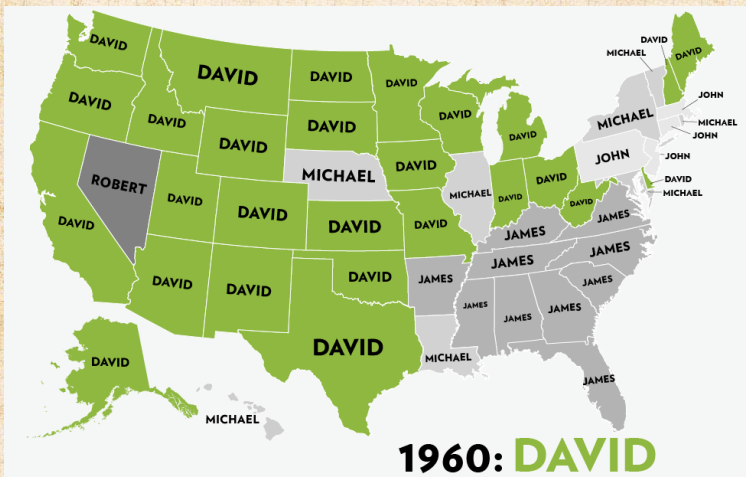
Groups

References

References





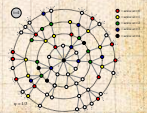


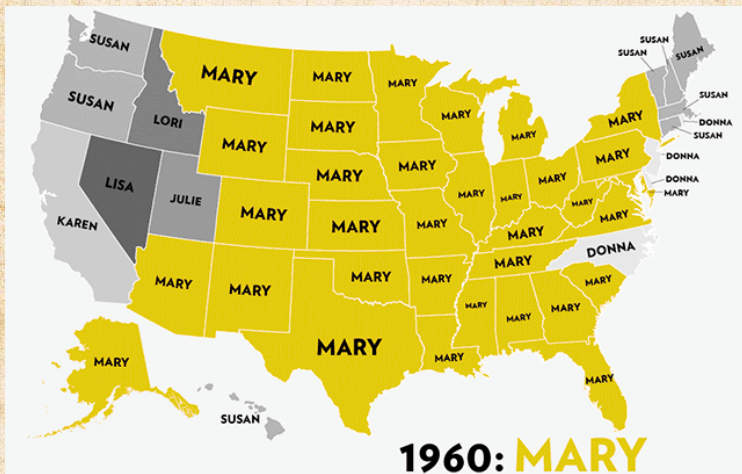
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

From the Atlantic ↗



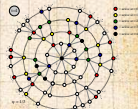


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Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



Richard Feynmann on the Social Sciences:

Social Contagion Models

Background

Granovetter's model

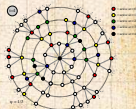
Network version

Final size

Spreading success

Groups

References

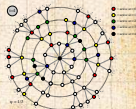


Sheldon Cooper on the Social Sciences:

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups



References



Things that spread well:

buzzfeed.com 



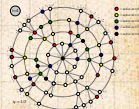
 Dangerously self aware: [11 Elements that make a perfect viral video.](#) 

+ News ...

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

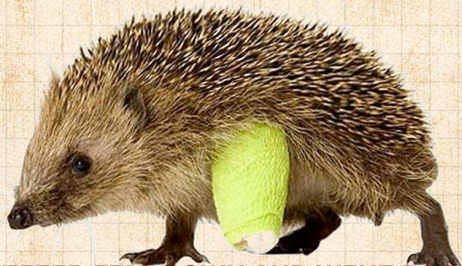


LOL + cute + fail + wtf:

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Oopsie!



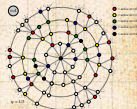
BUZZFEED FELL DOWN AND WENT BOOM.

Please try reloading this page. If the problem persists [let us know](#).

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Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



The whole lolcats thing:

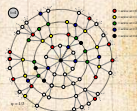
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Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



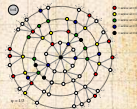
Some things really stick:

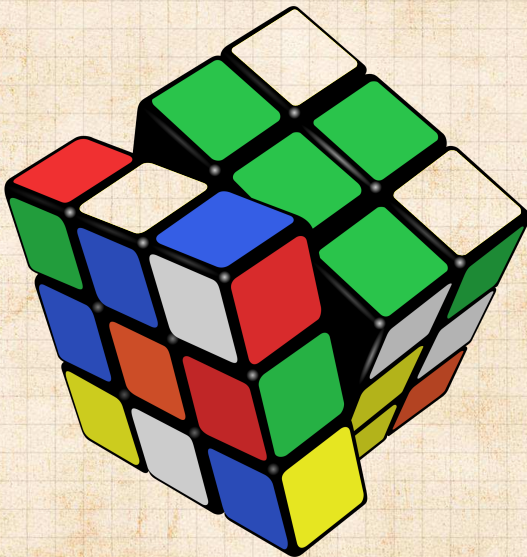


Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

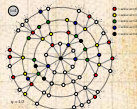




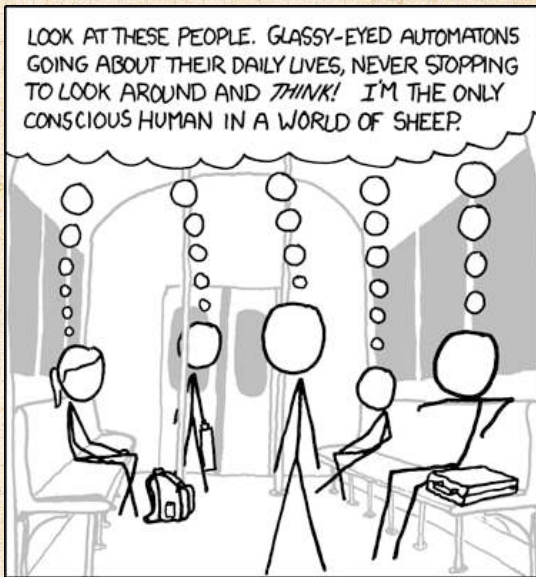
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



Why social contagion works so well:



Social Contagion Models

Background

Granovetter's model

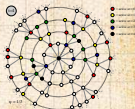
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Final size

Spreading success

Groups

References



<http://xkcd.com/610/>



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Social Contagion Models

Background

Granovetter's model

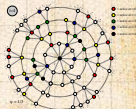
Network version

Final size

















Spreading success

Groups


References



Examples abound

-  fashion
-  striking
-  smoking  [7]
-  residential segregation [22]
-  iPhones and iThings
-  obesity  [6]
-  Harry Potter
-  voting
-  gossip
-  Rubik's cube 
-  religious beliefs
-  school shootings
-  **leaving lectures**

SIR and SIRS type contagion possible

-  Classes of behavior versus specific behavior :
dieting, horror movies, getting married, invading countries, ...

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Background

Granovetter's model

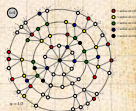
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Final size

Spreading success

Groups

References



Mixed messages: Please copy, but also, don't copy ...

Social Contagion Models

Background

Granovetter's model

Network version


Final size


Spreading success

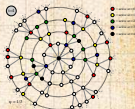
Groups

References

<http://www.youtube.com/watch?v=TgDxWNV4wWY?rel=0>

 [Cindy Harrell appeared](#) in the (terrifying) music video for Ray Parker Jr.'s [Ghostbusters](#).

 In [Stranger Things 2](#), Steve Harrington reveals his [Fabergé secret](#).



Market much?

