

# Overview of Complex Networks

## Principles of Complex Systems | @pocsvox

### CSYS/MATH 300, Fall, 2017

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Vermont Advanced Computing Core | University of Vermont



Complex  
Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

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- Physical networks
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References



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Overview of  
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Sealie & Lambie  
Productions



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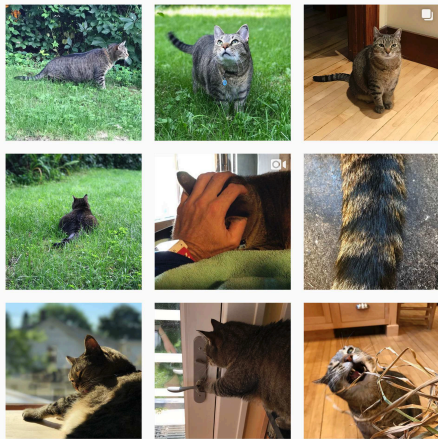
Physical networks  
Interaction networks  
Relational networks

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# These slides are also brought to you by:

## Special Guest Executive Producer: Pratchett



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 On Instagram at [pratchett\\_the\\_cat](https://www.instagram.com/pratchett_the_cat) 



# Outline

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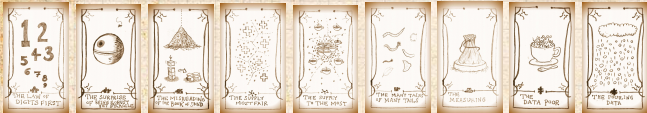
Interaction networks

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# net•work |'net,wɜrk|

noun

- 1 an arrangement of intersecting horizontal and vertical lines.
  - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- 2 a group or system of interconnected people or things : *a trade network.*
  - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
  - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj. ] *network television.*
  - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
  - a system of connected electrical conductors.

verb [ trans. ]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj. ] ( **networked** ) *networked workstations.*
- [ intrans. ] [often as n. ] ( **networking** ) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*



## Thesaurus deliciousness:

## network

noun

- a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- a network of lanes* MAZE, labyrinth, warren, tangle.
- a network of friends* SYSTEM, complex, nexus, web, webwork.

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From Keith Briggs's excellent etymological  
investigation: 



Opus  
reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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# Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

-  1658–: reticulate structures in animals
-  1839–: rivers and canals
-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

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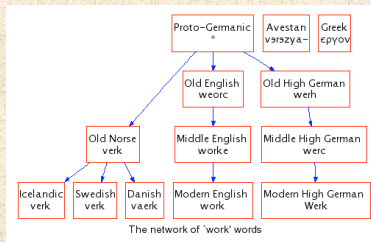
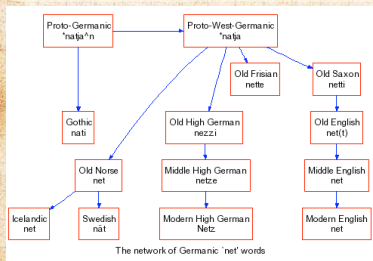



# Ancestry:

Net and Work are venerable old words:

 **'Net'** first used to mean spider web (King Ælfréd, 888).

 **'Work'** appear to have long meant purposeful action.



 **'Network'** = something built based on the idea of natural, flexible lattice or web.

 c.f., ironwork, stonework, fretwork.



## Key Observation:

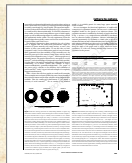
- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:




- Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>




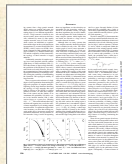
# Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks" 


Watts and Strogatz,  
Nature, **393**, 440–442, 1998. <sup>[14]</sup>

Times cited: ~ 34,435  (as of October 24, 2017)



"Emergence of scaling in random networks" 

Barabási and Albert,  
Science, **286**, 509–511, 1999. <sup>[2]</sup>

Times cited: ~ 29,621  (as of October 24, 2017)

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## Review articles:



“Complex Networks: Structure and Dynamics”

Boccaletti et al.,  
Physics Reports, **424**, 175–308, 2006. <sup>[3]</sup>

Times cited: ~ **7,689** (as of October 24, 2017)



“The structure and function of complex networks”

M. E. J. Newman,  
SIAM Rev., **45**, 167–256, 2003. <sup>[10]</sup>

Times cited: ~ **16,436** (as of October 24, 2017)



“Statistical mechanics of complex networks”

Albert and Barabási,  
Rev. Mod. Phys., **74**, 47–97, 2002. <sup>[1]</sup>

Times cited: ~ **19,104** (as of October 24, 2017)




