Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2017

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center | Vermont Advanced Computing Core | University of Vermont























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Models

Some important models:

- 1. Generalized random networks:
- 2. Small-world networks:
- 3. Generalized affiliation networks:
- 4. Scale-free networks;
- 5. Statistical generative models (p^*) .

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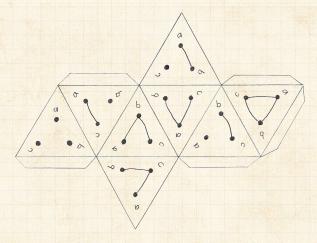








Random network generator for N=3:





Get your own exciting generator here .



 $As N \nearrow$, polyhedral die rapidly becomes a ball...

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Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
 - Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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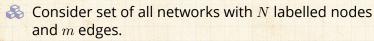
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Number of possible edges:

$$0 \le m \le {N \choose 2} = \frac{N(N-1)}{2}$$

$$2^{\binom{N}{2}} \sim e^{\frac{\ln_2}{2}N^2}$$

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Number of possible edges:

$$0 \le m \le {N \choose 2} = \frac{N(N-1)}{2}$$

- \clubsuit Limit of m=0: empty graph.
- Solution Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- $\ref{Number of possible networks with } N$ labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln_2 N^2}{2}}.$$

Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.

Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real worth links are usually costly so real networks are almost always sparse.

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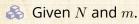
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How to build standard random networks:



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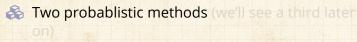




How to build standard random networks:



 \mathbb{A} Given N and m.



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How to build standard random networks:



 \mathbb{A} Given N and m.

Two probablistic methods (we'll see a third later) on)

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How to build standard random networks:

- \clubsuit Given N and m.
- Two probablistic methods (we'll see a third later on)
 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
 - 2. Take *N* nodes and add exactly *m* links by selecting edges without replacement.

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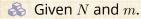
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 - 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
 - Useful for theoretical work.
 - 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.

Best for adding relatively small numbers of links (most cases).

1 and 2 are effectively equivalent for large N.

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A few more things:



For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

$$\langle k \rangle = rac{2 \langle m \rangle}{N}$$

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A few more things:



For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$



So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

$$= \frac{2}{N} p \frac{1}{2} N(N-1) = \frac{2}{N} p \frac{1}{2} N(N-1) = p(N-1).$$

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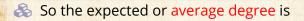


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$$=\frac{2}{N}p\frac{1}{2}N(N-1)=\frac{2}{\mathcal{N}}p\frac{1}{2}\mathcal{N}(N-1)$$

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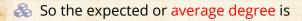


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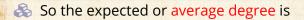


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Which is what it should be...

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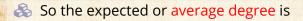


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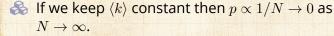


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Next slides:

Example realizations of random networks

N = 500

Vary m, the number of edges from 100 to 1000. Average degree (k) runs from 0.4 to 4. Look at full network plus the largest componen

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Next slides:

Example realizations of random networks



N = 500

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Next slides:

Example realizations of random networks



N = 500



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Example realizations of random networks



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Example realizations of random networks



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Look at full network plus the largest component.

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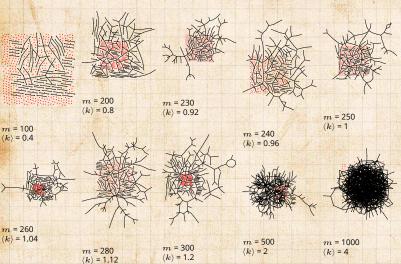
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Random networks: examples for N=500



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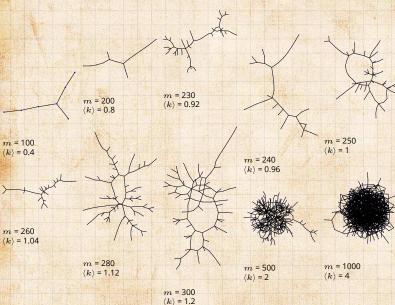






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Random networks: largest components