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Models

Background

Granovetter's model

Network version

Final size


Spreading success

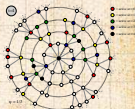
Groups

References

<http://www.youtube.com/watch?v=FEaCflp9qR4?rel=0>











Advertisement enjoyed during "Herstory of Dance" , Community S4E08, April 2013.



Framingham heart study:

Evolving network stories (Christakis and Fowler):

-  The spread of quitting smoking  ^[7]
-  The spread of spreading  ^[6]
-  Also: happiness  ^[11], loneliness, ...
-  The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives 

Social Contagion Models

Background

Granovetter's model

Network version





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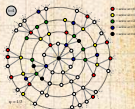
Spreading success

Groups

References

Controversy:

-  Are your friends making you fat?  (Clive Thomson, NY Times, September 10, 2009).
-  Everything is contagious  —Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).



Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?
Highly popularized by Gladwell^[12] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld)^[19]

Social Contagion Models

Background

Granovetter's model

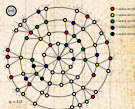
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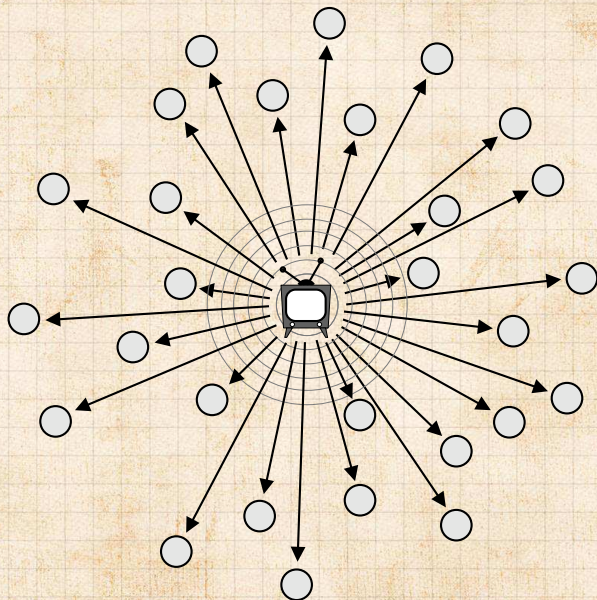
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Groups

References



The hypodermic model of influence



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Background

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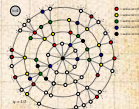
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Final size

Spreading success

Groups

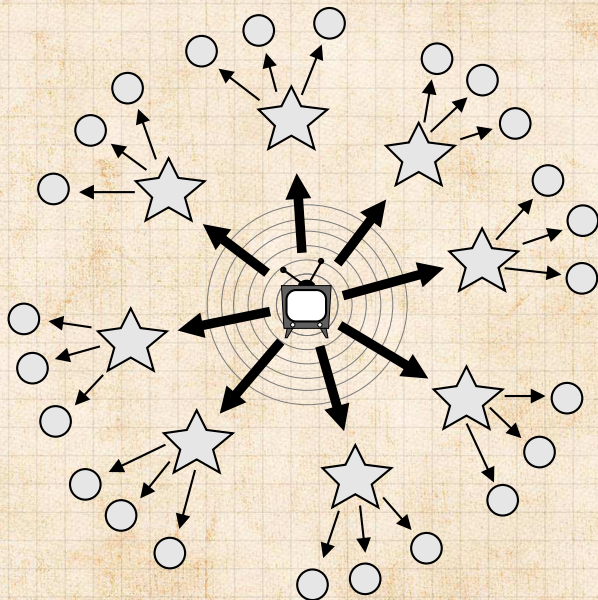
References



The two step model of influence [19]

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Social Contagion Models

Background

Granovetter's model

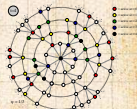
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Final size

Spreading success

Groups

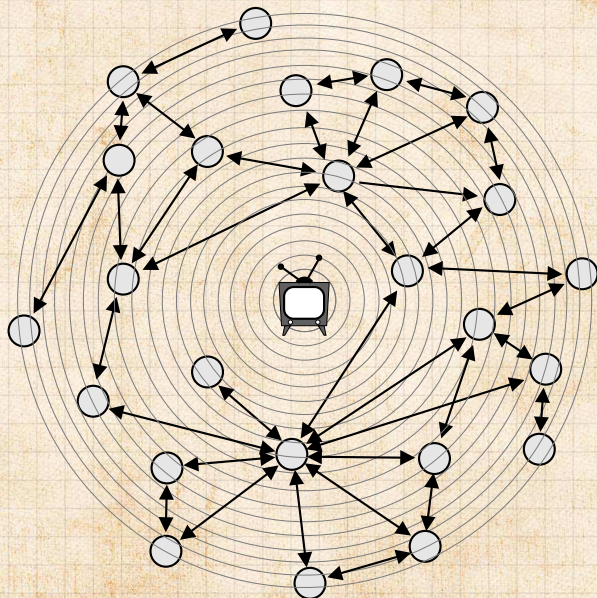
References



The general model of influence: the Social Wild

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Social Contagion Models

Background

Granovetter's model

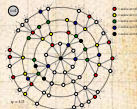
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Final size

Spreading success

Groups

References



Talking about the social wild:

Social Contagion Models

Background

Granovetter's model

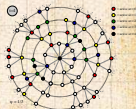
Network version

Final size

Spreading success

Groups

References



Why do things spread socially?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers:
homo narrativus ↗
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

Social Contagion Models

Background

Granovetter's model

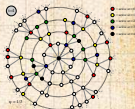
Network version

Final size

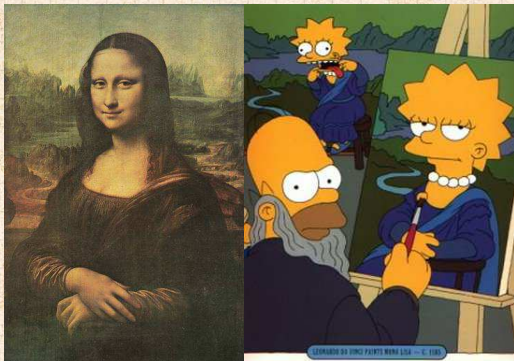
Spreading success

Groups

References



The Mona Lisa



Social Contagion Models

Background

Granovetter's model

Network version

Final size

Spreading success

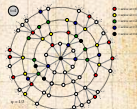
Groups

References

🧱 “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon

🧱 Not the world's greatest painting from the start...

🧱 Escalation through theft, vandalism, **parody**, ...



'Tattooed Guy' Was Pivotal in Armstrong Case [nytimes] ↗

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Social Contagion
Models

Background

Granovetter's model

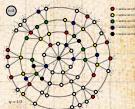
Network version

Final size

Spreading success

Groups

References



"... Leogrande's doping sparked a series of events
..."



The completely unpredicted fall of Eastern Europe:

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Social Contagion
Models

Background

Granovetter's model

Network version

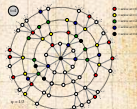
Final size

Spreading success

Groups

References

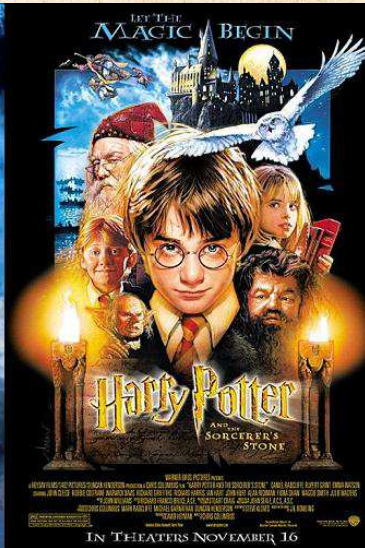
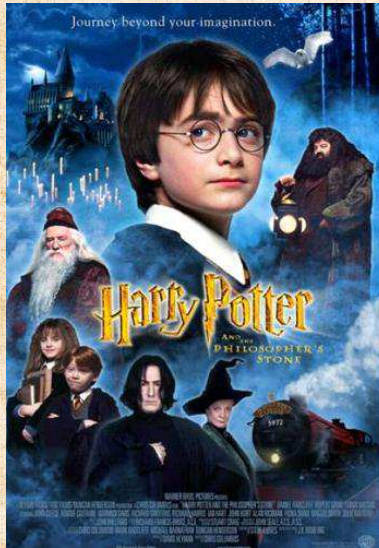
Timunr Kuran: ^[20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"



The dismal predictive powers of editors...

