Mechanisms for Generating Power-Law Size Distributions, Part 2

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2017

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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What's the Story?

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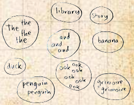

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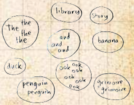
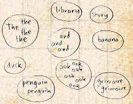


Outline



library
story
banana
duck
penguin penguin
grimoire grimoire

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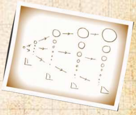
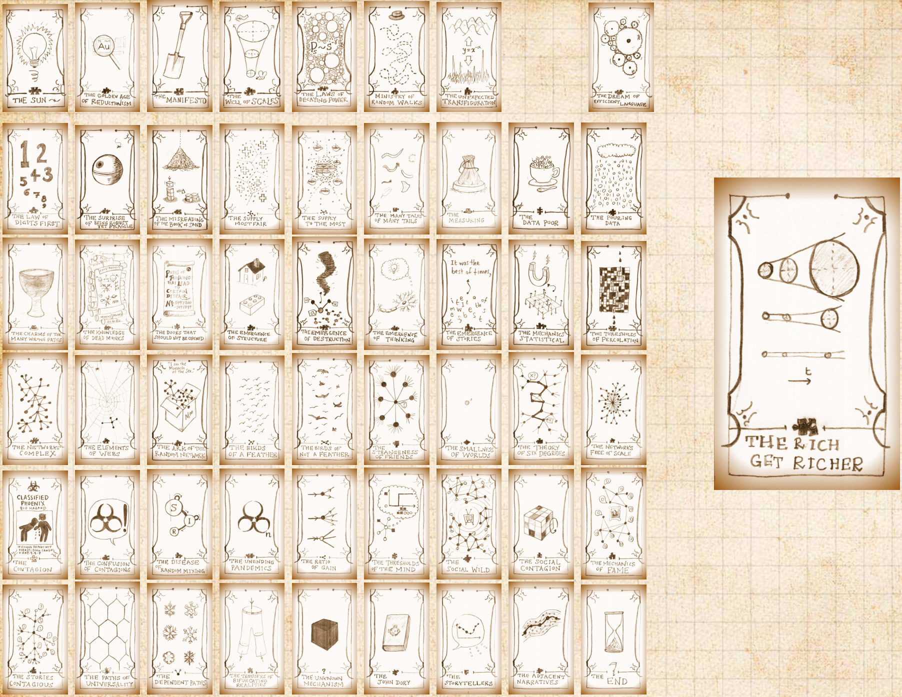
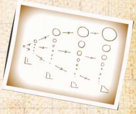



THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
MINISTRY OF RANDOM WALKS
THE UNEXPECTED TRANSFIGURATION
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE KNOWLEDGE OF DEAD MONKS
THE DOORS THAT SHOULD NOT BE OPENED
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE ARK OF THE RANDOM NETWORK
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE RATIO OF GAIN
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE MECHANICS OF FAME
THE STORIES CONTAGIOUS
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE JOHN DORY
THE STORYTELLERS
THE ADJACENT NARRATIVES
THE END
THE RICH GET RICHER

Aggregation:

- Random walks represent additive aggregation
- Mechanism: Random addition and subtraction
- Compare across realizations, no competition.
- Next: Random Additive/Copying Processes involving Competition.
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
- Competing mechanisms (trickiness)



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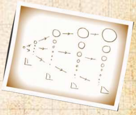

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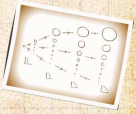

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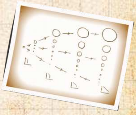

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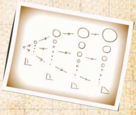

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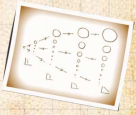
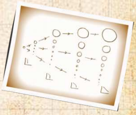


Pre-Zipf's law observations of Zipf's law

- 1910s: Word frequency examined re Stenography (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup [12].
- 1910s: Felix Auerbach pointed out the Zipfitude of city sizes in "Das Gesetz der Bevölkerungskonzentration" ("The Law of Population Concentration") [1].
- 1924: G. Udny Yule [31]:
  # Species per Genus (offers first theoretical mechanism)
- 1926: Lotka [17]:
  # Scientific papers per author (Lotka's law)



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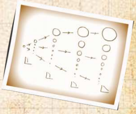

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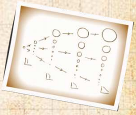

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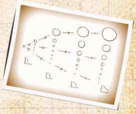

Theoretical Work of Yore:

- 1949: Zipf's "Human Behaviour and the Principle of Least-Effort" is published.[33]
- 1953: Mandelbrot[19]:
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- 1955: Herbert Simon[27, 33]:
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- 1999: Barabasi and Albert[4]:
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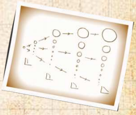

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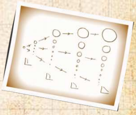

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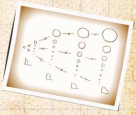

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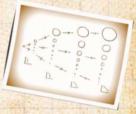

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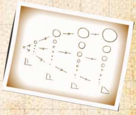
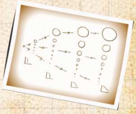


Herbert Simon (1916–2001):

- Political scientist (and much more)
- Involved in Cognitive Psychology, Computer Science, Public Administration, Economics, Management, Sociology
- Coined 'bounded rationality' and 'satisficing'
- Nearly 1000 publications (see Google Scholar)
- An early leader in Artificial Intelligence, Information Processing, Decision-Making, Problem-Solving, Attention Economics, Organization Theory, Complex Systems, And Computer Simulation Of Scientific Discovery.
- 1978 Nobel Laureate in Economics (his Nobel bio is here).



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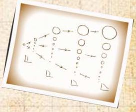

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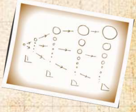

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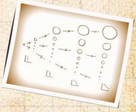

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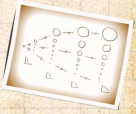

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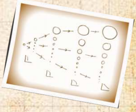
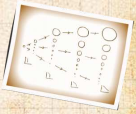


Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$
2. At time $t = 2, 3, 4, \ldots$, add a new elephant in one of two ways:
   - With probability ρ, create a new elephant with a new flavor
   - With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy
   - Elephants of the same flavor form a group



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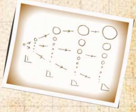

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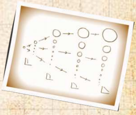

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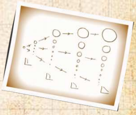

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     = Mutation/Innovation
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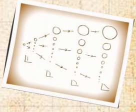

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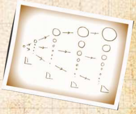
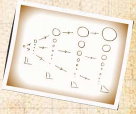


Random Competitive Replication:

Example: Words appearing in a language

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
- With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word

Note: This is a terrible way to write a novel.