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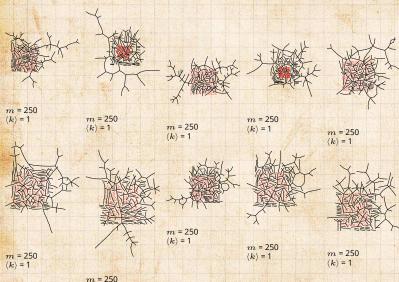






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Random networks: examples for N=500



 $\langle k \rangle = 1$

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Random networks: largest components

$$m$$
 = 250 $\langle k \rangle$ = 1

m = 250 $\langle k \rangle = 1$

m = 250

$$m$$
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angle$ = 1

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m = 250 $\langle k \rangle = 1$

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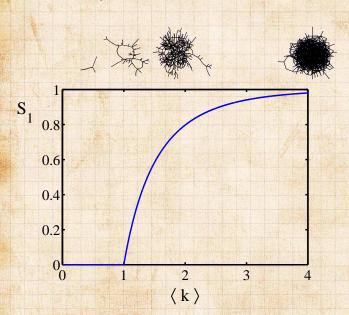
m = 250 $/L_1 - 1$

m = 250

 $\langle k \rangle = 1$

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Giant component



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For construction method 1, what is the clustering coefficient for a finite network?

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For construction method 1, what is the clustering coefficient for a finite network?

🎎 Consider triangle/triple clustering coefficient: 🖂

$$C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$$

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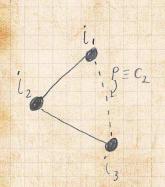




For construction method 1, what is the clustering coefficient for a finite network?

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Recall: C_2 = probability that two friends of a node are also friends.

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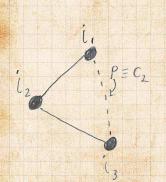
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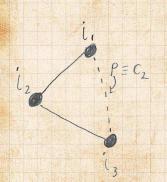




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- For construction method 1, what is the clustering coefficient for a finite network?
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- Recall: C_2 = probability that two friends of a node are also friends.
- Arr Or: C_2 = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p$$
.

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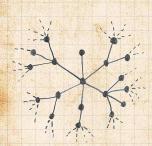
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So for large random networks $(N \to \infty)$, clustering drops to zero.

Key structura/ mat the of candom networks for that they locally reposition

No small look

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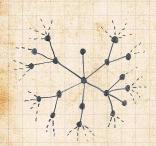
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- So for large random networks $(N \to \infty)$, clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks

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- No small loops.

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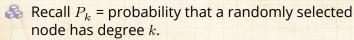
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Consider method 1 for constructing random networks: each possible link is realized with probability p.

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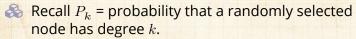
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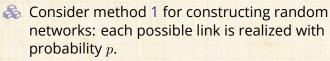
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 \implies Now consider one node: there are 'N – 1 choose k' ways the node can be connected to k of the other N-1 nodes.

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Therefore have a

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- \implies Now consider one node: there are 'N 1 choose k' ways the node can be connected to k of the other N-1 nodes.
- \clubsuit Each connection occurs with probability p, each non-connection with probability (1-p).
- Therefore have a binomial distribution :

$$P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$$

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Our degree distribution:

$$P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}$$

What happens as $N \to \infty$?

We must end up with the normal distribution right?

If p is fixed, then we would end up with a Gaussian with average degree $\langle k \rangle \simeq pN \to \infty$

But we want to keep $\langle k \rangle$ fixed..

So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = p(N-1) = \text{constant}$.

$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1}\right)^{N-1-k} \to \frac{\langle k \rangle^k}{k!} e^{-k}$$

This is a (k).

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This is a $\{(k), (k)\}$ with mean $\{(k), (k)\}$

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k! N-1

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Limiting form of P(k; p, N):

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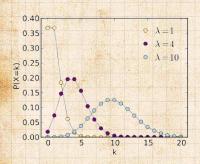
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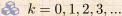


$$P(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}$$





 $\lambda > 0$





Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.



e.g.: phone calls/minute, horse-kick deaths.