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Social Contagion Models

Background

Granovetter's model

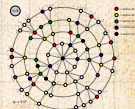
Network version

Final size

Spreading success

Groups


References




From a 2013 Believer Magazine [↗](#) interview with Maurice Sendak [↗](#):

BLVR: Did the success of *Where the Wild Things Are* ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention *Moby-Dick*. They're all going to talk about my first book, about ****ing maidens in Tahiti." He was right. No mention of *Moby-Dick* then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came *Moby-Dick* and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. *Moby-Dick* didn't get famous until 1930.

 Sendak named his dog Herman.

 The essential Colbert interview: [Pt. 1](#) [↗](#) and [Pt. 2](#) [↗](#).

Social Contagion
Models

Background

Granovetter's model

Network version

Final size

Spreading success

Groups

References



Drafting success in the NFL:

Top Players by Round, 1995-2012



1ST ROUND

Peyton Manning

1ST OVER ALL, 1998



2ND ROUND

Drew Brees

32ND PICK, 2001



3RD ROUND

Terrell Owens

89TH PICK, 1996



4TH ROUND

Jared Allen

126TH PICK, 2004



5TH ROUND

Zach Thomas

154TH PICK, 1996



6TH ROUND

Tom Brady

199TH PICK, 2000



7TH ROUND

Donald Driver

213TH PICK, 1999



Social Contagion Models

Background

Granovetter's model

Network version

Final size


Spreading success

Groups





References






Messing with social connections


 Ads based on message content
(e.g., Google and email)

 BzzAgent 

-  Harnessing of BzzAgents to directly market through social ties.
-  Generally: BzzAgents did not reveal their BzzAgent status and did not want to be paid.
-  NYT, 2004-12-05: "The Hidden (in Plain Sight) Persuaders" 

 One of Facebook's early advertising attempts:
Beacon 

 All of Facebook's advertising attempts.

 Seriously, Facebook. What could go wrong?

Social Contagion Models

Background

Granovetter's model

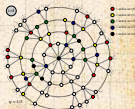
Network version

Final size

Spreading success

Groups

References



Getting others to do things for you

A very good book: 'Influence'^[8] by Robert Cialdini ↗

Six modes of influence:

1. **Reciprocation**: *The Old Give and Take... and Take*; e.g., Free samples, Hare Krishnas.
2. **Commitment and Consistency**: *Hobgoblins of the Mind*; e.g., Hazing.
3. **Social Proof**: *Truths Are Us*; e.g., Jonestown ↗, Kitty Genovese ↗ (contested).
4. **Liking**: *The Friendly Thief*; e.g., Separation into groups is enough to cause problems.
5. **Authority**: *Directed Deference*; e.g., Milgram's obedience to authority experiment. ↗
6. **Scarcity**: *The Rule of the Few*; e.g., Prohibition.

Social Contagion Models

Background

Granovetter's model

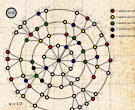
Network version


Final size


Spreading success

Groups


References




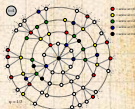
 Cialdini's modes are heuristics that help up us get through life.

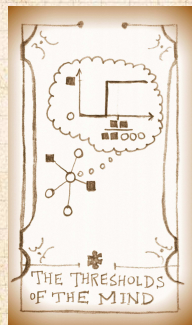
 Useful but can be leveraged...

Other acts of influence:

 Conspicuous Consumption (Veblen, 1912)

 Conspicuous Destruction (Potlatch)





Some important models:

- 📦 Tipping models—Schelling (1971) [22, 23, 24]
 - 📦 Simulation on checker boards
 - 📦 Idea of thresholds
 - 📦 Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.) [↗](#)
 - 📦 Explore the Netlogo [↗](#) online implementation [↗](#) [29]
- 📦 Threshold models—Granovetter (1978) [15]
- 📦 Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
 - 📦 Social learning theory, Informational cascades,...

Social Contagion Models

Background

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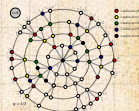
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Final size

Spreading success

Groups

References



Thresholds

- Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... **(unrealistic)**.
- Assumption: level of influence per person is uniform **(unrealistic)**.

Social Contagion Models

Background

Granovetter's model

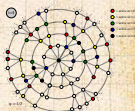
Network version

Final size







Spreading success

Groups

References



Some possible origins of thresholds:

-  Inherent, evolution-devised inclination to coordinate, to conform, to imitate. ^[1]
-  **Lack of information:** impute the worth of a good or behavior based on degree of adoption (social proof)
-  Economics: **Network effects** or **network externalities**
 -  Externalities = Effects on others not directly involved in a transaction
 -  Examples: telephones, fax machine, Facebook, operating systems
 -  An individual's utility increases with the adoption level among peers and the population in general

Social Contagion Models

Background

Granovetter's model

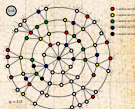
Network version

Final size

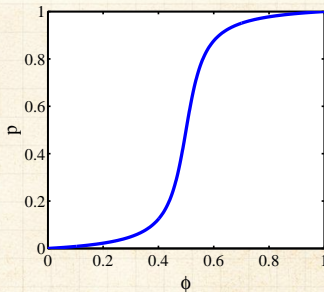
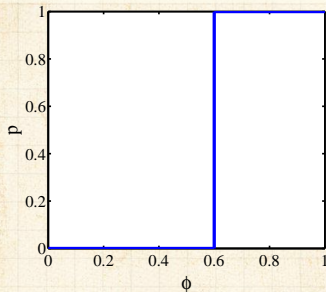
Spreading success


Groups


References




Threshold models—response functions



 Example threshold influence response functions:
deterministic and **stochastic**

 ϕ = fraction of contacts 'on' (e.g., rioting)

 Two states: S and I.

Social Contagion Models

Background

Granovetter's model

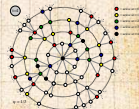
Network version

Final size

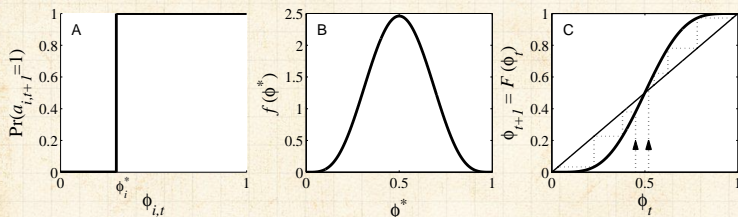
Spreading success

Groups

References



Action based on perceived behavior of others:



- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a **Critical mass model**

Social Contagion Models

Background

Granovetter's model

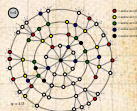
Network version

Final size

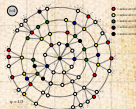
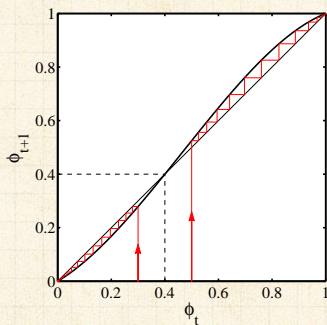
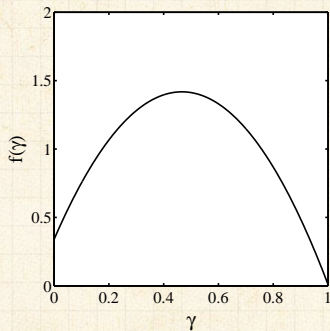
Spreading success

Groups

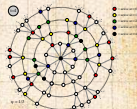
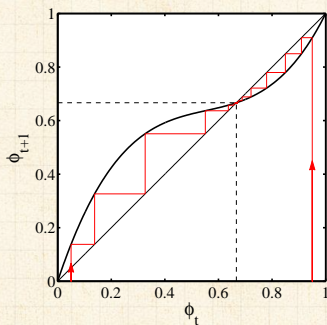
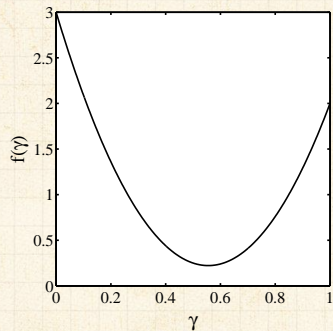
References



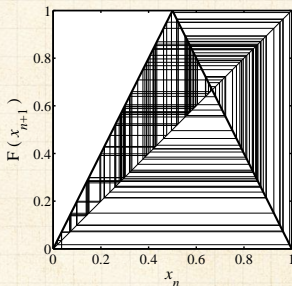
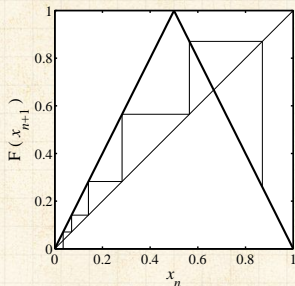
Another example of critical mass model:





Example of single stable state model:



Chaotic behavior possible [17, 16, 9, 18]



 Period doubling arises as map amplitude r is increased.

 Synchronous update assumption is crucial

Social Contagion Models

Background

Granovetter's model

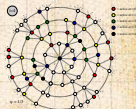
Network version

Final size

Spreading success

Groups

References



Implications for collective action theory:

1. Collective uniformity \nRightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes
3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves ↗.

Social Contagion Models

Background

Granovetter's model

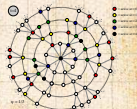
Network version

Final size

Spreading success

Groups

References



Many years after Granovetter and Soong's work:



"A simple model of global cascades on random networks"

D. J. Watts. Proc. Natl. Acad. Sci., 2002 [26]



Mean field model → network model



Individuals now have a limited view of the world

Social Contagion Models

Background

Granovetter's model

Network version

Final size

Spreading success

Groups

References

We'll also explore:



"Seed size strongly affects cascades on random networks" [14]

Gleeson and Cahalane, Phys. Rev. E, 2007.

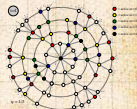


"Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks" [10] Dodds, Harris, and Payne, Phys. Rev. E, 2011



"Influentials, Networks, and Public Opinion Formation" [27]

Watts and Dodds, J. Cons. Res., 2007.



Threshold model on a network

Social Contagion Models

Background

Granovetter's model

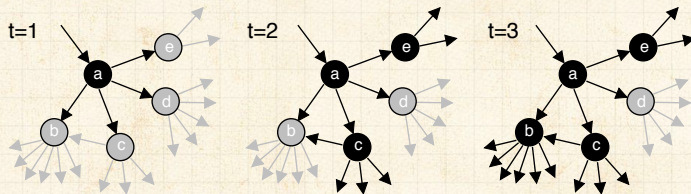
Network version


Final size

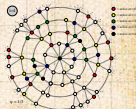
Spreading success

Groups

References



 All nodes have threshold $\phi = 0.2$.



Threshold model on a network

- Interactions between individuals now represented by a network.
- Network is **sparse**.
- Individual i has k_i contacts.
- Influence on each link is **reciprocal** and of **unit weight**.
- Each individual i has a fixed threshold ϕ_i .
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$.
- Individuals remain active when switched (no recovery = SI model).

Social Contagion Models

Background

Granovetter's model

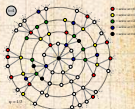
Network version

Final size

Spreading success

Groups

References



First study random networks:

- Start with N nodes with a degree distribution P_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a **cascade condition**

The Cascading Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

Social Contagion Models

Background

Granovetter's model

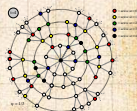
Network version

Final size

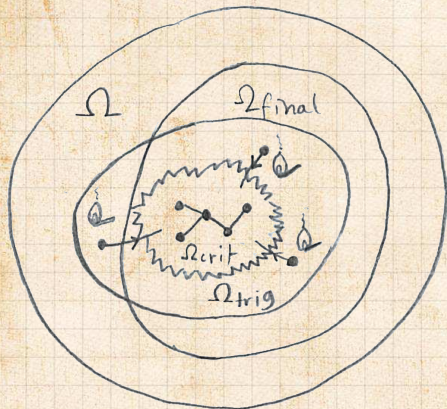
Spreading success


Groups


References





Example random network structure:



 $\Omega_{crit} = \Omega_{vuln} =$
critical mass =
global
vulnerable
component

 $\Omega_{trig} =$
triggering
component

 $\Omega_{final} =$
potential
extent of
spread

 $\Omega =$ entire
network

Social Contagion
Models

Background

Granovetter's model

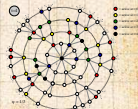
Network version

Final size

Spreading success

Groups


References





$$\Omega_{crit} \subset \Omega_{trig}; \Omega_{crit} \subset \Omega_{final}; \text{ and } \Omega_{trig}, \Omega_{final} \subset \Omega.$$

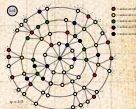


Follow active links

 An active link is a link connected to an activated node.


 If an infected link leads to **at least 1 more infected link**, then **activation spreads**.


 We need to understand which nodes can be activated when only one of their neighbors becomes active.



The most gullible


Vulnerables:


 We call individuals who can be activated by just one contact being active **vulnerables**


 The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

 Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$

 For global cascades on random networks, must have a *global cluster of vulnerables* [26]

 **Cluster of vulnerables = critical mass**

 Network story: 1 node \rightarrow critical mass \rightarrow everyone.

Social Contagion Models

Background

Granovetter's model

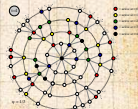
Network version

Final size




Spreading success

Groups

References



Back to following a link:

-  A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
-  Follows from there being k ways to connect to a node with degree k .
-  Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

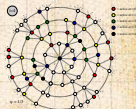
-  So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Social Contagion Models

[Background](#)[Granovetter's model](#)[Network version](#)[Final size](#)[Spreading success](#)[Groups](#)


References




Next: Vulnerability of linked node

 Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) d\phi'_*$$

 If linked node is **vulnerable**, it produces $k - 1$ new outgoing active links

 If linked node is **not vulnerable**, it produces **no** active links.

Social Contagion Models

Background

Granovetter's model

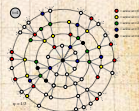
Network version

Final size


Spreading success

Groups

References



Putting things together:

 Expected number of active edges produced by an active edge:

$$R = \left[\sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1 - \beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}} \right]$$
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

Social Contagion Models

Background

Granovetter's model

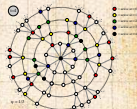
Network version

Final size

Spreading success


Groups


References

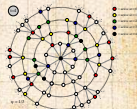


So... for random networks with fixed degree distributions, cascades take off when:


$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

 β_k = probability a degree k node is vulnerable.


