Properties of Complex Networks

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2017

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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Outline


THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
MINISTRY OF RANDOM WALKS
THE UNEXPECTED TRANSFIGURATION
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
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THE ADJACENT NARRATIVES
THE END


THE ELEMENTS OF WEBS

Outline

Properties of Complex Networks
- A problem
- Degree distributions
- Assortativity
- Clustering
- Motifs
- Concurrency
- Branching ratios
- Network distances
- Interconnectedness

Nutshell

References

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.
- And even when renderings somehow look good:
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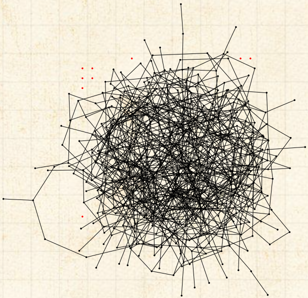

⇐ Typical hairball

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- number of edges m = 1000
- average degree $\langle k \rangle$ = 4

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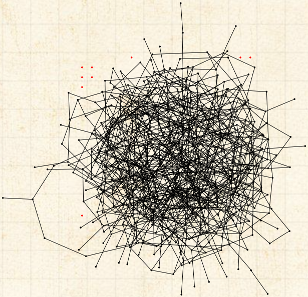

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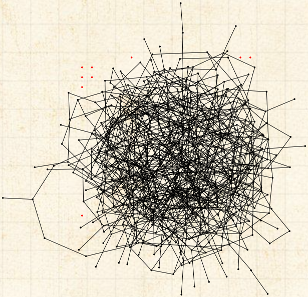

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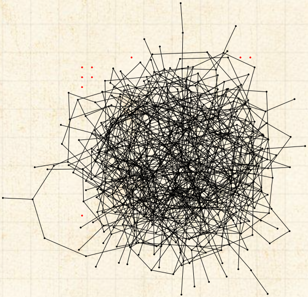

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Some key aspects of real complex networks:

- degree distribution*
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- interconnectedness
- robustness

- Plus coevolution of network structure and processes on networks.

∗ Degree distribution is the elephant in the room that we are now all very aware of...

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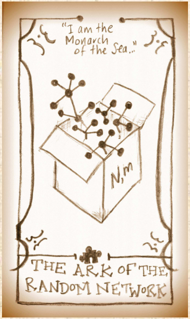

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N,m
THE ARK OF THE RANDOM NETWORK

Properties

1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k.
- k = node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:
  Insert question from assignment 7

$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'.
- link cost controls skew.
- hubs may facilitate or impede contagion.

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- Erdős-Rényi random networks are a *mathematical construct*.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.
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THE BIRDS OF NOT A FEATHER

Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network: [5] similar degree nodes connecting to each other.
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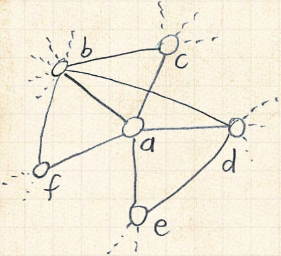
  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Outline

Local socialness:

4. Clustering:



- Your friends tend to know each other.
- Two measures (explained on following slides):

1. Watts & Strogatz [8]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

2. Newman [6]

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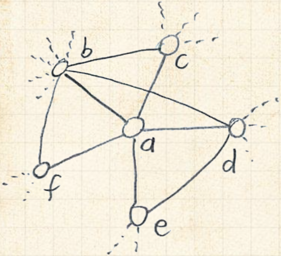

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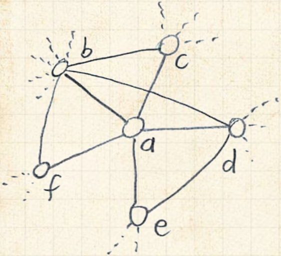

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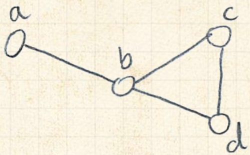

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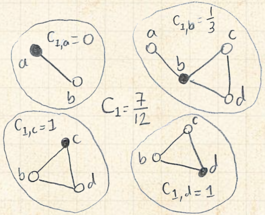

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- Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.