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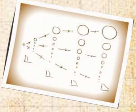

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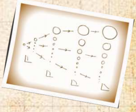

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Example: Words appearing in a language

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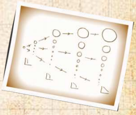

Random Competitive Replication:

Example: Words appearing in a language

- Consider words as they appear sequentially.
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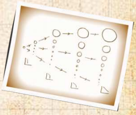

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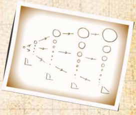

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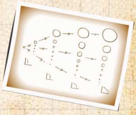

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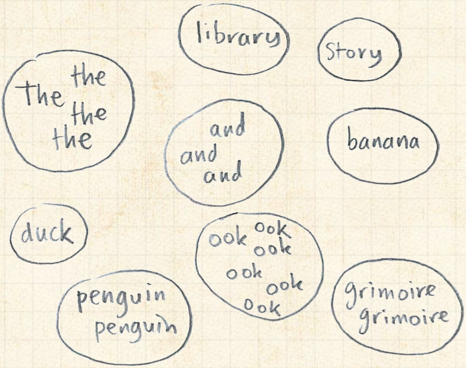
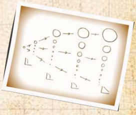


- 21 words used
- next word is new with prob ρ
- next word is a copy with prob $1-\rho$

prob:	next word:
6/21	ook
4/21	the
3/21	and
2/21	penguin
⋮	
1/21	library

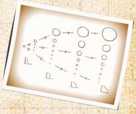


Some observations:

- Fundamental Rich-get-Richer story;
- Competition for replication between individual elephants is random;
- Competition for growth between groups of matching elephants is not random;
- Selection on groups is biased by size;
- Random selection sounds easy;
- Possible that no great knowledge of system needed (but more later ...).

Your free set of tofu knives:

- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.



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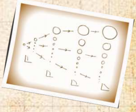

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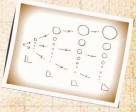

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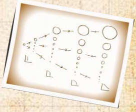

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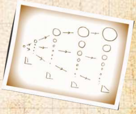

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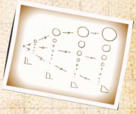

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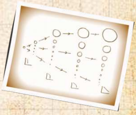

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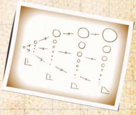

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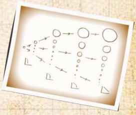

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- Steady growth of system: +1 elephant per unit time.
- Steady growth of distinct flavors at rate ρ
- We can incorporate
  1. Elephant elimination
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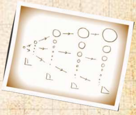

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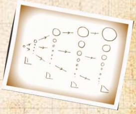

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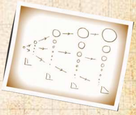

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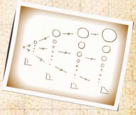

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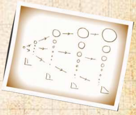

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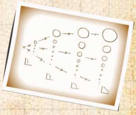

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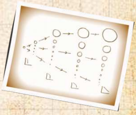

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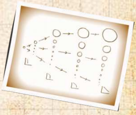

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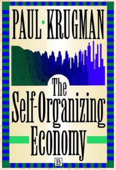
     (But mechanism for selection is not as simple...)



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Ch. 3: An Urban Mystery, p. 46

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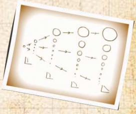

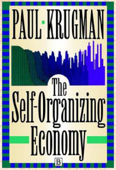

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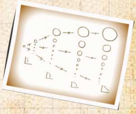

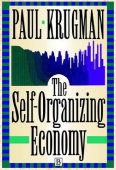

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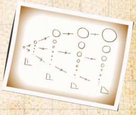

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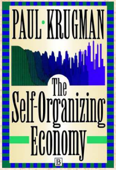

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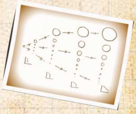

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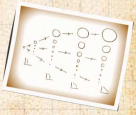
Rich-Get-Richer Mechanism
Simon's Model
Analysis
Words
Catchphrases
First Mover Advantage
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
References



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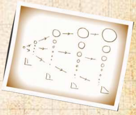
Definitions:

- k_i = size of a group i
- $N_{k,t}$ = # groups containing k elephants at time t.



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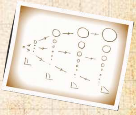

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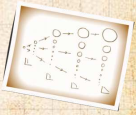
Basic question: How does $N_{k,t}$ evolve with time?



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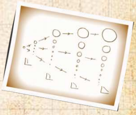

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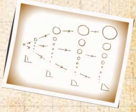
First: $\sum_k k N_{k,t} = t =$ number of elephants at time t

Random Competitive Replication:

$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

- $N_{k,t}$ size k groups
- $\Rightarrow kN_{k,t}$ elephants in size k groups
- t elephants overall



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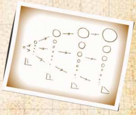

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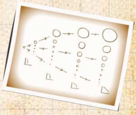

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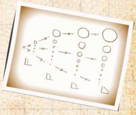

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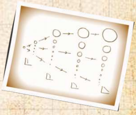
$$P_k(t) = \frac{kN_{k,t}}{t}.$$



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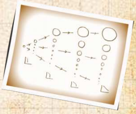
$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:

2. An elephant belonging to a group with $k-1$ elephants is replicated:



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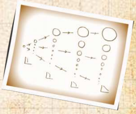

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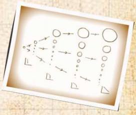

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$N_{k,t}$, the number of groups with k elephants, changes at time t if

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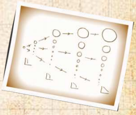

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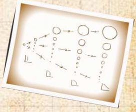

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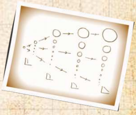

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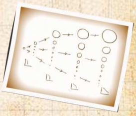

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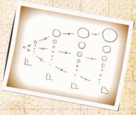

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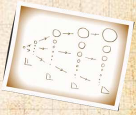

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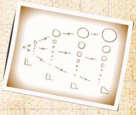

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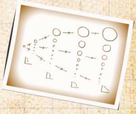

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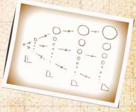

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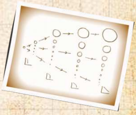