'Law of small numbers'

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Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

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Checking:

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

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$$\begin{split} \sum_{k=0}^{\infty} k P(k;\langle k \rangle) &= \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \\ &= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!} \\ &= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!} \\ &= \langle k \rangle e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^i}{i!} \end{split}$$

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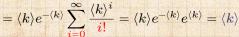


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$$\langle k \rangle = \sum_{k=0}^{\infty} k P(k; \langle k \rangle).$$

Checking:

$$\begin{split} \sum_{k=0}^{\infty} k P(k; \langle k \rangle) &= \sum_{k=0}^{\infty} k \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \\ &= e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^k}{(k-1)!} \\ &= \langle k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^{k-1}}{(k-1)!} \\ k \rangle e^{-\langle k \rangle} \sum_{k=1}^{\infty} \frac{\langle k \rangle^i}{(k-1)!} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle} e^{-\langle k \rangle} &= \langle k \rangle e^{-\langle k \rangle$$





In CocoNuTs, we find a different, crazier way of doing this...

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The variance of degree distributions for random networks turns out to be very important.

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- The variance of degree distributions for random networks turns out to be very important.
- & Using calculation similar to one for finding $\langle k \rangle$ we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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- & Using calculation similar to one for finding $\langle k \rangle$ we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$$



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 \red So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

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- The variance of degree distributions for random networks turns out to be very important.
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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

Variance is then

$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

- Note: This is a special property of Poisson distribution and can trip us up...

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Neural reboot (NR):

Unrelated: Feline elevation

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So... standard random networks have a Poisson degree distribution

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- So... standard random networks have a Poisson degree distribution
- \triangle Generalize to arbitrary degree distribution P_k .

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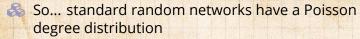
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Also known as the configuration model. [7]

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- So... standard random networks have a Poisson degree distribution
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- Can generalize construction method from ER random networks.

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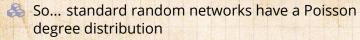
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 \mathbb{R} Generalize to arbitrary degree distribution P_k .

Also known as the configuration model. [7]

Can generalize construction method from ER random networks.

 $\red solution > 8$ Assign each node a weight w from some distribution P_w and form links with probability

 $P(\text{link between } i \text{ and } j) \propto w_i w_j$.

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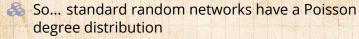
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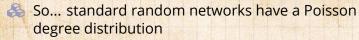
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1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

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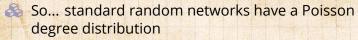
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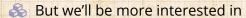
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1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

Examining mechanisms that lead to networks with certain degree distributions. PoCS | @pocsvox
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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

- N = 1000.
- $P_k \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_0 = 0$ (no isolated nodes).
- Vary exponent γ between 2.10 and 2.9
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random

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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:



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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:





$$Rrightarrow P_k \propto k^{-\gamma}$$
 for $k \geq 1$.

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$$Rrac{1}{4}$$
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Random networks: examples for N=1000

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 γ = 2.28 $\langle k \rangle$ = 2.306

 γ = 2.37 $\langle k \rangle$ = 2.504

 γ = 2.46 $\langle k \rangle$ = 1.856













References



 γ = 2.55 $\langle k \rangle$ = 1.712

 γ = 2.64 $\langle k \rangle$ = 1.6

 γ = 2.73 $\langle k \rangle$ = 1.862

 γ = 2.82 $\langle k \rangle$ = 1.386

 $\gamma = 2.91$ $\langle k \rangle = 1.49$



Random networks: largest components











 $\gamma = 2.37$ $\langle k \rangle = 2.504$

 $\gamma = 2.46$ $\langle k \rangle = 1.856$

 $\gamma = 2.1$ $\langle k \rangle = 3.448$

 $\langle k \rangle = 2.986$

 $\gamma = 2.28$ $\langle k \rangle = 2.306$











 $\gamma = 2.64$ $\langle k \rangle = 1.6$

 $\gamma = 2.73$ $\langle k \rangle = 1.862$

 $\gamma = 2.82$ $\langle k \rangle = 1.386$

 $\gamma = 2.91$ $\langle k \rangle = 1.49$

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Generalized random networks:

- Arbitrary degree distribution P_k .

