Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2017
Assignment 9 • code name: No! God, No! No, God, please, no!

Dispersed: Saturday, October 21, 2017.
Due: 11:59 pm, Friday, November 17, 2017.
Some useful reminders:
Deliverator: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds+pocs@uvm.edu
Office hours: 1:15 pm to 2:30 pm on Tuesday, 1:15 pm to 4:45 pm Thursday
Course website: http://www.uvm.edu/pdodds/teaching/courses/2017-08UVM-300
Bonus course notes: http://www.uvm.edu/pdodds/teaching/courses/2017-08UVM-300/docs/dewhurst-pocs-notes.pdf

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All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Please obey the basic life rule: Never use Excel. Or any Microsoft product except maybe Xbox (which sadly will likely not help you here.)

Graduate students are requested to use LaTeX (or related TeX variant).

Email submission: PDF only! Please name your file as follows (where the number is to be padded by a 0 if less than 10 and names are all lowercase):
`CSYS300assignment%02d$firstname-$lastname.pdf` as in
`CSYS300assignment06michael-palin.pdf`

Please submit your project's current draft in pdf format via email. Please use this file name format (all lowercase after CSYS):
`CSYS300project-$firstname-$lastname-YYYY-MM-DD.pdf` as in
`CSYS300project-lisa-simpson-1989-12-17.pdf` where the date is the date of submission (and not, say, your birthdate).

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1. $(3 + 3 + 3 + 3 + 3)$

   We take a look at the 80/20 rule, 1 per centers, and similar concepts.

   Take x to be the wealth held by an individual in a population of n people, and the number of individuals with wealth between x and $x + \mathrm{d}x$ to be approximately $N(x)\mathrm{d}x$.

Given a power-law size frequency distribution $N(x) = cx^{-\gamma}$ where $x_{\min} \ll x \ll \infty$, determine the value of γ for which the so-called 80/20 rule holds.

In other words, find γ for which the bottom 4/5 of the population holds 1/5 of the overall wealth, and the top 1/5 holds the remaining 4/5.

Assume the mean is finite, i.e., $\gamma > 2$.

(a) Determine the total wealth W in the system given $\int_{x_{\min}}^{\infty} \mathrm{d}xN(x) = n$.

(b) Imagine that $100q$ percent of the population holds $100(1-r)$ percent of the wealth.

Show γ depends on q and r as

$$\gamma = 1 + \frac{\ln \frac{1}{(1-q)}}{\ln \frac{1}{(1-q)} - \ln \frac{1}{r}}.$$

(c) Given the above, is every pairing of q and r possible?

(d) Find γ for the 80/20 requirement ($q = r = 4/5$).

(e) For the "80/20" γ you find, determine how much wealth $100q$ percent of the population possesses as a function of q and plot the result.