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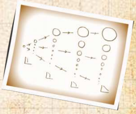
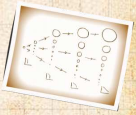


Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t}\right)$$



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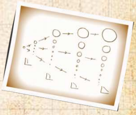

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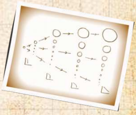
Random Competitive Replication:

Assume distribution stabilizes: $N_{k,t} = n_k t$
(Reasonable for t large)

- Drop expectations
- Numbers of elephants now fractional
- Okay over large time scales

- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k t}{\rho t} = \frac{n_k}{\rho}.$$



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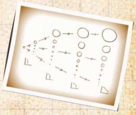

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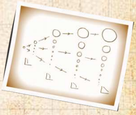

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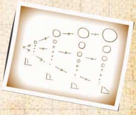

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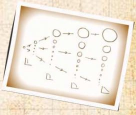

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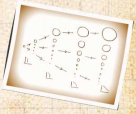
Stochastic difference equation:

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t}\right)$$

becomes

$$n_k(t+1) - n_k t = (1-\rho)\left((k-1)\frac{n_{k-1} t}{t} - k\frac{n_k t}{t}\right)$$



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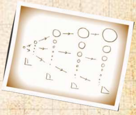

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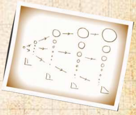

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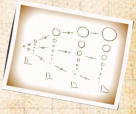
$$\Rightarrow n_k\left(1 + (1-\rho)k\right) = (1-\rho)(k-1)n_{k-1}$$



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We have a simple recursion:

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  Insert question from assignment 4
- For just the tail: Expand as a series of powers of $1/k$
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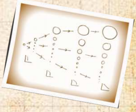

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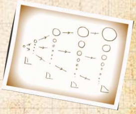

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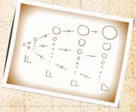

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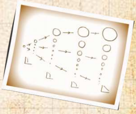

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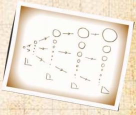

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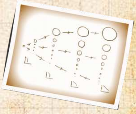

Micro-to-Macro story with ρ and γ measurable.

$$\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}$$

- Observe $2 < \gamma < \infty$ for $0 < \rho < 1$.
- For $\rho \simeq 0$ (low innovation rate):

$$\gamma \simeq 2$$

- 'Wild' power-law size distribution of group sizes, bordering on 'infinite' mean.
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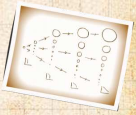
- All elephants have different flavors.
- Upshot: Tunable mechanism producing a family of universality classes.



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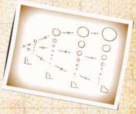

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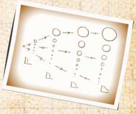

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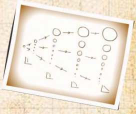

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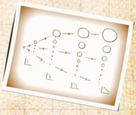

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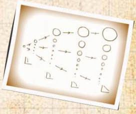

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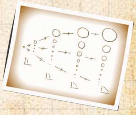

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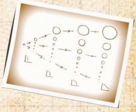

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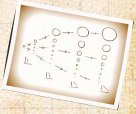

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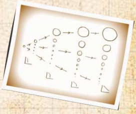

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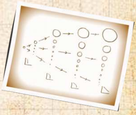

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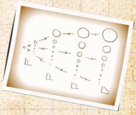

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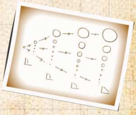

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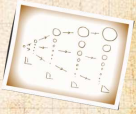

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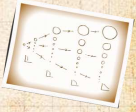

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We had one other equation:

$$\langle N_{1,t+1} - N_{1,t} \rangle = \rho - (1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

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- $$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$
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- Rearrange:
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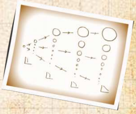

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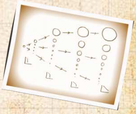

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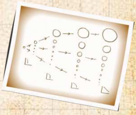

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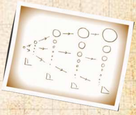

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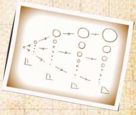

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(also = fraction of groups of size 1)

- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
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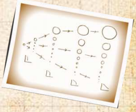
- Can show fraction of groups with two elephants $\sim 1/6$
- Model works well for large and small λ #awesome



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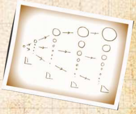

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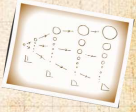

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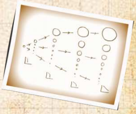

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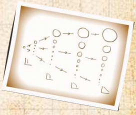

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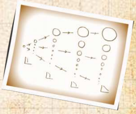

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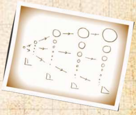

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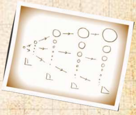

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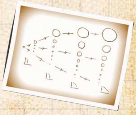
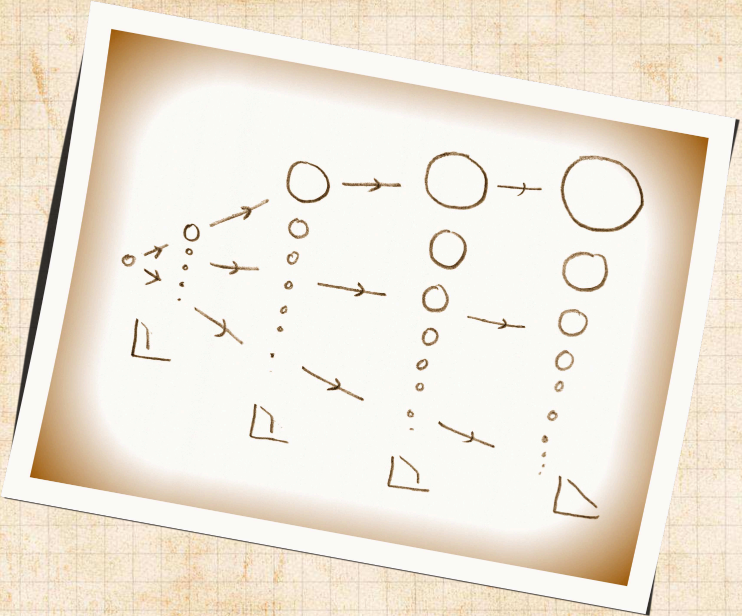
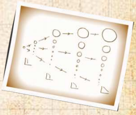




Outline



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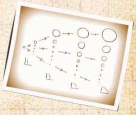

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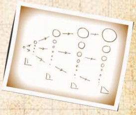

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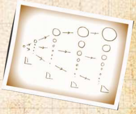
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870



Outline

Evolution of catch phrases:

- Yule's paper (1924) [31]:
  "A mathematical theory of evolution, based on the conclusions of Dr J. C. Willis, F.R.S."
- Simon's paper (1955) [27]:
  "On a class of skew distribution functions" (snore)

From Simon's introduction:

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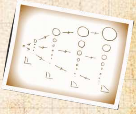
Its appearance is so frequent, and the phenomena so diverse, that one is led to conjecture that if these phenomena have any property in common it can only be a similarity in the structure of the underlying probability mechanisms.