# Popularity according to textbooks:

## Textbooks:




Mark Newman (Physics, Michigan)

"Networks: An Introduction" 



David Easley and Jon Kleinberg (Economics and  
Computer Science, Cornell)

"Networks, Crowds, and Markets: Reasoning About a  
Highly Connected World" 

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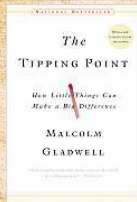
References



# Popularity according to books:

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Overview of  
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The Tipping Point: How Little Things can  
make a Big Difference—Malcolm  
Gladwell<sup>[7]</sup>

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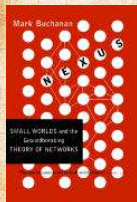
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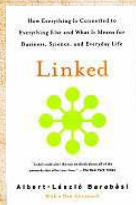
Nexus: Small Worlds and the  
Groundbreaking Science of  
Networks—Mark Buchanan



# Popularity according to books:

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Overview of  
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Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts<sup>[13]</sup>

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## Numerous others ...

-  **Complex Social Networks**—F. Vega-Redondo <sup>[12]</sup>
-  **Fractal River Basins: Chance and Self-Organization**—I. Rodríguez-Iturbe and A. Rinaldo <sup>[11]</sup>
-  **Random Graph Dynamics**—R. Durrette
-  **Scale-Free Networks**—Guido Caldarelli
-  **Evolution and Structure of the Internet: A Statistical Physics Approach**—Romu Pastor-Satorras and Alessandro Vespignani
-  **Complex Graphs and Networks**—Fan Chung
-  **Social Network Analysis**—Stanley Wasserman and Kathleen Faust
-  **Handbook of Graphs and Networks**—Eds: Stefan Bornholdt and H. G. Schuster <sup>[5]</sup>
-  **Evolution of Networks**—S. N. Dorogovtsev and J. F. F. Mendes <sup>[6]</sup>



# More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.\*
- A worthy goal: establish **mechanistic explanations**.

*\*If this is upsetting, maybe string theory is for you ...*

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# More observations

 Web-scale data sets can be overly **exciting**.


Witness:


 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 "The Unreasonable Effectiveness of Data,"  
Halevy et al. [8].

 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [15]

But:

 For scientists, description is only part of the battle.

 We still need to **understand**.

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# Super Basic definitions

**Nodes** = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

**Links** = Connections between nodes

 **Links** may be directed or undirected.

 **Links** may be binary or weighted.

Other spiffing words: vertices and edges.

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## Node degree = Number of links per node


 Notation: Node  $i$ 's degree =  $k_i$ .

  $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network =  $\langle k \rangle$   
(and sometimes  $z$ )

 Connection between number of edges  $m$  and  
average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

 Defn:  $\mathcal{N}_i$  = the set of  $i$ 's  $k_i$  neighbors

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## Adjacency matrix:

 We represent a directed network by a matrix  $A$  with link weight  $a_{ij}$  for nodes  $i$  and  $j$  in entry  $(i, j)$ .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)

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## So what passes for a complex network?

-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...

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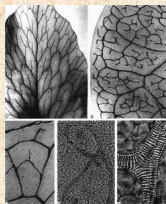



# Examples

## Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids



 **Distribution** (branching) versus **redistribution** (cyclical)

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# Examples


## Interaction networks


 The Blogosphere


 Biochemical networks


 Gene-protein networks

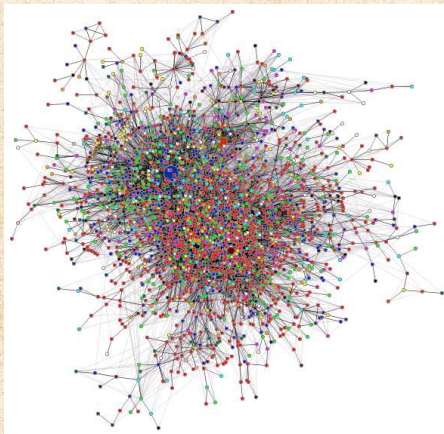
 Food webs: who eats whom


 The World Wide Web (?)

 Airline networks

 Call networks (AT&T)

 The Media



[datamining.typepad.com](http://datamining.typepad.com) 

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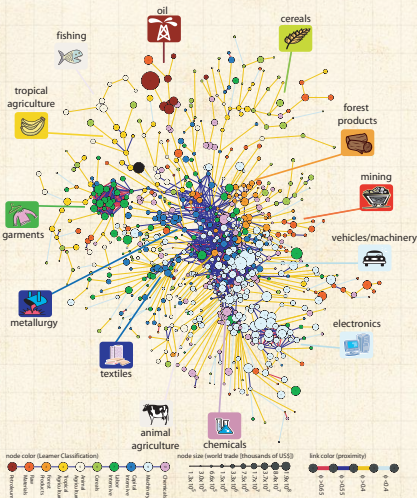
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# topics:

- 
 Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [9]
- 
 How do products depend on each other, and how does this network evolve?
- 
 How do countries depend on each other for water, energy, people (immigration), investments?