- Averaging over all nodes, we have:

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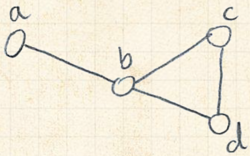

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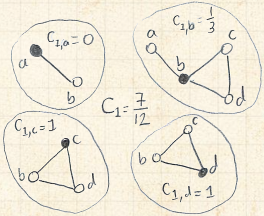

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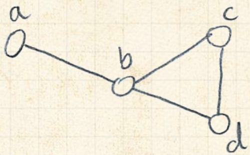

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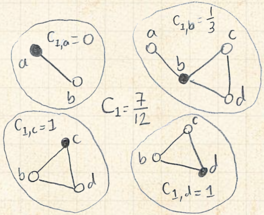

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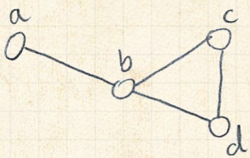

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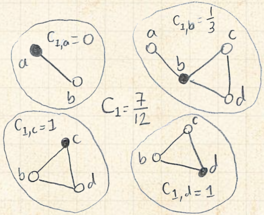

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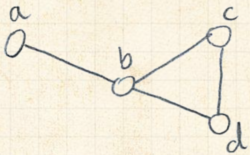

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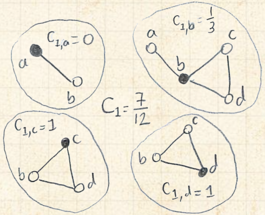

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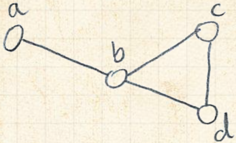
$$C_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i$$

Triples and triangles

Example network:



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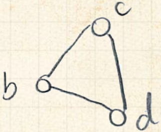

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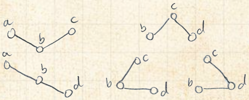

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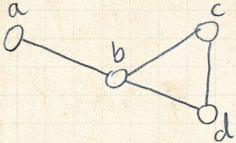

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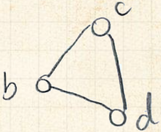

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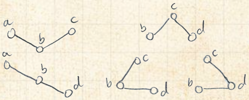

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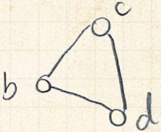

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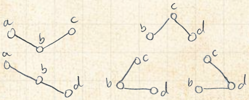

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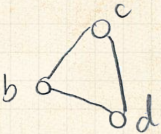

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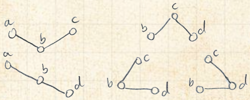

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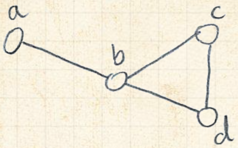

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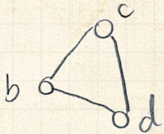

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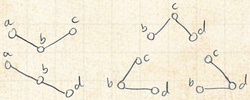

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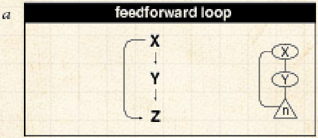

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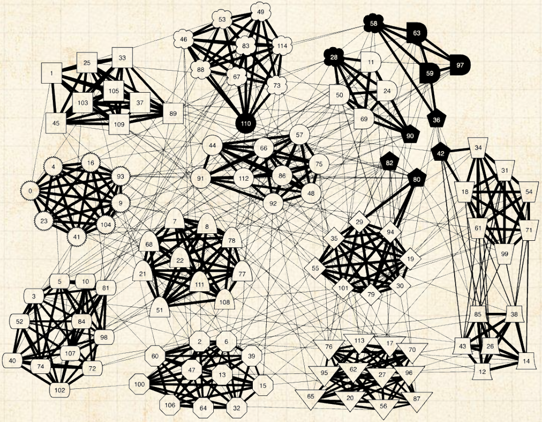
6. modularity and structure/community detection:



Clauset *et al.*, 2006 [2]: NCAA football

Outline

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7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [4]
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- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
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Properties

Interconnected networks and robustness (two for one deal):

"Catastrophic cascade of failures in interdependent networks"[1]. Buldyrev et al., Nature 2010.

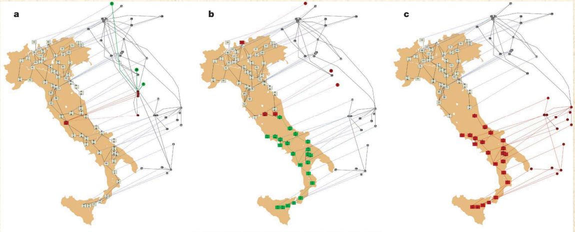

Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003[20]. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a,** One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network) at the next step are marked in green. **b,** Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c,** Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

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scale-free-networks,

Neural reboot (NR):

Mouse

https://www.youtube.com/v/GpYY9oz9qnI?rel=0

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