 P_k = probability a node has degree k .



Two special cases:

 (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

 (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

Social Contagion
Models

Background

Granovetter's model

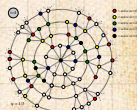
Network version

Final size

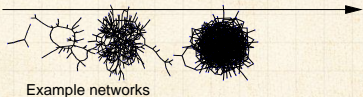
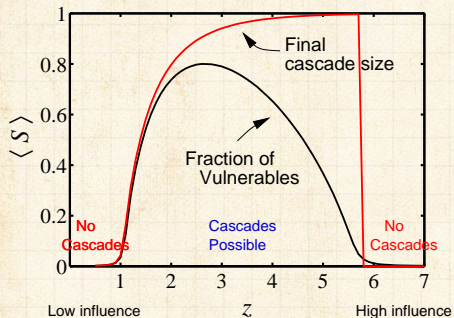
Spreading success

Groups

References



Cascades on random networks



Cascades occur only if size of max vulnerable cluster > 0 .



System may be 'robust-yet-fragile'.



'Ignorance' facilitates spreading.

Social Contagion Models

Background

Granovetter's model

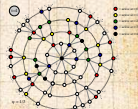
Network version

Final size

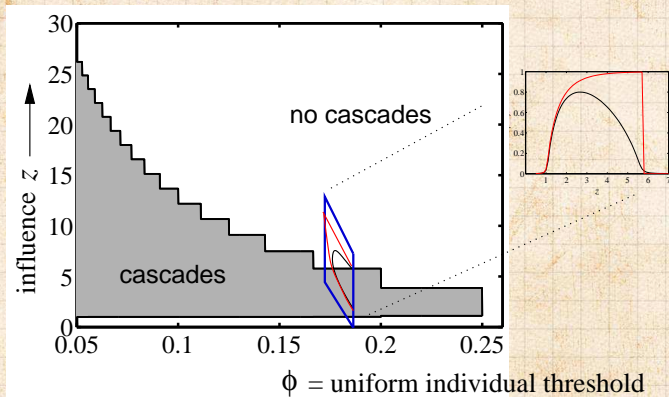
Spreading success

Groups

References



Cascade window for random networks



Social Contagion Models

Background

Granovetter's model

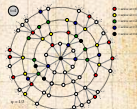
Network version

Final size

Spreading success

Groups

References



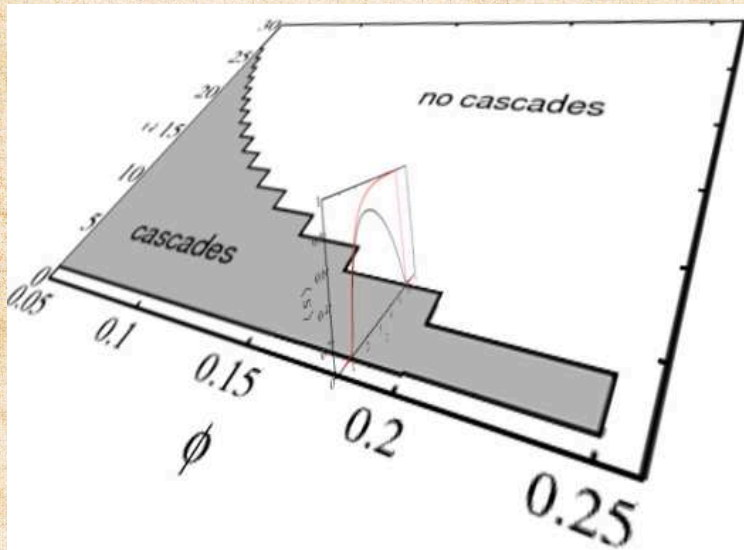
'Cascade window' widens as threshold ϕ decreases.



Lower thresholds enable spreading.



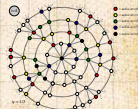
Cascade window for random networks



Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



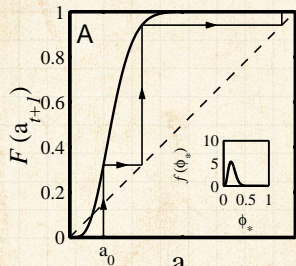
All-to-all versus random networks

Social Contagion Models

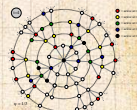
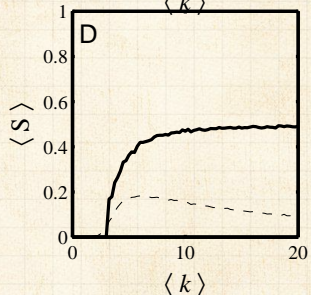
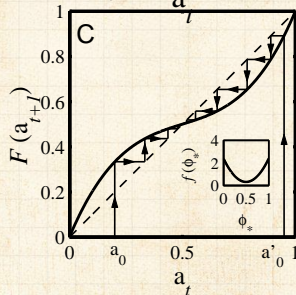
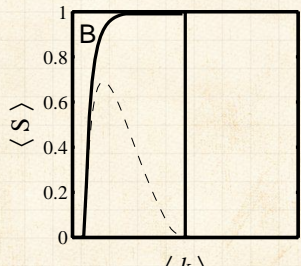
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

all-to-all networks

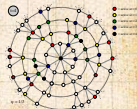


random networks



For our simple model of a uniform threshold:

1. **Low $\langle k \rangle$:** No cascades in poorly connected networks.
No global clusters of any kind.
2. **High $\langle k \rangle$:** Giant component exists but not enough vulnerables.
3. **Intermediate $\langle k \rangle$:** Global cluster of vulnerables exists.
Cascades are possible in **"Cascade window."**



Threshold contagion on random networks

Social Contagion Models

Background

Granovetter's model

Network version

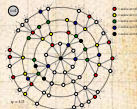
Final size

Spreading success

Groups

References

- Next: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem **beautifully solved** for infinite seed case by Gleeson and Cahalane:
"Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. ^[14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. ^[13]



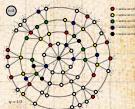
Determining expected size of spread:

- ☰ Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- ☰ Capitalize on local branching network structure of random networks (again)
- ☰ Now think about what must happen for a specific node i to become active at time t :
 - $t = 0$: i is one of the seeds (prob = ϕ_0)
 - $t = 1$: i was not a seed but enough of i 's friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = 2$: enough of i 's friends and friends-of-friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i .

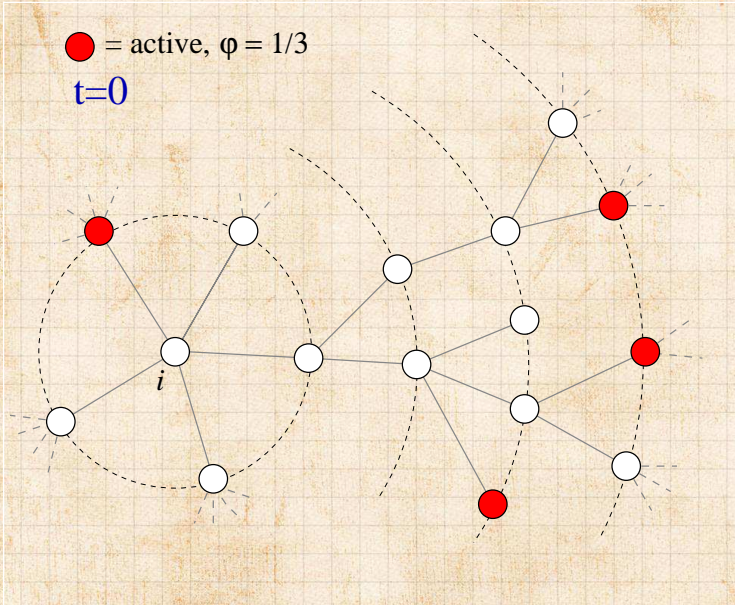
Social Contagion Models

[Background](#)[Granovetter's model](#)[Network version](#)[Final size](#)[Spreading success](#)[Groups](#)