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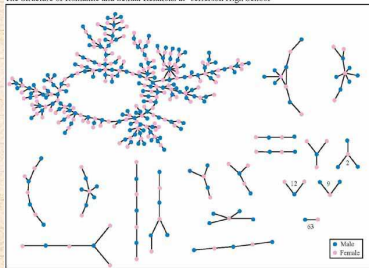


# Examples

## Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook ↗ twitter ↗,

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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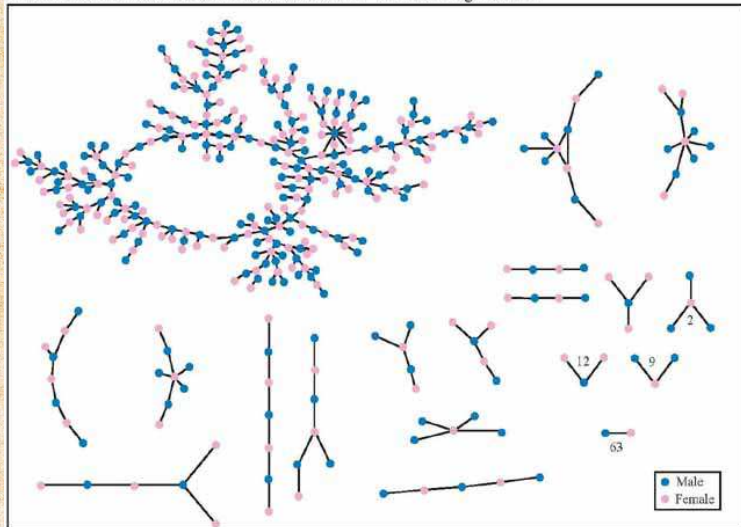
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Consumer purchases  
(Wal-Mart, Target, Amazon, ...)





Thesauri: Networks of words generated by  
meanings



Knowledge/Databases/Ideas



Metadata—Tagging: [bit.ly](http://bit.ly)  [flickr](http://flickr) 

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**common tags**    cloud | [list](#)

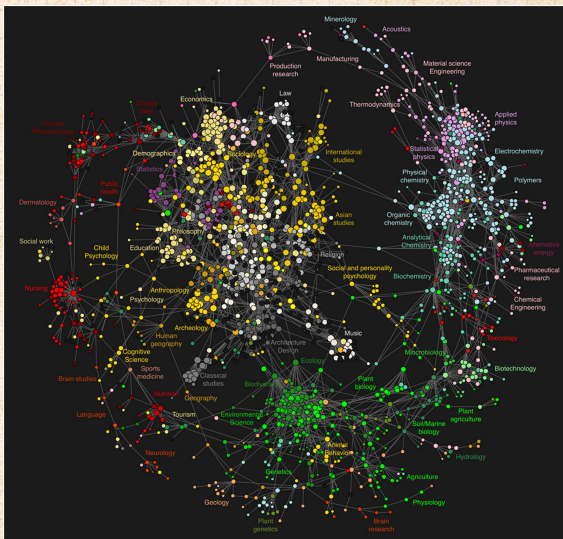
community    daily    dictionary    education    **encyclopedia**  
english    free    imported    info    information    internet    knowledge  
learning    news    **reference**    research    resource  
resources    search    tools    useful    web    web2.0    **wiki**  
**wikipedia**



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“Clickstream Data Yields High-Resolution Maps of Science”,  
Bollen et al. [4], 2009.



- [1] R. Albert and A.-L. Barabási.  
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[PLoS ONE, 4:e4803, 2009. pdf](#) 
- [5] S. Bornholdt and H. G. Schuster, editors.  
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- [6] S. N. Dorogovtsev and J. F. F. Mendes.  
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Oxford University Press, Oxford, UK, 2003.
- [7] M. Gladwell.  
The Tipping Point.  
Little, Brown and Company, New York, 2000.





- [8] A. Halevy, P. Norvig, and F. Pereira.  
The unreasonable effectiveness of data.  
[IEEE Intelligent Systems, 24:8–12, 2009. pdf](#)
- [9] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and  
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- [10] M. E. J. Newman.  
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[Communications on Pure and Applied  
Mathematics](#), 13:1–14, 1960. pdf 