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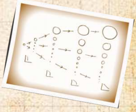

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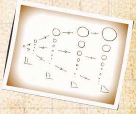

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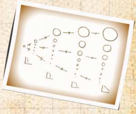

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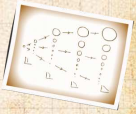

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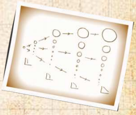

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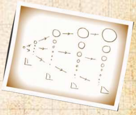

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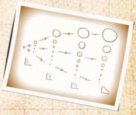

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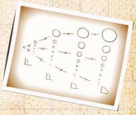

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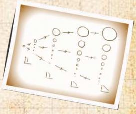

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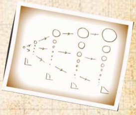

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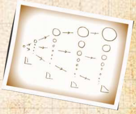

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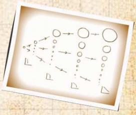

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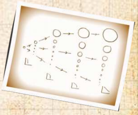

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- (Hath = suggested unit of purchasing power.)
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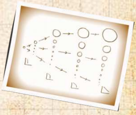

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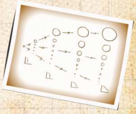

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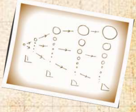

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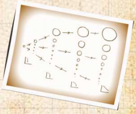

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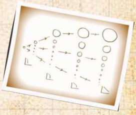

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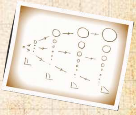

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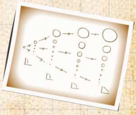

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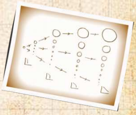

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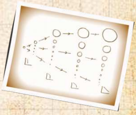

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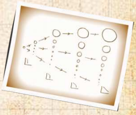

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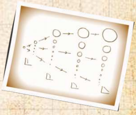

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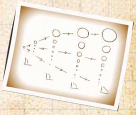

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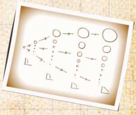

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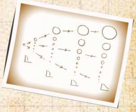

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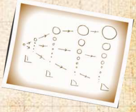

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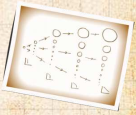

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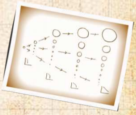

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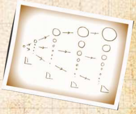

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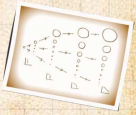

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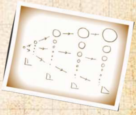

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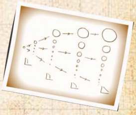

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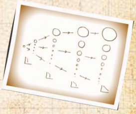
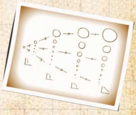


Another analytic approach: [10]

- Focus on how the nth arriving group typically grows.
- Analysis gives:

$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

- First mover is a factor $1/\rho$ greater than expected.
- Because ρ is usually close to 0, the first element is truly an elephant in the room.
- Appears that this has been missed for 60 years ...



Another analytic approach:[10]

- Focus on how the nth arriving group typically grows.
- Analysis gives:

$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)} \left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho} \left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

- First mover is a factor $1/\rho$ greater than expected.
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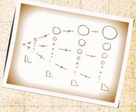

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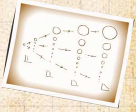
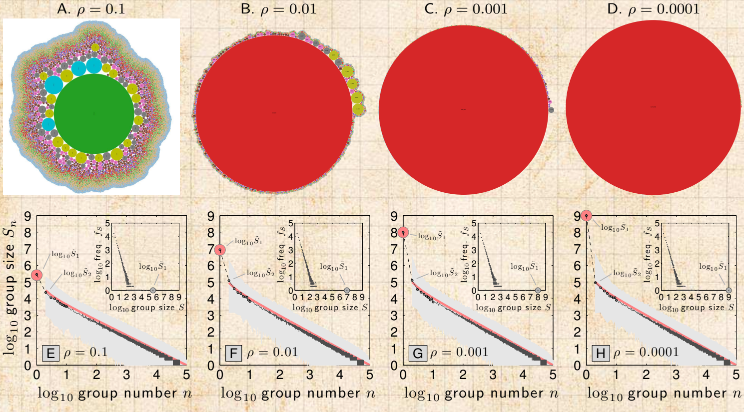
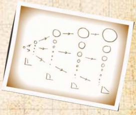


"Simon's fundamental rich-gets-richer model entails a dominant first-mover advantage"
Dodds et al.,
Available online at
http://arxiv.org/abs/0909.1104, 2016. [10]





See visualization at paper's online app-endices

Alternate analysis:

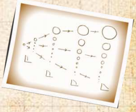
- Focus on the nth arriving group:

$$\left\langle S_{n,t+1} - S_{n,t} \right\rangle = (1-\rho_t) \cdot \frac{S_{n,t}}{t} \cdot (+1).$$

- For $t \geq t_n^{\text{init}}$, fix $\rho_t = \rho$ and shift t to $t-1$:

$$S_{n,t} = \left[1 + \frac{(1-\rho)}{t-1}\right] S_{n,t-1}$$

where $S_{n,t_n^{\text{init}}} = 1$.



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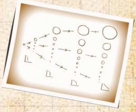
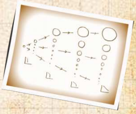




Betafication ensues:

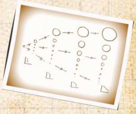
$$
\begin{aligned}
S_{n,t} &= \left[1 + \frac{(1-\rho)}{t-1}\right]\left[1 + \frac{(1-\rho)}{t-2}\right]\cdots\left[1 + \frac{(1-\rho)}{t_n^{\text{init}}}\right]\cdot 1 \\
&= \left[\frac{t+1-\rho}{t-1}\right]\left[\frac{t-\rho}{t-2}\right]\cdots\left[\frac{t_n^{\text{init}}+1-\rho}{t_n^{\text{init}}}\right] \\
&= \frac{\Gamma(t+1-\rho)\Gamma(t_n^{\text{init}})}{\Gamma(t_n^{\text{init}}+1-\rho)\Gamma(t)} \\
&= \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}.
\end{aligned}
$$

The first mover is really different:

- The issue is t_n^{init} in

$$S_{n,t} = \frac{\mathrm{B}(t_n^{\text{init}}, 1-\rho)}{\mathrm{B}(t, 1-\rho)}$$

- For $n \geq 2$ and $\rho \ll 1$, the nth group typically arrives at $t_n^{\text{init}} \simeq [\frac{n-1}{\rho}]$
- But $t_1^{\text{init}} = 1$ and the scaling is distinct in form.
- Simon missed the first mover by working on the size distribution.
- Contribution to $P_{k,t}$ of the first element vanishes as $t \to \infty$.
- Note: Does not apply to Barabási-Albert model.