 Create (unconnected) nodes with degree P_k .
 - create (unconnected) nodes with degrees sampled from P_k .
- Wire nodes together randomly
- Create ensemble to test deviations from

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Generalized random networks:



 \triangle Arbitrary degree distribution P_k .

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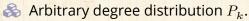
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Generalized random networks:



Create (unconnected) nodes with degrees sampled from P_k .

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Generalized random networks:

- \triangle Arbitrary degree distribution P_k .
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Generalized random networks:

- \triangle Arbitrary degree distribution P_k .
- Create (unconnected) nodes with degrees sampled from P_k .
- Wire nodes together randomly.
- Create ensemble to test deviations from randomness.

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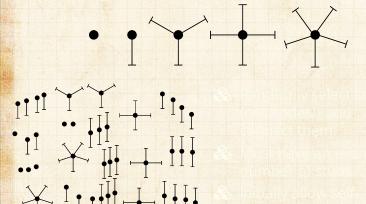






Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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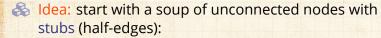
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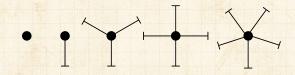


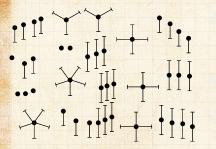




Phase 1:







Randomly select stub (not nodes!) and connect them.

Must have an even number of stubs.
Initially allow self- and

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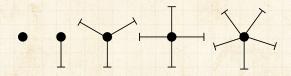


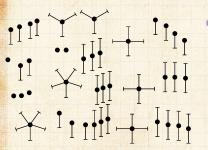




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Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them.

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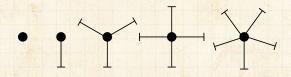


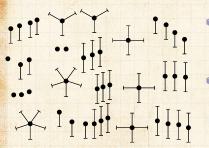




Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them.

Must have an even number of stubs.

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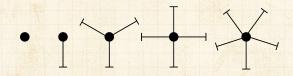


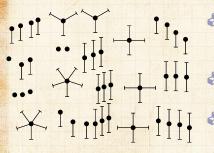




Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stubs (not nodes!) and connect them.

Must have an even number of stubs.

Initially allow self- and repeat connections.

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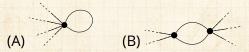


Building random networks: First rewiring

Phase 2:



Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



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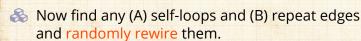


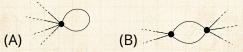




Building random networks: First rewiring

Phase 2:





Being careful: we can't change the degree of any node, so we can't simply move links around.

Simplest solution: randomly rewire two edges at a time.

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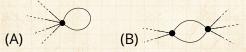


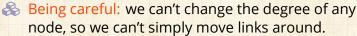
Building random networks: First rewiring

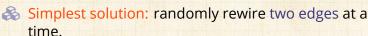
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Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.







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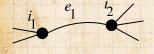
Random friends are strange

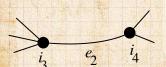
Largest component













Randomly choose two edges. (Or choose problem edge and a random edge)

Check to make sure edges ar

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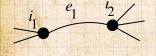
Motifs
Random friends are

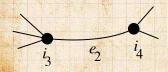
strange Largest component











- Randomly choose two edges. (Or choose problem edge and a random edge)
 - Check to make sure edges are disjoint.

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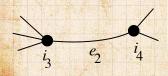


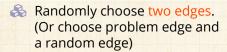












Check to make sure edges are disjoint.



Rewire one end of each edge.

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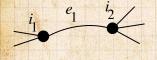
Generalized Random Networks

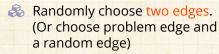
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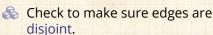


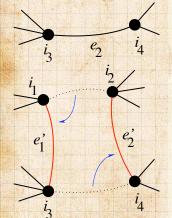












Rewire one end of each edge.

Node degrees do not change.

Works if e_1 is a self-loop or repeated edge.