References



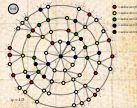
Expected size of spread



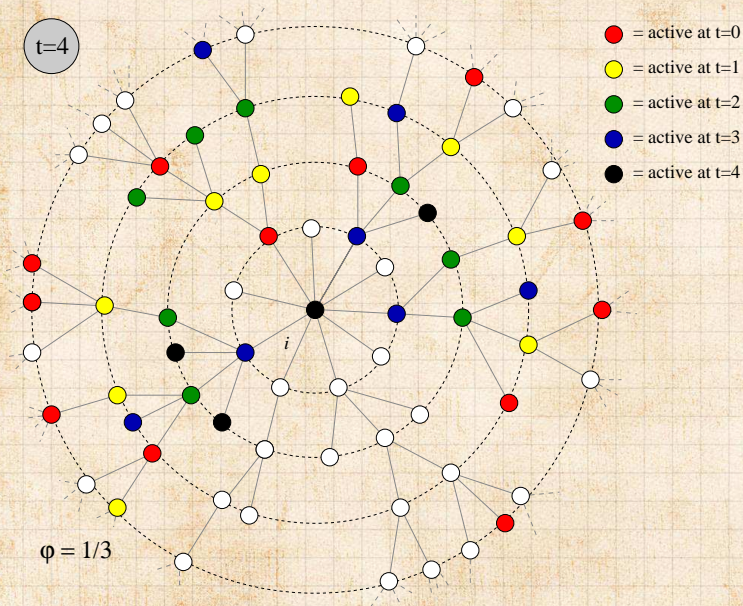
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



Expected size of spread



Social Contagion Models

Background

Granovetter's model

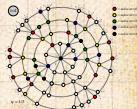
Network version

Final size

Spreading success

Groups

References



Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine **Pr**(node of degree k switching on at time t).
- Asynchronous updating can be handled too.

Social Contagion Models

Background

Granovetter's model

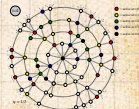
Network version

Final size

Spreading success

Groups

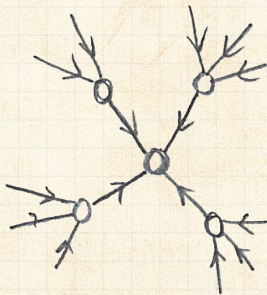
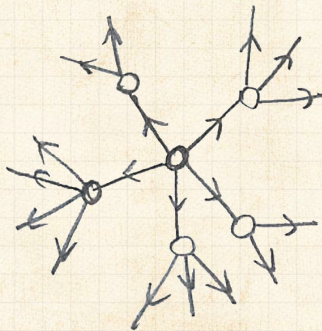
References



Expected size of spread

Pleasantness:

- ✖ Taking off from a single seed story is about **expansion** away from a node.
- ✖ Extent of spreading story is about **contraction** at a node.



Social Contagion Models

Background

Granovetter's model

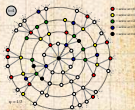
Network version

Final size


Spreading success

Groups


References





Expected size of spread


 **Notation:**


$\phi_{k,t} = \Pr$ (a degree k node is active at time t).


 **Notation:** $B_{kj} = \Pr$ (a degree k node becomes active if j neighbors are active).

 Our starting point: $\phi_{k,0} = \phi_0$.

 $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \Pr$ (j of a degree k node's neighbors were seeded at time $t = 0$).

 Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).

 Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0)$.

 Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}.$$

Social Contagion
Models

Background

Granovetter's model

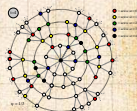
Network version

Final size

Spreading success

Groups

References



For general t , we need to know the probability an edge coming into a degree k node at time t is active.

Notation: call this probability θ_t .

We already know $\theta_0 = \phi_0$.

Story analogous to $t = 1$ case. For node i :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i j}.$$

Average over all nodes to obtain expression for ϕ_{t+1} :

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k j}.$$

So we need to compute θ_t ... massive excitement...

Social Contagion Models

Background

Granovetter's model

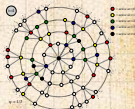
Network version

Final size

Spreading success


Groups

References





Expected size of spread


First connect θ_0 to θ_1 :


 $\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{kj}$$

 $\frac{k P_k}{\langle k \rangle} = R_k = \mathbf{Pr}$ (edge connects to a degree k node).

 $\sum_{j=0}^{k-1}$ piece gives \mathbf{Pr} (degree node k activates) of its neighbors $k - 1$ incoming neighbors are active.

 ϕ_0 and $(1 - \phi_0)$ terms account for state of node at time $t = 0$.

 See this all generalizes to give θ_{t+1} in terms of $\theta_t \dots$

Social Contagion Models

Background

Granovetter's model

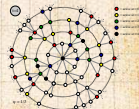
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Final size

Spreading success

Groups

References



Expected size of spread

Two pieces: edges first, and then nodes

$$1. \theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

with $\theta_0 = \phi_0$.

$$2. \phi_{t+1} =$$

$$\underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}}_{\text{social effects}}$$

Social Contagion
Models

Background

Granovetter's model

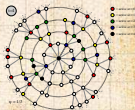
Network version

Final size

Spreading success

Groups

References



Expected size of spread

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}}$$

$$+(1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

$$= G(\theta_t; \phi_0)$$

Social Contagion Models

Background

Granovetter's model

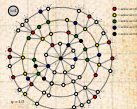
Network version

Final size

Spreading success

Groups

References



Expected size of spread:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

- If $\theta = 0$ is a fixed point of G (i.e., $G(0; \phi_0) = 0$) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

Social Contagion Models

Background

Granovetter's model

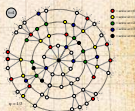
Network version

Final size

Spreading success

Groups

References



Expected size of spread:

In words:

- 🧱 If $G(0; \phi_0) > 0$, spreading must occur because some nodes turn on for free.
- 🧱 If G has an **unstable fixed point** at $\theta = 0$, then cascades are also always possible.

Non-vanishing seed case:

- 🧱 Cascade condition is more complicated for $\phi_0 > 0$.
- 🧱 If G has a **stable fixed point** at $\theta = 0$, and an **unstable fixed point** for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- 🧱 Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G .
- 🧱 A version of a critical mass model again.

Social Contagion
Models

Background

Granovetter's model

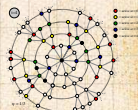
Network version

Final size

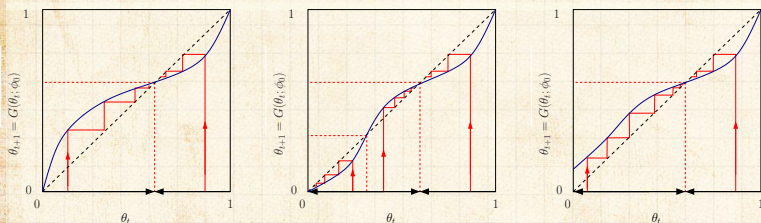
Spreading success

Groups

References



General fixed point story:



Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.

n.b., adjacent fixed points must have opposite stability types.

Important: Actual form of G depends on ϕ_0 .

So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G .

