

Overview of Complex Networks

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2017

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



1 of 43

Outline

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

References

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



4 of 43

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



2 of 43

net•work |'nɛt,wɜrk|

noun

- an arrangement of intersecting horizontal and vertical lines.
 - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- a group or system of interconnected people or things : *a trade network.*
 - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
 - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
 - a system of connected electrical conductors.

verb [trans.]

- connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*
- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
 - [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



7 of 43

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



3 of 43

Thesaurus deliciousness:

network

noun

- a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- a network of lanes* MAZE, labyrinth, warren, tangle.
- a network of friends* SYSTEM, complex, nexus, web, webwork.

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



8 of 43

Ancestry:

From Keith Briggs's excellent [etymological investigation](#):

- Opus reticulatum:
- A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network/>]

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 Overview of Complex Networks

Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References



9 of 43

Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mech** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>

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 Overview of Complex Networks

Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References



12 of 43

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658-: reticulate structures in animals
- 1839-: rivers and canals
- 1869-: railways
- 1883-: distribution network of electrical cables
- 1914-: wireless broadcasting networks

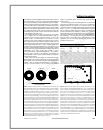
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 Overview of Complex Networks

Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References



10 of 43

Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"
 Watts and Strogatz, Nature, **393**, 440-442, 1998. [14]

Times cited: ~ 34,435 (as of October 24, 2017)



"Emergence of scaling in random networks"
 Barabási and Albert, Science, **286**, 509-511, 1999. [2]

Times cited: ~ 29,621 (as of October 24, 2017)

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Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References

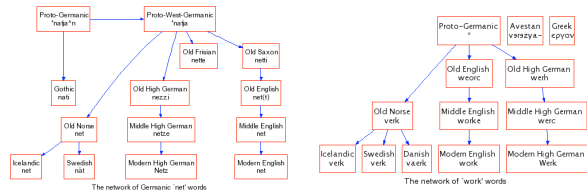


14 of 43

Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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 Overview of Complex Networks

Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References



11 of 43

Review articles:



"Complex Networks: Structure and Dynamics"
 Boccaletti et al., Physics Reports, **424**, 175-308, 2006. [3]

Times cited: ~ 7,689 (as of October 24, 2017)



"The structure and function of complex networks"
 M. E. J. Newman, SIAM Rev., **45**, 167-256, 2003. [10]

Times cited: ~ 16,436 (as of October 24, 2017)



"Statistical mechanics of complex networks"
 Albert and Barabási, Rev. Mod. Phys., **74**, 47-97, 2002. [1]

Times cited: ~ 19,104 (as of October 24, 2017)

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 Overview of Complex Networks

Complex Networks Basics
 Etymology
 Popularity
 Graph theory?
 Basic definitions
 Examples of Complex Networks
 Physical networks
 Interaction networks
 Relational networks
 References



15 of 43

Popularity according to textbooks:

Textbooks:

- 🔗 Mark Newman (Physics, Michigan)
"Networks: An Introduction" [↗](#)
- 🔗 David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
"Networks, Crowds, and Markets: Reasoning About a Highly Connected World" [↗](#)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks



🔗 16 of 43

Numerous others ...

- 🔗 Complex Social Networks—F. Vega-Redondo ^[12]
- 🔗 Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo ^[11]
- 🔗 Random Graph Dynamics—R. Durrett
- 🔗 Scale-Free Networks—Guido Caldarelli
- 🔗 Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- 🔗 Complex Graphs and Networks—Fan Chung
- 🔗 Social Network Analysis—Stanley Wasserman and Kathleen Faust
- 🔗 Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
- 🔗 Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes ^[6]

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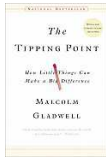
Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

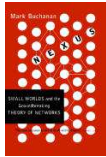


🔗 19 of 43

Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell ^[7]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks



🔗 17 of 43

More observations

- 🔗 But surely **networks aren't new** ...
 - 🔗 Graph theory is well established ...
 - 🔗 Study of social networks started in the 1930's ...
 - 🔗 So why all this 'new' research on networks?
 - 🔗 **Answer:** Oodles of Easily Accessible Data.
 - 🔗 We can now inform (alas) our theories with a much more measurable reality.*
 - 🔗 A worthy goal: establish **mechanistic explanations**.
- *If this is upsetting, maybe string theory is for you ...*

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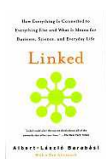
Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

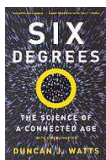


🔗 21 of 43

Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[13]

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks



🔗 18 of 43

More observations

- 🔗 Web-scale data sets can be overly **exciting**.

Witness:

- 🔗 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) [↗](#)
- 🔗 "The Unreasonable Effectiveness of Data," Halevy et al. ^[8]
- 🔗 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" ^[15]

But:

- 🔗 For scientists, description is only part of the battle.
- 🔗 We still need to understand.

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks



🔗 22 of 43

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

Links may be directed or undirected.
Links may be binary or weighted.

Other spiffing words: vertices and edges.

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



24 of 43

Examples

So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



27 of 43

Super Basic definitions

Node degree = Number of links per node

Notation: Node i 's degree = k_i .