Same as finding on/off/on/off 4-cycles, and rotating them. PoCS | @pocsvox Random

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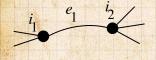
Random friends are strange

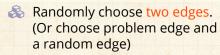
Largest component



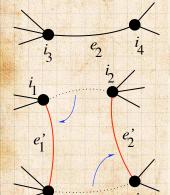








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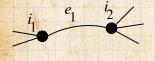
Random friends are Largest component

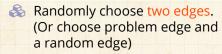


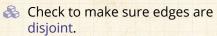


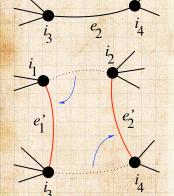


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Phase 2:



Use rewiring algorithm to remove all self and repeat loops.

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Phase 2:



Use rewiring algorithm to remove all self and repeat loops.

Phase 3:



Randomize network wiring by applying rewiring algorithm liberally.

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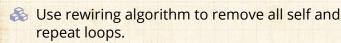
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Phase 2:



Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

 \aleph Rule of thumb: # Rewirings $\simeq 10 \times \# \text{ edges}^{[5]}$.

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Random sampling



Problem with only joining up stubs is failure to randomly sample from all possible networks.

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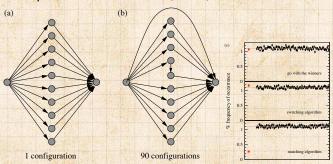


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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

🍪 Example from Milo et al. (2003) [5]:



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 \mathbb{R} What if we have P_k instead of N_k ?

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- \bigotimes What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.

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- \bigotimes What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .

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- \bigotimes What if we have P_k instead of N_k ?
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- \bigotimes What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .
- Easy to do exactly numerically since k is discrete.
- Arr Note: not all P_{k} will always give nodes that can be wired together.

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ldea of motifs [8] introduced by Shen-Orr, Alon et al. in 2002.

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- [8] Idea of motifs [8] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

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Random Networks

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Random Networks

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- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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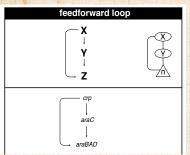
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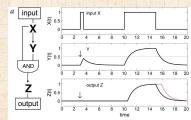
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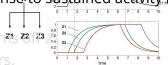






Z only turns on in response to sustained activity in

X.



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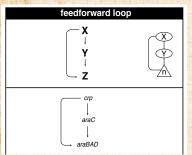


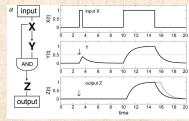






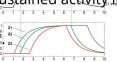






Z only turns on in response to sustained activity X.

Turning off X rapidly turrs でff32。



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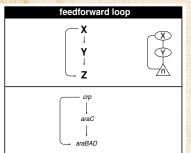
Random friends are

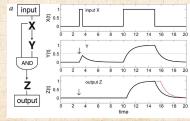












Z only turns on in response to sustained activity X.

る Turning off X rapidly turns 奇f 之.

Analogy to elevator doors.

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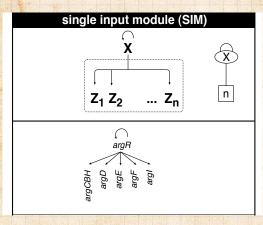
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Master switch.

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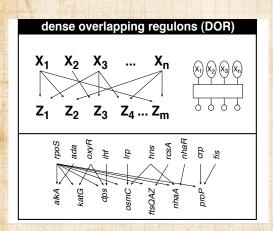
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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

A For more, see work carried out by Wiggins et al. at Columbia.

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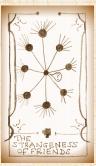












The degree distribution P_k is fundamental for our description of many complex networks

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The degree distribution P_k is fundamental for our description of many complex networks

& Again: P_k is the degree of randomly chosen node.

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- The degree distribution P_k is fundamental for our description of many complex networks
- \mathbb{R} Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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- Again: P_{l} is the degree of randomly chosen node.
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- \mathbb{A} Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.