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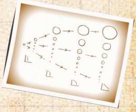

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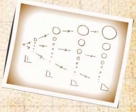

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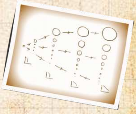

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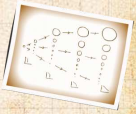


Variability:

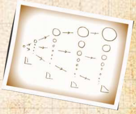
- The probability that the nth arriving group, if of size $S_{n,t} = k$ at time t, first replicates at time $t + \tau$:

$$\textbf{Pr}(S_{n,t+\tau} = k+1 \,|\, S_{n,t+i} = k) \;\; \text{for } i = 0, \ldots, \tau - 1$$

$$= \prod_{i=0}^{\tau-1} \left[1 - (1-\rho)\frac{k}{t+i} \right] \cdot (1-\rho)\frac{k}{t+\tau}$$

$$= k \frac{B(\tau, t)}{B(\tau, t-(1-\rho))} \frac{1-\rho}{t+\tau} \propto \frac{\tau^{-(1-\rho)k}}{t+\tau}.$$

- Upshot: nth arriving group starting at size 1 will on average wait for an infinite time to replicate.



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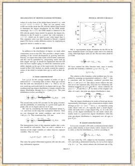
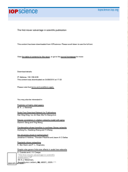
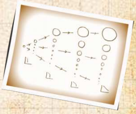




"Organization of Growing Random Networks"

Krapivsky and Redner,
Phys. Rev. E, **63**, 066123, 2001. [15]



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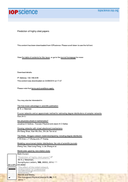

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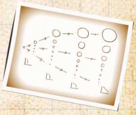

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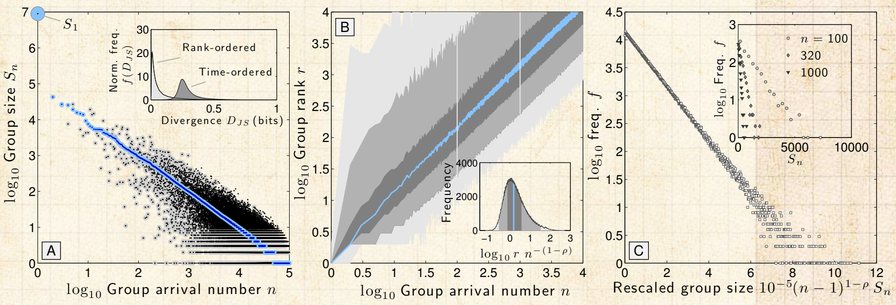

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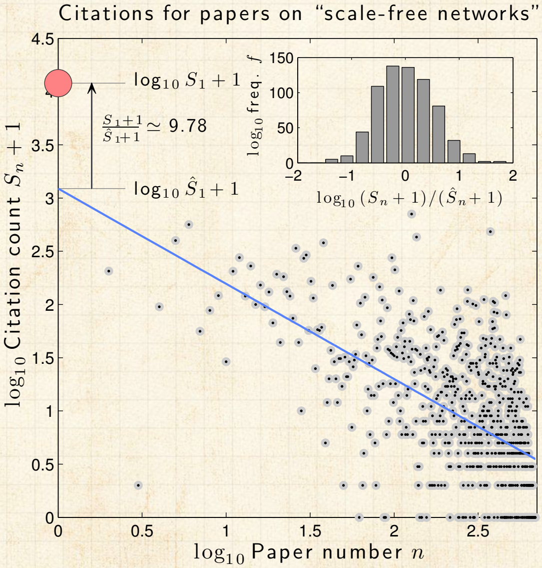
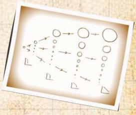
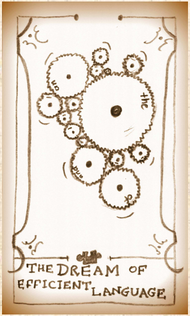
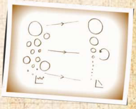




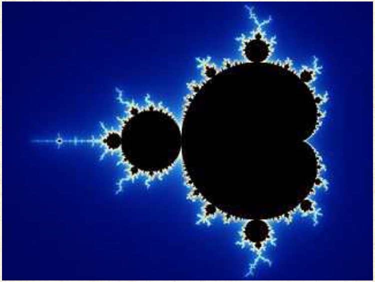

THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
MINISTRY OF RANDOM WALKS
THE UNEXPECTED TRANSFIGURATION
THE RICH GET RICHER
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MISREADING OF THE BOOK OF SAND
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE MANY TALES OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE KNOWLEDGE OF DEAD MONKS
THE DOORS THAT SHOULD NOT BE OPENED
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
It was the best of times,
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE ARK OF THE RANDOM NETWORK
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
CLASSIFIED
PHOENIX
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE RATIO OF GAIN
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE MECHANICS OF FAME
THE STORIES CONTAGIOUS
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE JOHN DORY
THE STORYTELLERS
THE ADJACENT NARRATIVES
THE END


THE DREAM OF EFFICIENT LANGUAGE

Outline



Benoît Mandelbrot

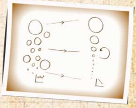


- Mandelbrot = father of fractals
- Mandelbrot = almond bread
- Bonus Mandelbrot set action: here.

Another approach:

Benoît Mandelbrot

- Derived Zipf's law through optimization [19]
- Idea: Language is efficient
- Communicate as much information as possible for as little cost
- Need measures of information (H) and average cost (C)...
- Language evolves to maximize H/C, the amount of information per average cost.
- Equivalently: minimize C/H.
- Recurring theme: what role does optimization play in complex systems?



