PoCS What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2017
Assignment 6 • code name: Ghee Buttersnaps AKA The Heater

Dispersed: Friday, October 6, 2017.
Due: 11:59 pm, Friday, October 13, 2017.
Some useful reminders:
Deliverator: Peter Dodds
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Office hours: 1:15 pm to 2:30 pm on Tuesday, 1:15 pm to 4:45 pm Thursday
Course website: http://www.uvm.edu/pdodds/teaching/courses/2017-08UVM-300
Bonus course notes: http://www.uvm.edu/pdodds/teaching/courses/2017-08UVM-300/docs/dewhurst-pocs-notes.pdf

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All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Please obey the basic life rule: Never use Excel.

Graduate students are requested to use LaTeX (or related TeX variant).

Email submission: PDF only! Please name your file as follows (where the number is to be padded by a 0 if less than 10 and names are all lowercase):
`CSYS300assignment%02d$firstname-$lastname.pdf` as in
`CSYS300assignment06michael-palin.pdf`

Please submit your project's current draft in pdf format via email. Please use this file name format (all lowercase after CSYS):
`CSYS300project-$firstname-$lastname-YYYY-MM-DD.pdf` as in
`CSYS300project-lisa-simpson-1989-12-17.pdf` where the date is the date of submission (and not, say, your birthdate).

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1. The 1-d theoretical percolation problem:

   Consider an infinite 1-d lattice forest with a tree present at any site with probability p.

   (a) Find the distribution of forest sizes as a function of p. Do this by moving along the 1-d world and figuring out the probability that any forest you enter will extend for a total length ℓ.

(b) Find p_c, the critical probability for which a giant component exists.

Hint: One way to find critical points is to determine when certain average quantities explode. Compute $\langle l \rangle$ and find p such that this expression goes boom (if it does).

2. Show analytically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.

   Hint—Real-space renormalization gets it done.:
   `http://www.youtube.com/watch?v=J1kbU5U7QqU`

3. (3 + 3)

   Coding, it's what's for breakfast:

   (a) Percolation in two dimensions (2-d) on a simple square lattice provides a classic, nutritious example of a phase transition.

   Your mission, whether or not you choose to accept it, is to code up and analyse the L by L square lattice percolation model for varying L.

   Take L = 20, 50, 100, 200, 500, and 1000.

   (Go higher if you feel L = 1000 is for mere mortals.)

   (Go lower if your code explodes.)

   Let's continue with the tree obsession. A site has a tree with probability p, and a sheep grazing on what's left of a tree with probability $1 - p$.

   Forests are defined as any connected component of trees bordered by sheep, where connections are possible with a site's four nearest neighbors on a lattice.

   Each square lattice is to be considered as a landscape on which forests and sheep co-exist.

   Do not bagelize (or doughnutize) the landscape (no periodic boundary conditions—boundaries are boundaries).

   (Note: this set up is called site percolation. Bond percolation is the alternate case when all links between neighboring sites exist with probability p.)

   Steps:

   i. For each L, run N_{tests}=100 tests for occupation probability p moving from 0 to 1 in increments of 10^{-2}. (As for L, you may use a smaller or larger increment depending on how things go.)

   ii. Determine the fractional size of the largest connected forest for each of the N_{tests}, and find the average of these, S_{avg}.

   iii. On a single figure, for each L, plot the average S_{avg} as a function of p.

(b) Comment on how $S_{\rm avg}(p; N)$ changes as a function of L and estimate the critical probability p_c (the percolation threshold).

Helpful reuse of code (intended for black and white image analysis): You can use Matlab's bwconncomp to find the sizes of components. Very nice.

4. (3 + 3)

   (a) Using your model from the previous question and your estimate of p_c, plot the distribution of forest sizes (meaning cluster sizes) for $p \simeq p_c$ for the largest L your code and psychological makeup can withstand. (You can average the distribution over separate simulations.)

   Comment on what kind of distribution you find.

   (b) Repeat the above for $p = p_c/2$ and $p = p_c + (1 - p_c)/2$, i.e., well below and well above p_c.

   Produce plots for both cases, and again, comment on what you find.