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- Now choosing nodes based on their degree (i.e., size):



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$$Q_k \propto k P_k$$

Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k' P_{k'}}$$

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$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k' P_{k'}} = \frac{kP_k}{\langle k \rangle}.$$

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Big deal: Rich-get-richer mechanism is built into this selection process.

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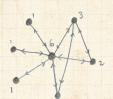
Random friends are Largest component







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Probability of randomly selecting a node of degree k by choosing from nodes:

$$P_1 = 3/7$$
, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.

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Probability of randomly selecting a node of degree k by choosing from nodes:

$$P_1 = 3/7, P_2 = 2/7, P_3 = 1/7, P_6 = 1/7.$$



Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:

$$Q_1 = 3/16$$
, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$.

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Probability of randomly selecting a node of degree k by choosing from nodes:

$$P_1 = 3/7, P_2 = 2/7, P_3 = 1/7, P_6 = 1/7.$$



Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:

$$Q_1 = 3/16, Q_2 = 4/16,$$

 $Q_3 = 3/16, Q_6 = 6/16.$



Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$R_0 = 3/16 \; R_1 = 4/16,$$

 $R_2 = 3/16, \; R_5 = 6/16.$



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 \mathbb{R} For random networks, Q_k is also the probability that a friend (neighbor) of a random node has kfriends.

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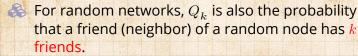
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 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends.

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 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends.



$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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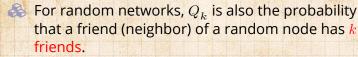
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 R_k = probability that a friend of a random node has k other friends.



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3

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

 \clubsuit Equivalent to friend having degree k+1.

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 \mathbb{R} For random networks, Q_{k} is also the probability that a friend (neighbor) of a random node has kfriends.

 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends.

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- \clubsuit Equivalent to friend having degree k+1.
- Natural question: what's the expected number of other friends that one friend has?

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 \mathbb{R}_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}$$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$, is true for all random networks, independent of degree distribution.

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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Again, neatness of results is a special property of the Poisson distribution.

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- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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 \mathbb{A} In fact, R_k is rather special for pure random networks ...

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Substituting

$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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#samesies.

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Reason #1:

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_E$$

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Reason #1:



Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle^{\frac{1}{16}} (\langle k^2 \rangle)^{\frac{1}{16}} \langle k^2 \rangle^{\frac{1}{16}} \langle$$

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Reason #1:



Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R = \langle k \rangle \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$$

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Key: Average depends on the 1st and 2nd moments of P_k and not just the 1st moment.

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- Three peculiarities:
 - 1. We might guess $\langle k_2 \rangle = \langle k \rangle (\langle k \rangle 1)$ but it's actually $\langle k(k-1) \rangle$.
 - 2. If P_k has a large second moment then $\langle k_2 \rangle$ will be big.
 - 3. Your friends really are different from you...
 - 4. See also: class size paradoxes (nod to: Gelman)

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More on peculiarity #3:



 \triangle A node's average # of friends: $\langle k \rangle$

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = 7k$$

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$$\frac{\langle k^2 \rangle}{\langle k \rangle} = 76$$

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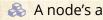
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Comparison:

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2}$$

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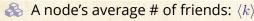
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$$\frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle \frac{\langle k^2 \rangle}{\langle k \rangle^2} = \langle k \rangle \frac{\sigma^2 + \langle k \rangle^2}{\langle k \rangle^2} = \langle k \rangle \left(1 + \frac{\sigma^2}{\langle k \rangle^2} \right)$$

- So only if everyone has the same degree (variance= $\sigma^2 = 0$) can a node be the same as its friends
- Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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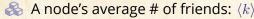
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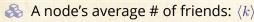
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Deferences









"Generalized friendship paradox in complex networks: The case of scientific collaboration"
Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014. [3]

Your friends really are monsters #winners:1

Go on Mixt me: Friends have more coauthors, citations, and publications.

Other hornfic studies: your connections on Twitter have more followers than you, are happy than you, , more sexual partners than you, ...

The hope: Maybe they have more enemies and diseases too

Research possibility: The Frenemy Paradox.

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¹Some press here [MIT Tech Review].