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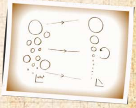

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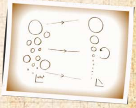

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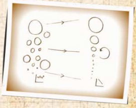

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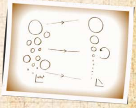

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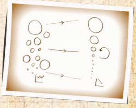

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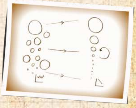

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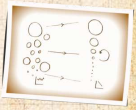

The Quickening—Mandelbrot v. Simon:

There Can Be Only One:

- Things there should be only one of: Theory, Highlander Films.
- Feel free to play Queen's It's a Kind of Magic in your head (funding remains tight).

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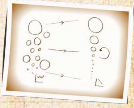

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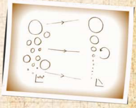
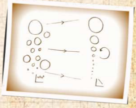


We were born to be Princes of the Universe

vs.

Mandelbrot vs. Simon:

- Mandelbrot (1953): "An Informational Theory of the Statistical Structure of Languages"[19]
- Simon (1955): "On a class of skew distribution functions"[27]
- Mandelbrot (1959): "A note on a class of skew distribution functions: analysis and critique of a paper by H.A. Simon"[20]
- Simon (1960): "Some further notes on a class of skew distribution functions"[28]



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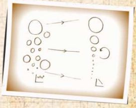

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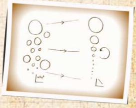

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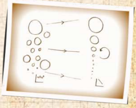

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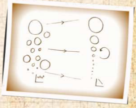

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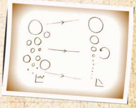

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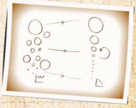

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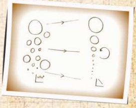

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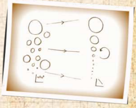

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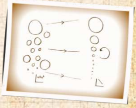

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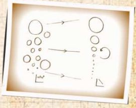

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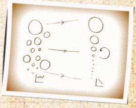

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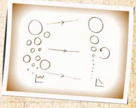

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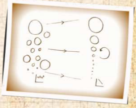

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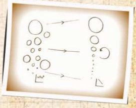
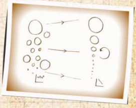
"You can't do this to me, I WENT TO COLLEGE!" "You weak minded fool!" "You just lost your brain privileges," etc.



Two theories enter, one theory leaves



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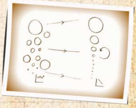
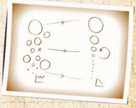


Zipfarama via Optimization:

Mandelbrot's Assumptions:

- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
- Words appear randomly according to this distribution (obviously not true...)
- Words = composition of letters is important
- Alphabet contains m letters
- Words are ordered by length (shortest first)



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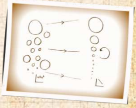

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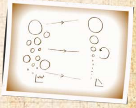

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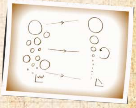

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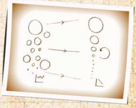

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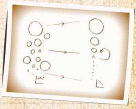


Zipfarama via Optimization:

Word Cost

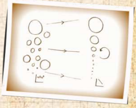
- Length of word (plus a space)
- Word length was irrelevant for Simon's method

Objection

- Real words don't use all letter sequences

Objections to Objection

- Maybe real words roughly follow this pattern (?)
- Words can be encoded this way
- Na na na-na naaaaa...



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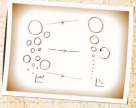

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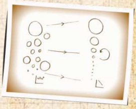

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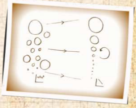

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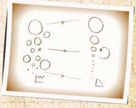

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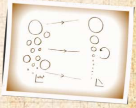

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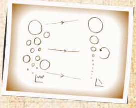

Zipfarama via Optimization:

Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \log_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

- Word length of 2^kth word: $= k + 1$
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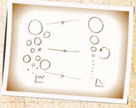

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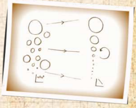

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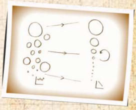

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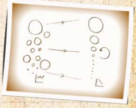

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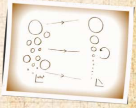

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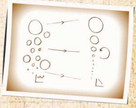


Zipfarama via Optimization:

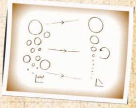
Total Cost C

- Cost of the ith word: $C_i \simeq 1 + \log_m i$
- Cost of the ith word plus space: $C_i \simeq 1 + \log_m (i+1)$
- Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i+1)$
- Simplify base of logarithm:

$$C'_i \simeq \log_m (i+1) = \frac{\log_e (i+1)}{\log_e m}$$

- Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C'_i \propto \sum_{i=1}^{n} p_i \log_e (i+1)$$



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- Cost of the ith word: $C_i \simeq 1 + \log_m i$
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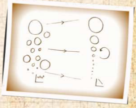

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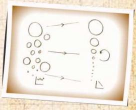

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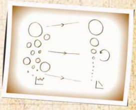

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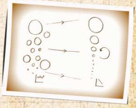

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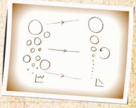

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Information Measure

- Use Shannon's Entropy (or Uncertainty):

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- (allegedly) von Neumann suggested 'entropy'...
- Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence
- $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others
- If $p_i = 1/2$, need only 1 bit ($\log_2 1/p_i = 1$)
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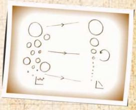

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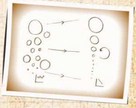

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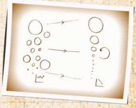

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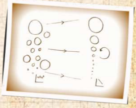

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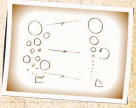

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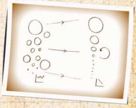

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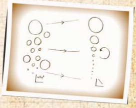

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- Use a slightly simpler form:

$$H = -\sum_{i=1}^{n} p_i \log_e p_i / \log_e 2 = -g \sum_{i=1}^{n} p_i \log_e p_i$$

where $g = 1/\log_e 2$

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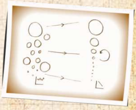


Zipfarama via Optimization:

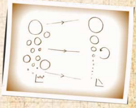
- Minimize

$$F(p_1, p_2, \ldots, p_n) = C/H$$

subject to constraint

$$\sum_{i=1}^{n} p_i = 1$$

- Tension:
(1) Shorter words are cheaper



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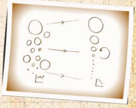

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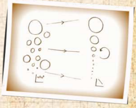
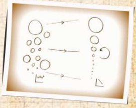
- Tension:
(1) Shorter words are cheaper
(2) Longer words are more informative (rarer)



Outline



Zipfarama via Optimization:

Time for Lagrange Multipliers:

- Minimize

$$\Psi(p_1, p_2, \ldots, p_n) =$$

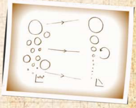
$$F(p_1, p_2, \ldots, p_n) + \lambda G(p_1, p_2, \ldots, p_n)$$

where

$$F(p_1, p_2, \ldots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^{n} p_i \log_e(i+1)}{-g \sum_{i=1}^{n} p_i \log_e p_i}$$

and the constraint function is

$$G(p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i - 1 (= 0)$$



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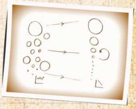

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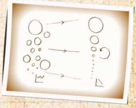
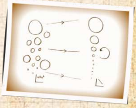
Insert question from assignment 5



Zipfarama via Optimization:

Some mild suffering leads to:

- $$p_j = e^{-1-\lambda H^2/gC}(j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$
- A power law appears [applause]: $\alpha = H/gC$
- Next: sneakily deduce λ in terms of g, C, and H.
- Find
$$p_j = (j+1)^{-H/gC}$$



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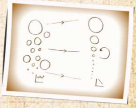

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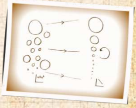

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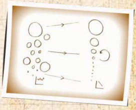

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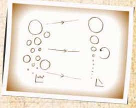

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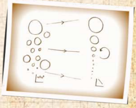
Finding the exponent

- Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

- As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function
- Gives $\alpha \simeq 1.73$ (> 1, too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)
- If cost function changes ($j + 1 \to j + a$) then exponent is tunable
- Increase a, decrease α



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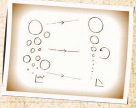

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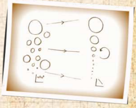

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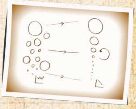

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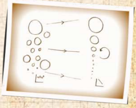

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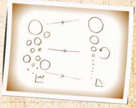

Zipfarama via Optimization:

Finding the exponent

- Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

- As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function
- Gives $\alpha \simeq 1.73$ (> 1, too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)
- If cost function changes ($j + 1 \to j + a$) then exponent is tunable
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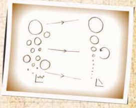
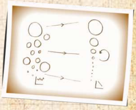


Zipfarama via Optimization:

All told:

- Reasonable approach: Optimization is at work in evolutionary processes
- But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness,...
- Mandelbrot's argument is not super convincing
- Exponent depends too much on a loose definition of cost



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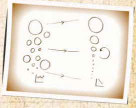

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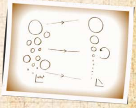

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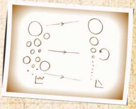
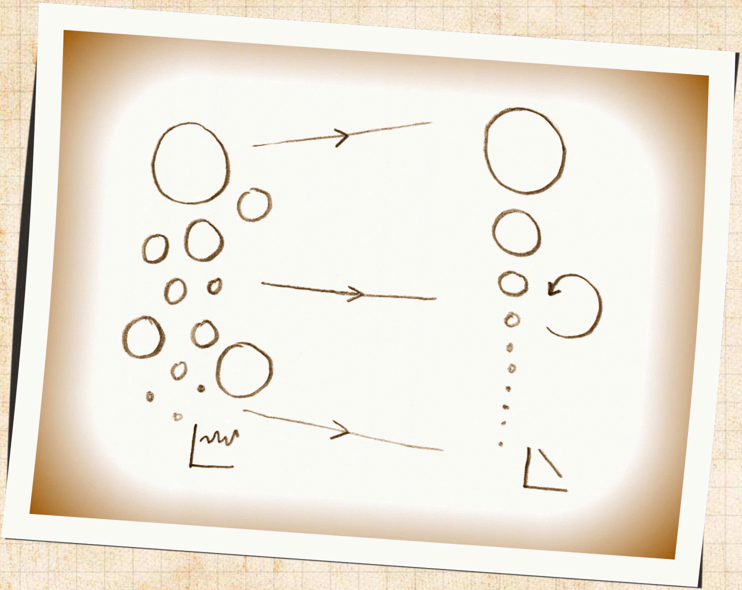
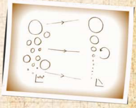




From the discussion at the end of Mandelbrot's paper:

- A. S. C. Ross: "M. Mandelbrot states that 'the actual direction of evolution (sc. of language) is, in fact, towards fuller and fuller utilization of places'. We are, in fact, completely without evidence as to the existence of any 'direction of evolution' in language, and it is axiomatic that we shall remain so. Many philologists would deny that a 'direction of evolution' could be theoretically possible; thus I myself take the view that a language develops in what is essentially a purely random manner."
- Mandelbrot: "As to the 'fundamental linguistic units being the least possible differences between pairs of utterances' this is a logical consequence of the fact that two is the least integer greater than one."



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Reconciling Mandelbrot and Simon

- Mixture of local optimization and randomness
- Numerous efforts...

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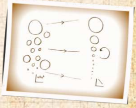

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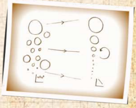

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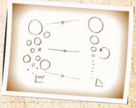

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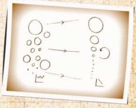

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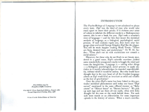
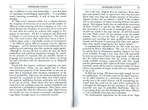
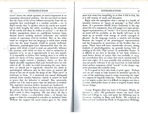
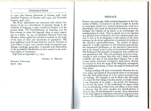

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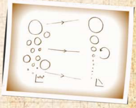

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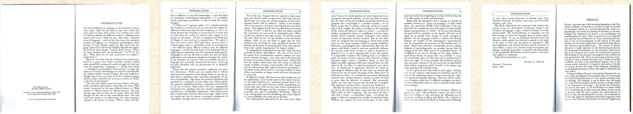

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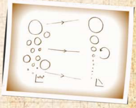

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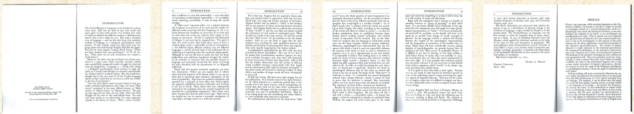

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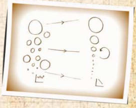

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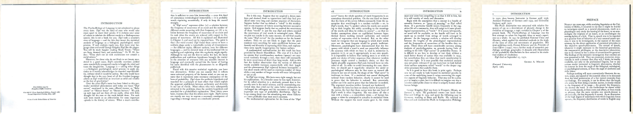

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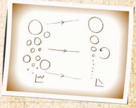