Social Contagion Models

Background

Granovetter's model

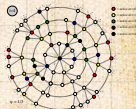
Network version

Final size

Spreading success

Groups

References

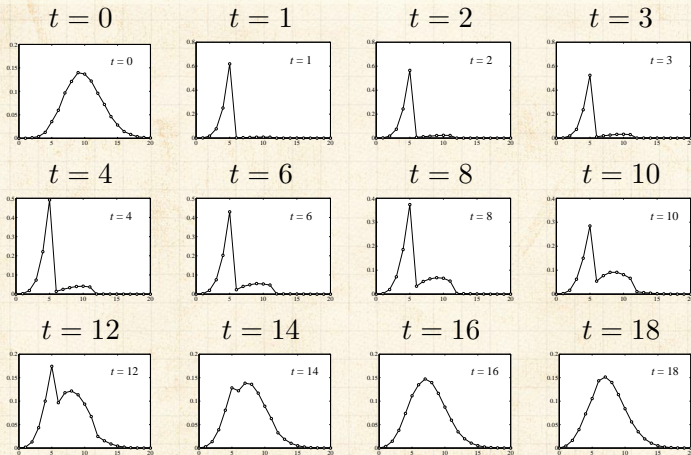


Early adopters—degree distributions

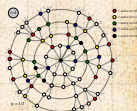
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

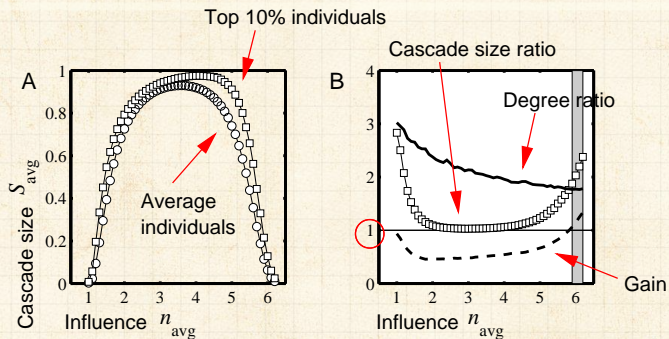
References



$P_{k,t}$ versus k

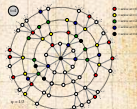


The multiplier effect:

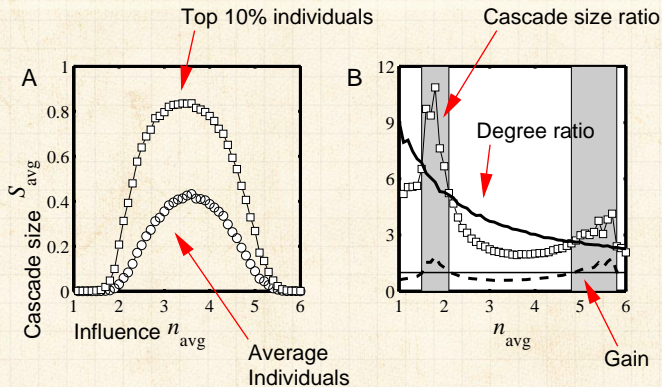


🧱 Fairly uniform levels of individual influence.

🧱 Multiplier effect is mostly below 1.



The multiplier effect:



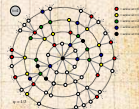
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

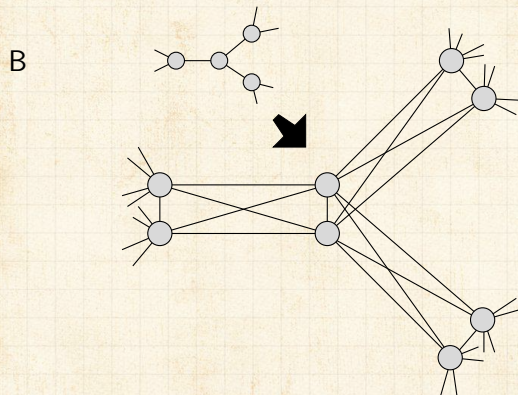
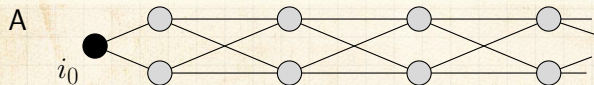
References



Skewed influence distribution example.



Special subnetworks can act as triggers

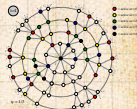


 $\phi = 1/3$ for all nodes

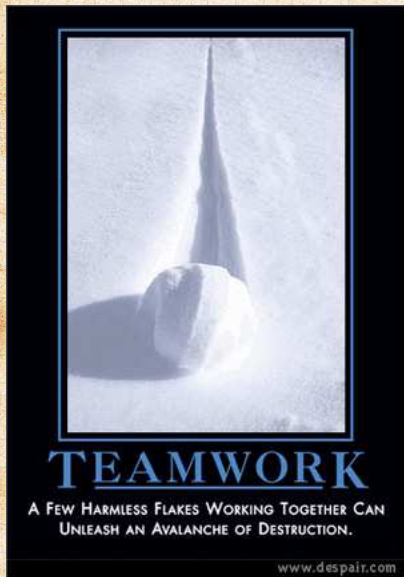
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success**
- Groups

References



The power of groups...

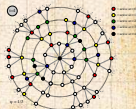


“A few harmless flakes working together can unleash an avalanche of destruction.”

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References





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Social Contagion
Models

Background

Granovetter's model

Network version

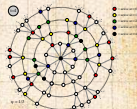
Final size

Spreading success

Groups

References

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: **group structure**

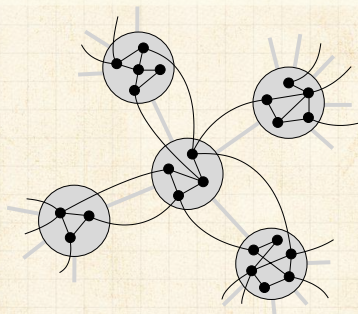


Group structure—Ramified random networks

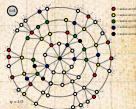
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups**

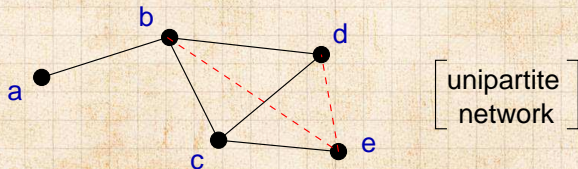
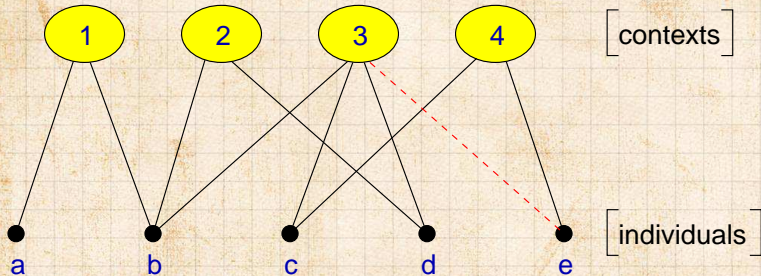
References



p = intergroup connection probability
 q = intragroup connection probability.