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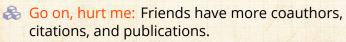
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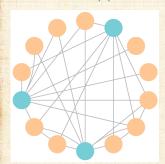




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Related disappointment:





Nodes see their friends' color choices.

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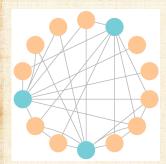






¹https://www.washingtonpost.com/graphics/business/ wonkblog/majority-illusion/

Related disappointment:



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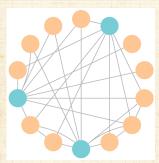




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Related disappointment:



- Nodes see their friends' color choices.
- Which color is more popular?1
- Again: thinking in edge space changes everything.

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(Big) Reason #2:



 $\langle k \rangle_{R}$ is key to understanding how well random networks are connected together.

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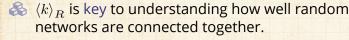
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(Big) Reason #2:



🙈 e.g., we'd like to know what's the size of the largest component within a network.

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Note: Component = Cluster

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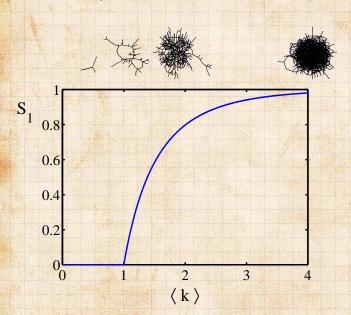
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Giant component



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Giant component:



A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} >$$

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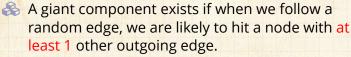
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Giant component:



Equivalently, expect exponential growth in node number as we move out from a random node.

All of this is the same as requiring $\langle k \rangle_R > 1$ Giant-component condition (or percolation condition):

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} >$$

Again, see that the second moment is an essentia part of the story.

Equivalent statement: $\langle k^2 \rangle > 2\langle k \rangle$

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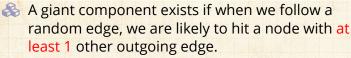
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For random networks, we know local structure is pure branching.

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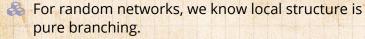
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Successful spreading is a contingent on single edges infecting nodes.

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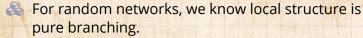
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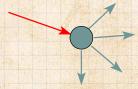


Successful spreading is : contingent on single edges infecting nodes.

Success



Failure:



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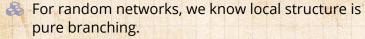
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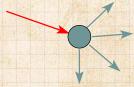


Successful spreading is a contingent on single edges infecting nodes.

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Focus on binary case with edges and nodes either infected or not.

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- For random networks, we know local structure is pure branching.
- Successful spreading is a contingent on single edges infecting nodes.

Success







- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?

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We need to find: [2]

R = the average # of infected edges that one random infected edge brings about.

& Call **R** the gain ratio.

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 prob. of connecting to a degree k node

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(k-1)# outgoing infected edges

 $\underbrace{B_{k1}}_{\text{Prob. of infection}}$

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 $+\sum_{k=0}^{\infty}\frac{kP_k}{\langle k\rangle}$

$$B_{k1}$$
Prob. of infection

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$$\underbrace{(k-1)}_{\text{\# outgoing infected edges}}$$

$$+\sum_{k=0}^{\infty} \frac{\widehat{kP_k}}{\langle k \rangle} \bullet \underbrace{0}_{\begin{subarray}{c} \# \text{ outgoing infected} \\ \text{ edges} \end{subarray}} \bullet \underbrace{(1-B_{k1})}_{\begin{subarray}{c} \text{Prob. of} \\ \text{no infection} \end{subarray}}$$

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Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

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Case 1-Rampant spreading:

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 \clubsuit Case 1-Rampant spreading: If $B_{k_1} = 1$

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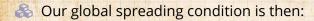
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$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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Good: This is just our giant component condition again.

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 \triangle Case 2—Simple disease-like: If $B_{k,1} = \beta < 1$

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Solution Case 2—Simple disease-like: If $B_{k_1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

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3 Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

 $A fraction (1-\beta) of edges do not transmit infection.$

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8 Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

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- Analogous phase transition to giant component case but critical value of $\langle k \rangle$ is increased.