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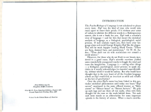
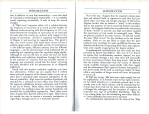
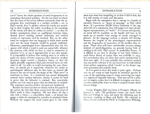
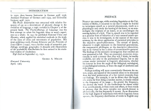

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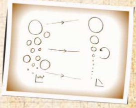
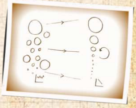


INTRODUCTION

The Psycho-Biology of Language is not calculated to please every taste. Zipf was the kind of man who would take roses apart to count their petals; if it violates your sense of values to tabulate the different words in a Shakespearean sonnet, this is not a book for you. Zipf took a scientist's view of language — and for him that meant the statistical analysis of language as a biological, psychological, social process. If such analysis repels you, then leave your language alone and avoid George Kingsley Zipf like the plague. You will be much happier reading Mark Twain: "There are liars, damned liars, and statisticians." Or W. H. Auden: "Thou shalt not sit with statisticians nor commit a social science."

However, for those who do not flinch to see beauty murdered in a good cause, Zipf's scientific exertions yielded some wonderfully unexpected results to boggle the mind and tease the imagination. Language *is* — among other things — a biological, psychological, social process; to apply statistics to it merely acknowledges its essential unpredictability, without which it would be useless. But who would have thought that in the very heart of all the freedom language allows us Zipf would find an invariant as solid and reliable as the law of gravitation?



Put it this way. Suppose that we acquired a dozen monkeys and chained them to typewriters until they had produced some very long and random sequence of characters. Suppose further that we defined a "word" in this monkey-text as any sequence of letters occurring between successive spaces. And suppose finally that we counted the occurrences of these "words" in just the way Zipf and others counted the occurrences of real words in meaningful texts. When we plot our results in the same manner, we will find exactly the same "Zipf curves" for the monkeys as for the human authors. Since we are not likely to argue that the poor monkeys were searching for some equilibrium between uniformity and diversity in expressing their ideas, such explanations seem equally inappropriate for human authors.

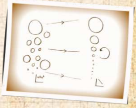
A mathematical rationalization for this result has been provided by Benoit Mandelbrot. The crux of it is that if we assume that word-boundary markers (spaces) are scattered randomly through a text, then there will necessarily be more occurrences of short than long words. Add to this fact the further observation that the variety of different words available increases exponentially with their length and the phenomenon Zipf reported becomes inescapable: a few short words will be used an enormous number of times while a vast number of longer words will occur infrequently or not at all.

So Zipf was wrong. His facts were right enough, but not his explanations. In a broader sense he was right, however, for he called attention to a stochastic process that is frequently seen in the social sciences, and by accumulating statistical data that cried out for some better explanation he challenged his colleagues and his successors to explore an important new type of probability distribution. Zipf belongs among those rare but stimulating men whose failures are more profitable than most men's successes.

So who's right?

Bornholdt and Ebel (PRE), 2001:
"World Wide Web scaling exponent from Simon's 1955 model" [5].

- Show Simon's model fares well.
- Recall ρ = probability new flavor appears.
- Alta Vista crawls in approximately 6 month period in 1999 give $\rho \simeq 0.10$
- Leads to $\gamma = 1 + \frac{1}{1-\rho} \simeq 2.1$ for in-link distribution.
- Cite direct measurement of γ at the time: 2.1 ± 0.1 and 2.09 in two studies.



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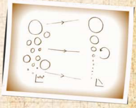

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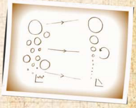

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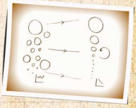

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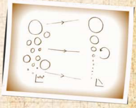

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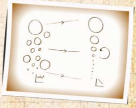
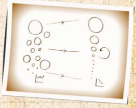


Outline



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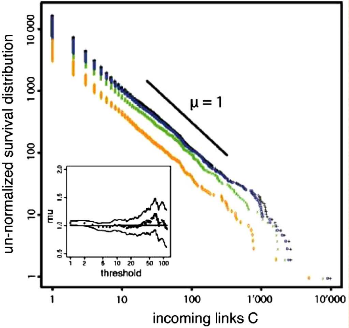
Recent evidence for Zipf's law...



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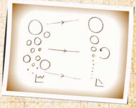

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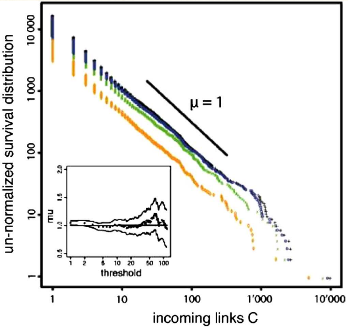

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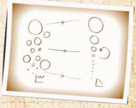

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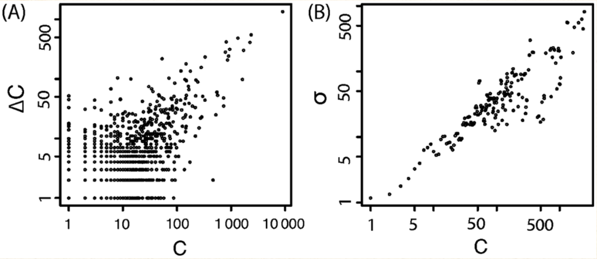
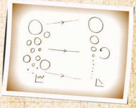


FIG. 2. Left panel: Plots of ΔC versus C from the Etch release (15.08.2007) to the latest Lenny version (05.05.2008) in double logarithmic scale. Only positive values are displayed. The linear regression $\Delta C = R \times C + C_0$ is significant at the 95% confidence level, with a small value $C_0 = 0.3$ at the origin and $R = 0.09$. Right panel: same as left panel for the standard deviation of ΔC.

- Rough, approximately linear relationship between C number of in-links and ΔC.



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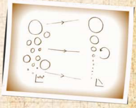

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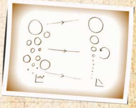

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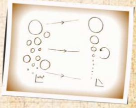

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Neural reboot (NR):

Walking with a baby robin:

https://www.youtube.com/v/CxiDTwvsLbA?rel=0

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Krugman and Simon

- "The Self-Organizing Economy" (Paul Krugman, 1996) [16]
- Krugman touts Zipf's law for cities, Simon's model
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Who needs a hug?

From Berry [3]

- Déjà vu, Mr. Krugman. Been there, done that. The Simon-Ijiri model was introduced to geographers in 1958 as an explanation of city size distributions, the first of many such contributions dealing with the steady states of random growth processes, ...
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