$k_i = 0, 1, 2, \dots$

Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)

Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

Defn: \mathcal{N}_i = the set of i 's k_i neighbors

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

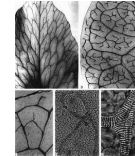


25 of 43

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids



Distribution (branching) versus **redistribution** (cyclical)

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



29 of 43

Super Basic definitions

Adjacency matrix:

We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(n.b., for numerical work, we always use sparse matrices.)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

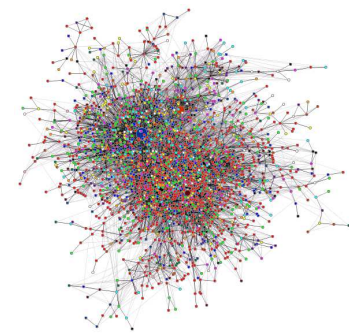


26 of 43

Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media



datamining.typepad.com

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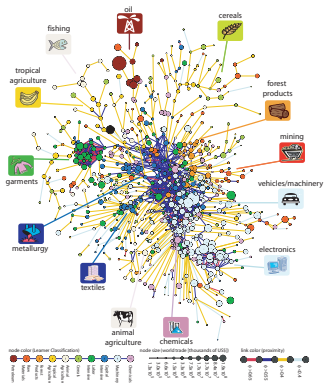
Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



31 of 43

topics:

- Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [9]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



32 of 43

Examples

- Relational networks
- Consumer purchases (Wal-Mart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: bit.ly flickr

common tags [cloud](#) | [list](#)

[community](#) [daily](#) [dictionary](#) [education](#) [encyclopedia](#) [english](#) [free](#) [imported](#) [info](#) [information](#) [internet](#) [knowledge](#) [learning](#) [news](#) [reference](#) [research](#) [resource](#) [resources](#) [search](#) [tools](#) [useful](#) [web](#) [web2.0](#) [wiki](#) [wikipedia](#)

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Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References

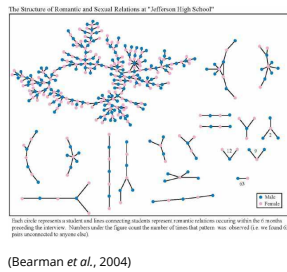


36 of 43

Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- [facebook](#) [twitter](#)



(Bearman et al., 2004)

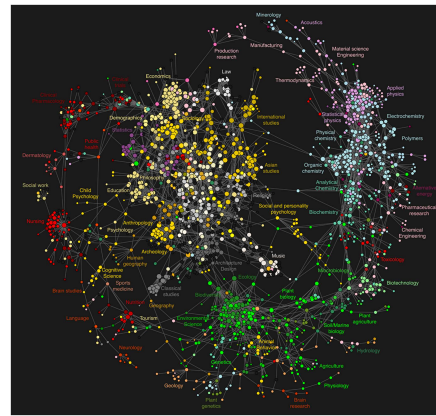
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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



33 of 43

Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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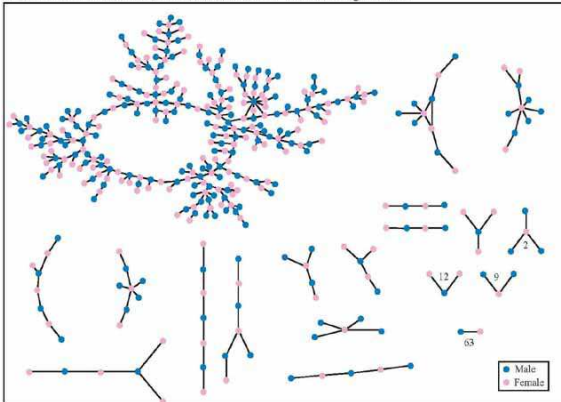
Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



37 of 43

Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



34 of 43

References

- [1] R. Albert and A.-L. Barabási. Statistical mechanics of complex networks. *Rev. Mod. Phys.*, 74:47–97, 2002. [pdf](#)
- [2] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–511, 1999. [pdf](#)
- [3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. *Physics Reports*, 424:175–308, 2006. [pdf](#)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions
Examples of Complex Networks
Physical networks
Interaction networks
Relational networks
References



39 of 43

References II

- [4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
Clickstream data yields high-resolution maps of science.
[PLoS ONE, 4:e4803, 2009. pdf](#)
- [5] S. Bornholdt and H. G. Schuster, editors.
Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.
- [6] S. N. Dorogovtsev and J. F. F. Mendes.
Evolution of Networks.
Oxford University Press, Oxford, UK, 2003.
- [7] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



40 of 43

References V

- [15] E. Wigner.
The unreasonable effectiveness of mathematics in the natural sciences.
[Communications on Pure and Applied Mathematics, 13:1-14, 1960. pdf](#)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



43 of 43

References III

- [8] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
[IEEE Intelligent Systems, 24:8-12, 2009. pdf](#)
- [9] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman.
The product space conditions the development of nations.
[Science, 317:482-487, 2007. pdf](#)
- [10] M. E. J. Newman.
The structure and function of complex networks.
[SIAM Rev., 45\(2\):167-256, 2003. pdf](#)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



41 of 43

References IV

- [11] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambridge, UK, 1997.
- [12] F. Vega-Redondo.
Complex Social Networks.
Cambridge University Press, 2007.
- [13] D. J. Watts.
Six Degrees.
Norton, New York, 2003.
- [14] D. J. Watts and S. J. Strogatz.
Collective dynamics of 'small-world' networks.
[Nature, 393:440-442, 1998. pdf](#)

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Overview of Complex Networks

Complex Networks Basics
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks

[References](#)



42 of 43