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Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

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- $A fraction (1-\beta) of edges do not transmit infection.$
- Analogous phase transition to giant component case but critical value of $\langle k \rangle$ is increased.
- Aka bond percolation .

Resulting degree Astronton P.

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3 Case 2—Simple disease-like: If $B_{k1} = \beta < 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k-1) \bullet \beta > 1.$$

- $A fraction (1-\beta) of edges do not transmit infection.$
- Analogous phase transition to giant component case but critical value of $\langle k \rangle$ is increased.
- Aka bond percolation .
- $\red {\Bbb R}$ Resulting degree distribution $\tilde P_k$:

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} {i \choose k} (1-\beta)^{i-k} P_i.$$

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 \Longrightarrow Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

$$\langle k \rangle_{R}^{\mathbf{c}} = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

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Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \langle k \rangle$$

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 \Leftrightarrow Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.

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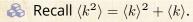
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- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- \Leftrightarrow When $\langle k \rangle < 1$, all components are finite.

Fine example of a continuous phase $\langle k \rangle = 1$ marks the critical point of the system.

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 \Leftrightarrow e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

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So giant component always exists for these kinds of networks.

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- $\mbox{\&}$ Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.

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And how big is the largest component?



\mathbb{A} Define S_1 as the size of the largest component.

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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And how big is the largest component?



Define S_1 as the size of the largest component.



Consider an infinite ER random network with average degree $\langle k \rangle$.

Let's find S_1 with a back-of-the-envelope argument.

Define has the probability that a randomly chosen node does not belong to the largest component.

Simple connection: $\delta = 1 + S_1$

Dirty trick: If a randomly chosen node is not part of the largest component, then none of its neighbors are.

So

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

Substitute in Poisson distribution...

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Substitute in Poisson distribution...

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Substitute in Poisson distribution...

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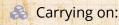
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$$\frac{\delta}{\delta} = \sum_{k=0}^{\infty} P_k \delta^k$$

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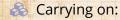
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$$\delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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Carrying on:

$$\begin{split} \delta &= \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ &= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \end{split}$$

Now subdirecte to 1 = 1 - 7 and rearrings to

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Carrying on:

$$\begin{split} \delta &= \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ &= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \\ &= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1-\delta)}. \end{split}$$

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and rearrange to

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Carrying on:

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Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$

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We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}.$

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- We can figure out some limits and details for $S_1 = 1 e^{-\langle k \rangle S_1}$.
- \Leftrightarrow First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}.$$

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$$\langle k \rangle = \frac{1}{S_1} {\rm ln} \frac{1}{1 - S_1}. \label{eq:self-local}$$

 \clubsuit As $\langle k \rangle \to 0$, $S_1 \to 0$.

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- $As \langle k \rangle \to \infty$, $S_1 \to 1$.

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- \Leftrightarrow As $\langle k \rangle \to \infty$, $S_1 \to 1$.
- \Leftrightarrow Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

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- $\red {\$}$ Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.
- $\red {\$}$ Only solvable for $S_1>0$ when $\langle k\rangle>1$.

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- Really a transcritical bifurcation. [9]

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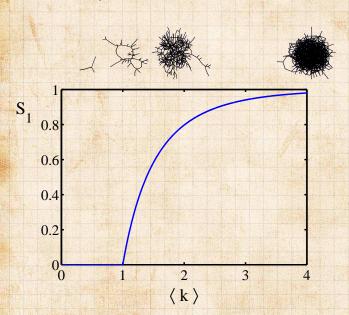
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Our dirty trick only works for ER random networks.

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The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.

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But we know our friends are different from us...

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But we know our friends are different from us...



Works for ER random networks because $\langle k \rangle = \langle k \rangle_B$.

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- But we know our friends are different from us...
- Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability δ' for the chance that an edge leads to the giant (infinite) component.
 - We can sort many things out with sensible probabilistic arguments...
 - More detailed investigations will profit from a spot of Generating function ology.
 - CocoNuTs. We figure out the final size and complete dynamics.

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