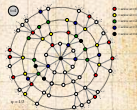
Bipartite networks



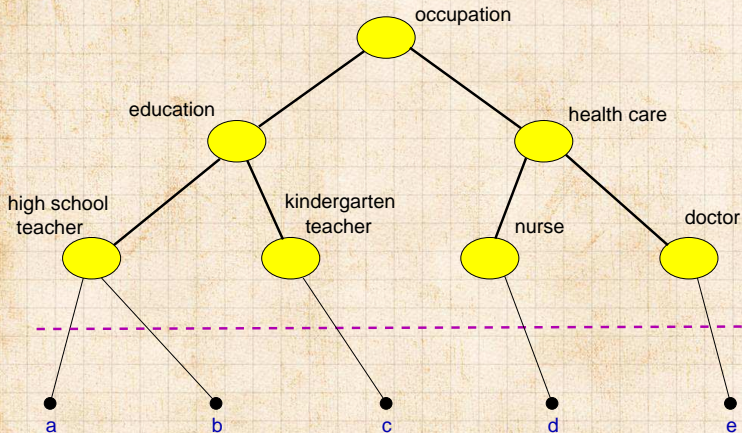
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



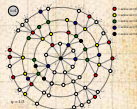
Context distance



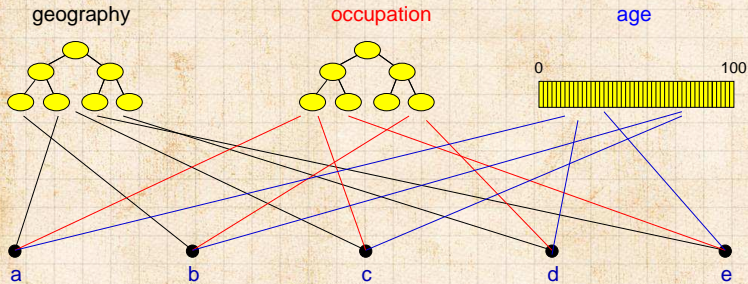
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



Generalized affiliation model

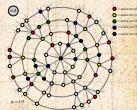


(Blau & Schwartz, Simmel, Breiger)

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



Generalized affiliation model networks with triadic closure



Connect nodes with probability $\propto \exp^{-\alpha d}$
where

α = homophily parameter

and

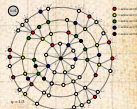
d = distance between nodes (height of lowest common ancestor)



τ_1 = intergroup probability of friend-of-friend connection



τ_2 = intragroup probability of friend-of-friend connection



Cascade windows for group-based networks

Social Contagion Models

Background

Granovetter's model

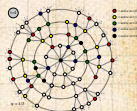
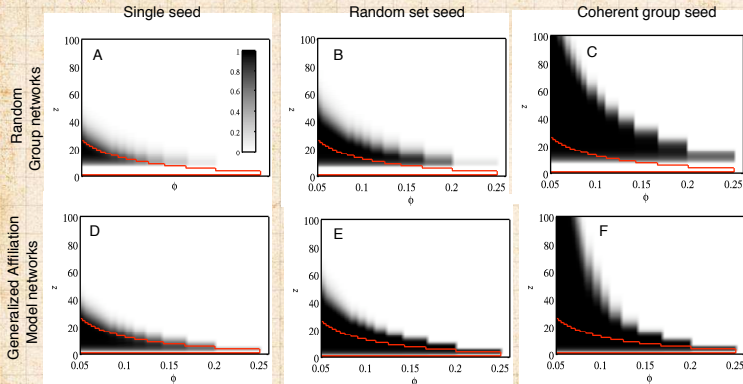
Network version

Final size

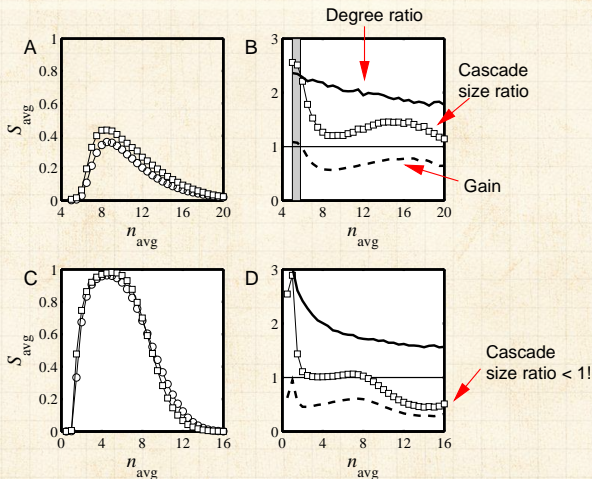
Spreading success

Groups

References



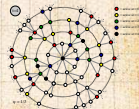
Multiplier effect for group-based networks:




Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

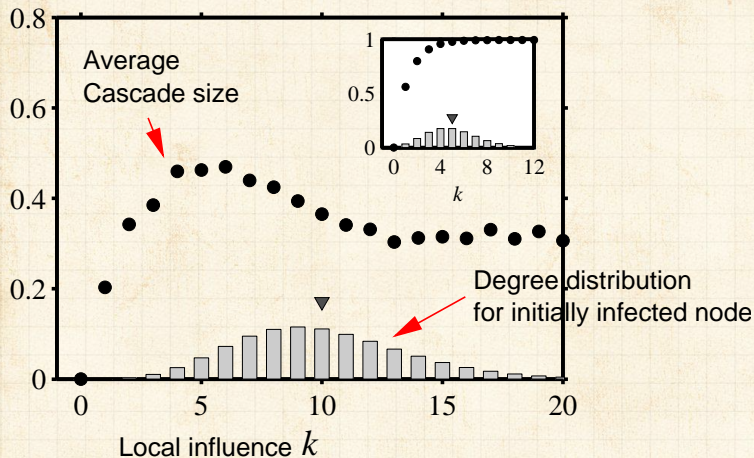
References



 Multiplier almost always below 1.




Assortativity in group-based networks




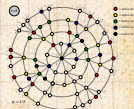
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References

 The most connected nodes aren't always the most 'influential.'

 Degree assortativity is the reason.



Summary

- 🧱 **'Influential vulnerables'** are key to spread.
- 🧱 Early adopters are mostly vulnerables.
- 🧱 Vulnerable nodes important but not necessary.
- 🧱 Groups may greatly facilitate spread.
- 🧱 Seems that cascade condition is a global one.
- 🧱 Most extreme/unexpected cascades occur in highly connected networks
- 🧱 'Influentials' are posterior constructs.
- 🧱 Many potential influentials exist.

Social Contagion Models

Background

Granovetter's model

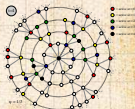
Network version

Final size

Spreading success

Groups

References



Implications

- Focus on **the influential vulnerables**.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only **simple ideas** can spread by word-of-mouth.
(Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be **passive** = free (yo-yo's, fashion), or **active** = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

Social Contagion Models

Background

Granovetter's model

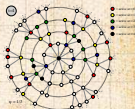
Network version


Final size

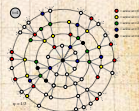
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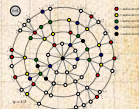




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Social Contagion
Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



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Social Contagion
Models

Background

Granovetter's model

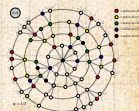
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Final size

Spreading success

Groups

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Social Contagion
Models

Background

Granovetter's model

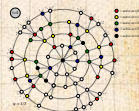
Network version

Final size

Spreading success

Groups

References

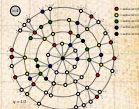


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Social Contagion
Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups


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Social Contagion
Models

Background

Granovetter's model

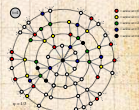
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Final size

Spreading success

Groups

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Social Contagion
Models

Background

Granovetter's model

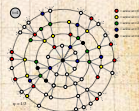
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Final size


Spreading success

Groups

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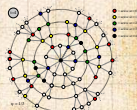
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Social Contagion
Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

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Social Contagion Models

[Background](#)

[Granovetter's model](#)

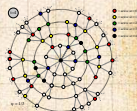
[Network version](#)

[Final size](#)

[Spreading success](#)

[Groups](#)

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